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A SERIES OF TEXTBOOKS FOR PERSONS ENGAGED IN ENGINEERING PROFESSIONS, TRADES, AND VOCATIONAL OCCUPATIONS OR FOR THOSE WHO DESIRE INFORMATION CONCERNING THEM. FULLY ILLUSTRATED

GEOMETRY AND MENSURATION
ELECTRIC WIRING AND BELLWORK
DESIGN OF BEAMS
DESIGN OF COLUMNS
DESIGN OF ROOF TRUSSES
DESIGN OF PLATE GIRDERS

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1921

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

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In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed. At the end of the volume will be found a complete index, so that any subject treated can be quickly found.

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CONTENTS

	Section	Page
GEOMETRY AND MENSURATION		
Plane Surfaces and Areas	4	1
The Measurement of Arcs and Angles	4	4
Similar Polygons and Triangles	4	19
Geometrical Principles of the Circle	4	23
Inscribed and Circumscribed Polygons	4	28
Reading Drawings	4	31
Conversion Tables	4	33
Mensuration of Plane Surfaces	4	34
Sectors and Segments of Circles	4	47
The Ellipse	4	50
Area of Any Plane Figure	4	51
Areas of Irregular Figures	4	52
Mensuration of Solids	4	59
The Prismoidal Formula	4	68
Symmetrical and Similar Figures	4	73
ELECTRIC WIRING AND BELLWORK		
Elements of Electricity and Magnetism	5	1
Electrodynamics	5	6
Practical Application of Electricity and Magnetism	5	36
Storage Batteries	5	42
Electric-Light and Power Wiring	6	1
Materials for Light and Power Wiring	6	7
Systems of Distribution	6	19
Determining the Size of Wires	6	32
Installation of Wires	6	53
Wiring for Minor Electric Appliances	6	74
Electric Clocks	6	87
Electric Gas Lighting	6	88
Standard Symbols for Wiring Plans	6	92

	Section	Page
DESIGN OF BEAMS		
Elements of Mechanics	7	1
The Composition of Forces	7	4
The Resolution of Forces	7	11
Moments of Forces	7	15
The Lever	7	21
Center of Gravity	7	24
Loads Carried by Structures	7	25
Stresses and Strains	7	31
Strength of Building Materials	7	35
Beams	7	42
Stresses in Beams	7	47
Strength of Beams	7	61
Deflection of Beams	7	82
Flitch-Plate Girders	7	85
Trussed Beams	7	88
Reinforced Concrete	7	95
DESIGN OF COLUMNS		
Wooden Columns	8	1
Cast-Iron Columns	8	4
Steel Columns	8	9
Detailing of Structural Columns and Connections	8	35
Reinforced-Concrete Columns	8	46
Tables of Steel Shapes	8	49
DESIGN OF ROOF TRUSSES		
Loads on Roof Trusses	9	1
Graphic Statics	9	8
Determination of Stresses in the Fink Truss	9	39
Strength of Rivets and Pins	9	47
Truss Design	9	65
The Design of a Structural-Steel Roof Truss	9	77
DESIGN OF PLATE GIRDERS		
Construction and Design	10	1
Principles of Design	10	4
Practical Problems	10	37

GEOMETRY AND MENSURATION

(PART 1)

PLANE SURFACES AND AREAS

PRELIMINARY DEFINITIONS

1. A **point** indicates position only; it has neither length, breadth, nor thickness.

2. A **line** has only one dimension—length.

3. A **straight line,** Fig. 1, is one that does not change its direction throughout its whole length. A straight line is also frequently called a **right line.**

FIG. 1

4. A **curved line,** Fig. 2, changes its direction at every point.

FIG. 2

5. A **broken line,** Fig. 3, is one made up wholly of straight lines lying in different directions.

FIG. 3

Unless otherwise specified, the word *line* is understood to mean a *straight line.*

6. A **surface** has the shape of a body but is without thickness. A surface has but two dimensions—length and breadth.

7. A **flat surface, plane surface,** or simply a **plane,** is a surface such that a straight line between any two of its points lies wholly in the surface. If a straightedge is laid

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on a plane surface in any direction, every point of the straightedge will touch the surface.

8. A **figure** is any combination of points and lines. A figure that lies entirely in one plane is a **plane figure.** In referring to a figure, a point is designated by a letter placed conveniently near it; thus, in Fig. 1, the left end of the line is referred to as "the point A." The entire line is referred to as "the line AB," the letters A and B designating two points, usually the ends of the line. If a line is broken or curved, as many points are named as are considered necessary to designate the line.

9. **Geometry** is that branch of mathematics which treats of the properties of lines, angles, surfaces, and volumes.

10. **Mensuration** is that part of geometry which treats of the measurement of lines, surfaces, and solids.

ANGLES AND PERPENDICULARS

11. An **angle,** Fig. 4, is the opening between two straight lines that meet in a point. The two straight lines are the **sides,** and the point where the lines meet is the **vertex** of the angle. Thus, in Fig. 4, the straight lines OA and OB form an angle at the point O; the lines OA and OB are the sides of this angle, and the point O is its vertex.

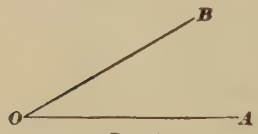

FIG. 4

12. An angle is usually referred to by naming a letter on each of its sides and a third letter at the vertex, the letter at the vertex being placed between the other two. Thus, the angle in Fig. 4 is called angle AOB or angle BOA.

An angle may also be designated by a letter placed between its sides near the vertex. Thus, the two angles XCY and YCZ, Fig. 5, may be referred to as the angles A and B, respectively.

An angle that stands alone, that is, an angle whose vertex is not the vertex of any other angle, may be designated by

naming the letter at its vertex. For example, the angle in Fig. 4 may be called the angle O.

13. Two angles, as A and B, Fig. 5, having the same vertex and a common side CY, are called **adjacent angles.**

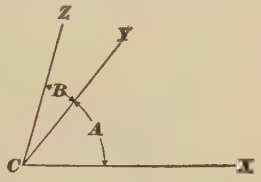

Fig. 5

14. Any angle may be thought of as being formed, or **generated,** by a line turning about the vertex as a pivot, from the position of one side to the position of the other. Thus, the angle AOB, Fig. 4, may be conceived as generated by a line turning about O from the position OA to the position OB. The size of the angle does not depend on the length of the sides, which are supposed to be of indefinite length, but on the opening between the sides; or, what is the same thing, on the amount of turning necessary to bring one side to the position of the other.

15. If a straight line, as AB, Fig. 6, meets another straight line, as CD, so as to make with it two equal adjacent angles, each of these angles is a **right angle,** and the first line is said to be **perpendicular** to the second. The point where the first line meets the second is called the **foot** of the perpendicular. It is evident that all right angles are equal.

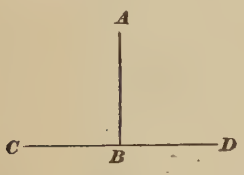

Fig. 6

The edges of the tongue of a carpenter's square are perpendicular to those of the body, or long blade, and the angles thus formed are right angles. Thus, in Fig. 7, AB is perpendicular to BC, and the angle at B is a right angle; DE is perpendicular to EF, and the angle at E is a right angle.

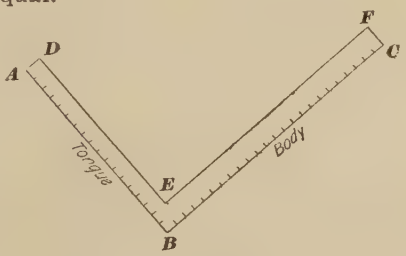

Fig. 7

THE MEASUREMENT OF ARCS AND ANGLES

FIG. 8

16. A **circle,** Fig. 8, is a plane figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

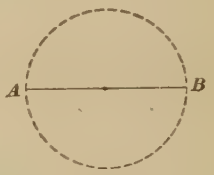

FIG. 9

17. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference, as AB, Fig. 9.

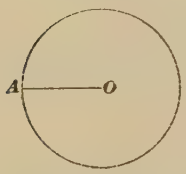

FIG. 10

18. The **radius** of a circle, OA, Fig. 10, is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of the word radius is **radii.** All radii of any circle are equal in length.

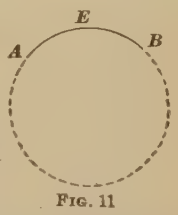

FIG. 11

19. An **arc** of a circle is any part of its circumference, as AEB, Fig. 11.

20. The whole circumference of a circle is divided into 360 equal parts, and each of these equal parts is called a **degree of arc;** a degree of arc is divided into 60 equal parts called **minutes of arc;** and a minute of arc is divided into 60 equal parts called **seconds of arc.** Degrees, minutes, and seconds of arc are used as units for measuring circular arcs.

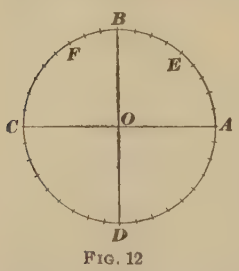

FIG. 12

21. In Fig. 12, the circumference is divided into 36 equal parts; hence, each length of arc between two consecutive division lines equals $360 \div 36 = 10$ degrees. Arc AEB is one-quarter of a circumference. A quarter of a circumference is called a **quadrant;** its length is $360 \div 4$

= 90 degrees. The arc EBF extends over seven divisions; hence, its length is $7 \times 10 = 70$ degrees.

22. Angles are measured by finding the number of degrees in the arc of the circle lying between the lines forming the sides of the angle, the center of the circle being at the vertex of the angle. For example, in Fig. 13, the sides of the angle CAB cut off an arc of 40 degrees from the circumference whose center is at A; hence, angle CAB is 40 degrees in magnitude.

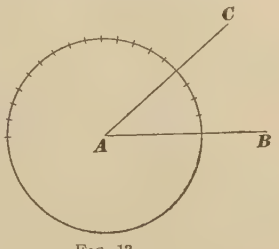

FIG. 13

Since the sides of a right angle whose vertex is at the center of a circle include between them an arc of 90 degrees, each right angle is equal to 90 degrees.

Degrees, minutes, and seconds, both of angle and of arc, are indicated by the marks (°), (′), (″); thus, 32° 15′ 10″ is read *thirty-two degrees fifteen minutes and ten seconds.*

In practice, angles are commonly laid off or measured by means of a *protractor* or a similar instrument. An explanation of the use of the protractor is given in works on drawing.

23. An **oblique angle** is any angle that is not a right angle. An **acute angle** is an oblique angle that is less than a right angle. An **obtuse angle** is an oblique angle that is greater than a right angle. In Fig. 14, BOC and AOC are oblique angles, BOC being an acute angle, and AOC an obtuse angle.

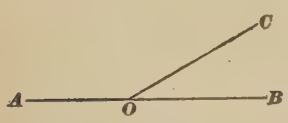

FIG. 14

In carpentry, an angle other than a right angle is known as a **bevel angle.**

24. Two angles are said to be **complementary** when their sum is equal to one right angle, or 90°. Each of two complementary angles is called the **complement** of the other. Thus, in Fig. 15, in which AB is perpendicular to BD, the angles M and N are complementary, their sum being equal to the right angle ABD.

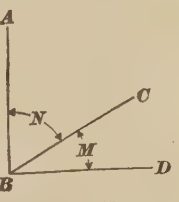

FIG. 15

EXAMPLE.—If angle CBD, Fig. 15, is 31°, what is the complementary angle ABC?

SOLUTION.—Since ABD is a right angle, the angle ABC is
90° − angle CBD = 90° − 31° = 59°. Ans.

25. Two angles are said to be **supplementary** when their sum is equal to two right angles, or 180°. Each of two supplementary angles is called the **supplement** of the other. In Fig. 16, AOD and DOB are supplementary angles, their sum being evidently equal to the sum of the two right angles POB and POA.

FIG. 16

EXAMPLE.—In Fig. 16, angle AOD is 45°; what is the supplementary angle DOB?

SOLUTION.—Since angle AOD + angle DOB = 180°,
Angle DOB = 180° − angle AOD = 180° − 45° = 135°. Ans.

26. The sum of all the angles formed on the same side of a straight line about the same point in the line is equal to two right angles, or 180°.

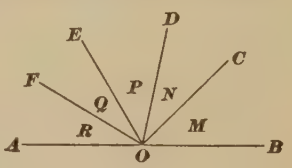

FIG. 17

EXAMPLE.—In Fig. 17, assuming that R = 30°, Q = 25°, P = 40°, and N = 28°, find M.

SOLUTION.—Since $R + Q + P + N + M$ = 180°,
M = 180° − $(R + Q + P + N)$ = 180° − (30° + 25° + 40° + 28°)
= 180° − 123° = 57°. Ans.

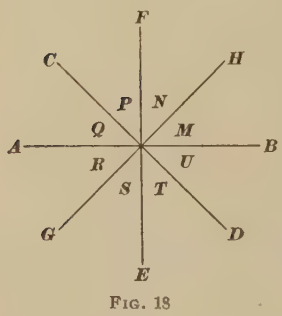

FIG. 18

27. The sum of all the angles formed in the same plane about one point is equal to four right angles. Thus, in Fig. 18,

$M + N + P + Q + R + S + T + U$
= four right angles = 360°.

EXAMPLE.—If the angles N, M, U, etc., Fig. 18, are equal to one another, what is the magnitude of each angle?

SOLUTION.—There are eight angles and their sum equals 360°. Hence, each angle is
360° ÷ 8 = 45°. Ans.

28. When two lines, as AB and CD, Fig. 19, cut or cross each other, they are said to **intersect.** Their common point O is called their **point of intersection,** or simply their **intersection.**

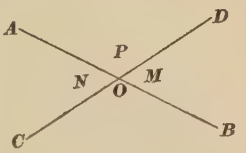

Fig. 19

29. Two intersecting straight lines determine four angles having a common vertex. Any one of these angles and the angle on the opposite side of both lines, as the angles M and N, Fig. 19, are called **vertical angles** with respect to each other. Vertical angles may also be defined as those having a common vertex and in which the sides of the one are the prolongations of the sides of the other.

Since M and N are each the supplement of P, they are equal to each other. Any angle is equal to its vertical angle.

PARALLELS

30. **Parallel lines,** Fig. 20, are straight lines that lie in the same plane and never meet, however far they are produced. Any two parallel lines have the same direction and are everywhere equally distant from each other.

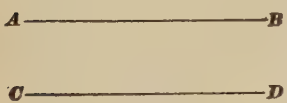

Fig. 20

31. A **horizontal line,** Fig. 21, is a line parallel to the horizon, or to the surface of still water.

32. A **vertical line,** Fig. 21, is a line perpendicular to a horizontal line; it has the direction of a plumb-line.

Fig. 21

33. In a drawing, it is usual to speak of lines drawn from left to right, or from right to left, as horizontal. In a diagram in a book, a line parallel to the printed lines is called a horizontal line, and a line perpendicular to the horizontal line is said to be vertical.

34. When two parallel lines, as PQ and RS, Fig. 22, are cut by a third line, as XY, the angles denoted by the capital letters are equal to one another; that is, $A = B = C = D$. The angles designated by the small letters are also equal to one another: $a = b = c = d$.

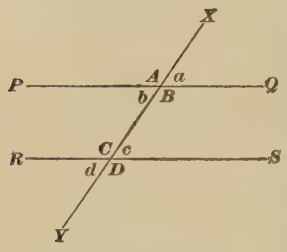

FIG. 22

35. If the corresponding sides of two angles are parallel and lie in the same or in opposite directions, the angles are equal. Thus, if the side AB, Fig. 23 or Fig. 24, is parallel to the side DE, and if the side BC is

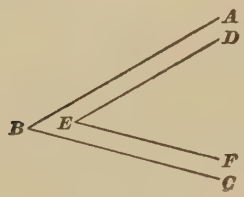

FIG. 23

FIG. 24

parallel to the side EF, the angle E is equal to the angle B

36. But if one of the sides of one angle lies in the same direction as the corresponding side of the other angle and the other two corresponding sides lie in opposite directions, the angles are supplementary. Thus, in Fig. 25, GH is parallel to and lies in the same direction as DE and HI is parallel to but lies in the opposite direction to EF; hence, angle GHI is the supplement of DEF.

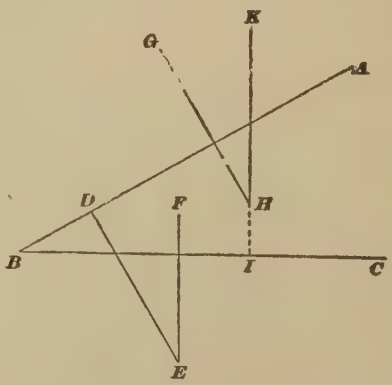

FIG. 25

37. If two sides of an angle are perpendicular to two sides of another angle, the two angles are equal or supplementary. Thus, if DE

and GH, Fig. 25, are perpendicular to BA, and if EF and HK are perpendicular to BC, then will angle E = angle B angle GHK; also GHI is the supplement of ABC.

EXAMPLES FOR PRACTICE

1. In Fig. 26, find the number of degrees in the angle EOA.
Ans. 156°

2. In Fig. 26, find the number of degrees in the arc AD.
Ans. 113°

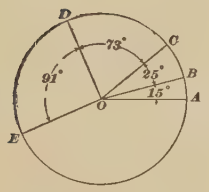

FIG. 26

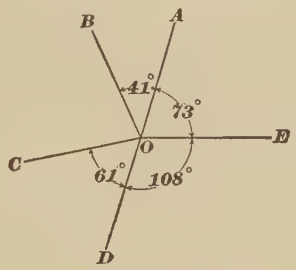

FIG. 27

3. In Fig. 27, find the number of degrees in the angle BOC.
Ans. 77°

4. In a wheel with fourteen spokes spaced equal distances apart, what is the angle included between the center lines of any two adjacent spokes? Ans. $25\frac{5}{7}°$

5. If one straight line meets another straight line so as to form an angle equal to 137°, what does its adjacent angle equal? Ans. 43°

6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°

PLANE FIGURES

38. A **plane figure** is any part of a plane surface bounded by straight or curved lines.

39. When a plane figure is bounded by *straight lines*, it is called a **polygon.** The bounding lines are called the *sides*, and the length of the broken line that bounds it (or the whole distance around it) is called the **perimeter** of the polygon.

40. The angles formed by the sides are called the *angles* of the polygon. Thus, $ABCDE$, Fig. 28, is a polygon. AB, BC, etc. are the sides; EAB, ABC, etc. are the angles; and the length of the broken line $ABCDEA$ is the perimeter.

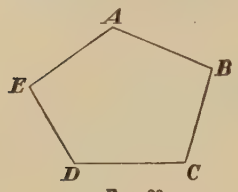

FIG. 28

41. Polygons are classified according to the number of their sides: One of three sides is called a **triangle;** one of four sides, a **quadrilateral;** one of five sides, a **pentagon;** one of six sides, a **hexagon;** one of seven sides, a **heptagon;** one of eight sides, an **octagon;** one of ten sides, a **decagon;** one of twelve sides, a **dodecagon;** etc.

42. Equilateral polygons are those in which the sides are all equal. Thus, in Fig. 29, $AB = BC = CD = DA$; hence, $ABCD$ is an equilateral polygon.

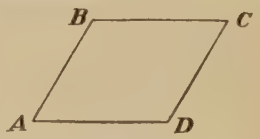

FIG. 29

FIG. 30

43. An **equiangular polygon** is a polygon whose angles are all equal. Thus, in Fig. 30, angle A = angle B = angle D = angle C; hence, $ABDC$ is an equiangular polygon.

44. A **regular polygon** is a polygon in which all the sides and all the angles are equal. Thus, in Fig. 31, $AB = BD = DC = CA$; and angle A = angle B = angle D = angle C; hence, $ABDC$ is a regular polygon. A regular polygon having four sides is called a *square*. Some other regular polygons are shown in Fig. 32.

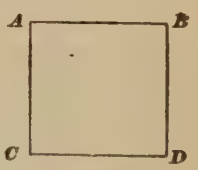

FIG. 31

FIG. 32

THE TRIANGLE

45. Triangles are named according to their sides as *isosceles*, *equilateral*, and *scalene* triangles, and according to their angles as *right* and *oblique* triangles.

FIG. 33

46. An **isosceles triangle**, Fig. 33, is one having two of its sides equal.

47. An **equilateral triangle**, Fig. 34, is one that has its three sides equal.

FIG. 34

48. A **scalene triangle**, Fig. 35, is one having no two of its sides equal.

FIG. 35

49. A **right-angled triangle**, Fig. 36, is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse**; when the *sides* of a right-angled triangle are referred to, the short sides are usually meant. For brevity, a right-angled triangle is termed a **right triangle.**

FIG. 36

50. An **oblique-angled**, or **oblique, triangle**, Fig. 37, is one that has no right angle.

FIG. 37

51. The **base** of any triangle is the side on which the triangle is supposed to stand. Any side of a triangle may be taken as the base.

The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base, or to the base extended. Thus, in Figs. 38 and 39, if the side AC is the base of the triangle, the line

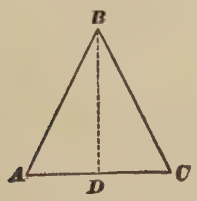

FIG. 38

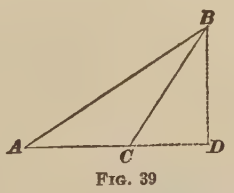

FIG. 39

BD is the altitude. In a right triangle, if one of the short sides is taken as the base, the other short side will be the altitude of the triangle.

52. In an isosceles triangle, the angles opposite the equal sides are equal. Thus, in Fig. 40, $AB = BC$; hence, angle C = angle A.

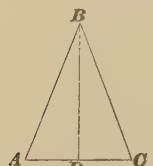

FIG. 40

In any isosceles triangle, if a perpendicular be drawn from the vertex opposite the unequal side to that side, it bisects (cuts in halves) the side. Thus, AC, Fig. 40, is the unequal side in the isosceles triangle ABC; hence, the perpendicular BD bisects AC, or $AD = DC$.

If two angles of any triangle are equal, the triangle is isosceles.

53. In any triangle, the sum of the three angles is equal to two right angles, or 180°. Thus, in Fig. 41, the sum of the angles at A, B, and C = two right angles; that is, $A + B + C = 180°$. Hence, if any two angles of a triangle are given, the third may be found by subtracting the sum of the two from 180°.

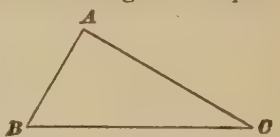

FIG. 41

EXAMPLE.—If, in Fig. 41, $A = 90°$ and $C = 35°$, how many degrees are there in B?

SOLUTION.—Since the sum of A, B, and C is 180°, B is equal to the difference between 180° and the sum of A and C. Therefore,

$$B = 180° - (90° + 35°) = 180° - 125° = 55°. \text{ Ans.}$$

54. In any right triangle there can be but one right angle, and since the sum of all the angles equals two right angles, it is evident that the sum of the two acute angles must be equal to a right angle. Therefore, if in any right triangle one acute angle is known, the other can be found by subtracting the known angle from 90°. Thus, ABC, Fig. 42, is a right triangle, right angled at C. Then, the angles $A + B$ = one right angle. If $A = 40° 30'$, $B = 90° - 40° 30' = 49° 30'$.

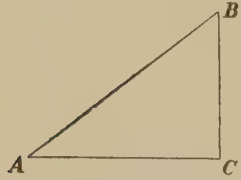

FIG. 42

RISE, RUN, AND PITCH

55. In roof framing and other branches of carpentry, angles may not be expressed directly by the number of degrees contained in them, but indirectly by the *rise* and *run* or by the *pitch*. These terms will now be explained. In Fig. 43, AB and BC represent two rafters placed in position. The **rise** is the vertical distance BD, or the elevation of B above A or C. The **run** of the rafter AB is the distance AD, or the horizontal distance from A, the lower end of AB, to a plumb-line dropped from B. Similarly, the run of the rafter CB is the distance CD. The **pitch** is the proportion that the rise bears to the span, or to the distance AC, and is expressed by $\frac{BD}{AC}$. If BD is 8 feet and AC 24 feet, then the pitch of AB will be $\frac{8}{24}$, or $\frac{1}{3}$.

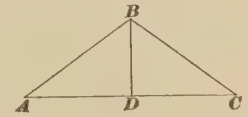

FIG. 43

56. The pitch of rafters or other inclined pieces is often expressed as the rise for a run of 1 foot. Thus, a rise of 8 inches to the foot means that the difference in elevation, for each foot measured horizontally, is 8 inches. See Fig. 44.

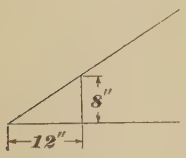

FIG. 44

57. Fig. 45 further illustrates the use of the terms rise,

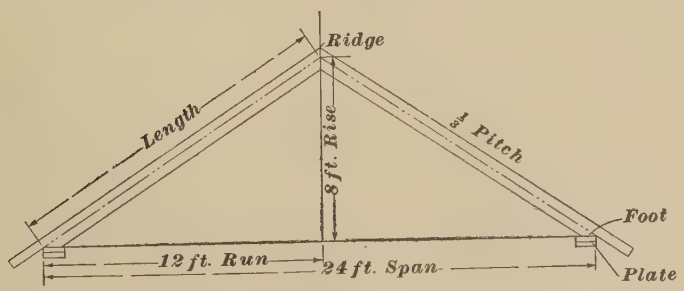

FIG. 45

run, and pitch. It will be noticed that the length of the

rafter is measured on a line passing through the outside edge of the plate parallel to the edge of the rafter, and that the run is measured from the same point in the plate to a point directly below the ridge.

EXAMPLES FOR PRACTICE

1. Find the number of degrees in each of the angles B and C of the triangle $A\,B\,C$, Fig. 46. Ans. $\begin{cases} B = 36° \\ C = 108° \end{cases}$

2. How many degrees are there in each of the acute angles of an isosceles right triangle? Ans. 45°

3. One of the acute angles of a right triangle contains 30°; how many degrees are there in the other acute angle? Ans. 60°

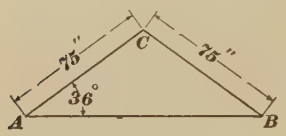

FIG. 46

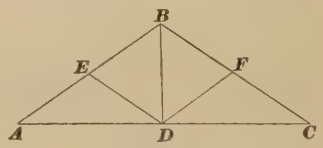

FIG. 47

4. Fig. 47 represents a roof truss. Angle $A = 35°$ and $A\,E = E\,B = E\,D$; find the number of degrees in angles $A\,E\,D$, $E\,D\,A$, and $A\,B\,D$. Ans. $\begin{cases} A\,E\,D = 110° \\ E\,D\,A = 35° \\ A\,B\,D = 55° \end{cases}$

5. The rise of a rafter is 8 feet 6 inches and the run is 10 feet; what is the rise per foot? Ans. 10.2 in., or $10\frac{1}{4}$ in., nearly

6. The roof of a building 26 feet wide is $\frac{1}{3}$ pitch; what is: (a) the height of the ridge above the foot of the rafters? (b) the rise per foot? Ans. $\begin{cases} (a)\ 8 \text{ ft. } 8 \text{ in.} \\ (b)\ 8 \text{ in.} \end{cases}$

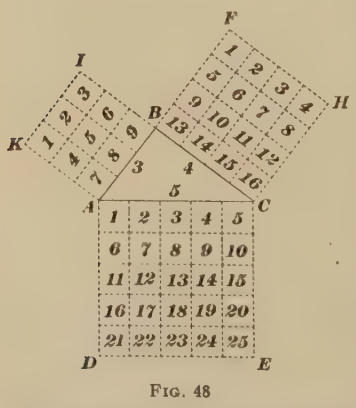

FIG. 48

PRINCIPLE OF THE RIGHT TRIANGLE

58. In any right triangle, the square described on the hypotenuse is equal to the sum of the squares described on the other two sides. If $A\,B\,C$, Fig. 48, is a right triangle, right angled at B, then the square described on the hypotenuse $A\,C$ is equal to the sum

of the squares described on the sides AB and BC; consequently, if the lengths of the sides AB and BC are known, the length of the hypotenuse can be found by adding the squares of the lengths of the sides AB and BC, and then extracting the square root of the sum.

Rule.—*To find the hypotenuse of a right triangle, add together the squares of the sides and extract the square root of the sum.*

Or, $$c = \sqrt{a^2 + b^2}$$

in which a = length of one short side;
b = length of other short side;
c = length of hypotenuse.

EXAMPLE 1.—In Fig. 49, BC is a brace joining the two pieces AB and AC, which are perpendicular to each other. Find the length of BC, B being 24 inches from A, and C 36 inches from A.

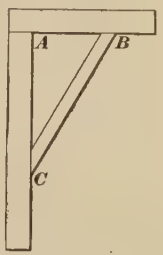

FIG. 49

SOLUTION.— ABC is a right triangle, of which AB and AC are the sides and BC the hypotenuse. The length of BC is found by adding the squares of the lengths of AB and AC and extracting the square root of the sum. Thus, the square of AB equals

$$24^2 = 24 \times 24 = 576$$

The square of AC is

$$36^2 = 36 \times 36 = 1{,}296$$

The sum of these squares equals $576 + 1{,}296 = 1{,}872$; the square root of $1{,}872 = 43.27$. The length of BC is therefore 43.27 in. Ans.

EXAMPLE 2.—The width of a house is 20 feet, and the roof is half pitch—that is, the height of the gable is equal to one-half the width; what is the length of a rafter?

SOLUTION.—Let one-half the width be called a = 10 ft.; the height of the gable, b, is one-half of 20, or 10 ft.; and the hypotenuse, c. Substituting these values in the formula,

$$c = \sqrt{a^2 + b^2} = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14 \text{ ft.} \quad \text{Ans.}$$

59. When one side and the hypotenuse of a right triangle are known, the other side may be found by applying the following rule:

Rule.—*To find one side of a right triangle when the other side and the hypotenuse are known, subtract the square of the given side from the square of the hypotenuse and extract the square root of the difference.*

Then, using letters having the same meaning as those given in the formula of Art. 58,

$$a = \sqrt{c^2 - b^2} \text{ or } b = \sqrt{c^2 - a^2}$$

EXAMPLE 1.—It is desired to ascertain the height of the ceiling in a room. One end of a 10-foot pole is set in the angle of the ceiling and wall, and the other end is 6 feet from the base of the wall; what is the height of the ceiling?

SOLUTION.—In this problem, the 10-ft. pole is the hypotenuse c, and the short side b is 6 ft. Hence, applying the formula,

$$a = \sqrt{c^2 - b^2} = \sqrt{100 - 36} = 8 \text{ ft.},$$

which is the height of the ceiling. Ans.

EXAMPLE 2.—The length of each of the equal sides of an isosceles triangle is 10 inches, and the length of the base is 12 inches; find the length of the perpendicular from the vertex to the base.

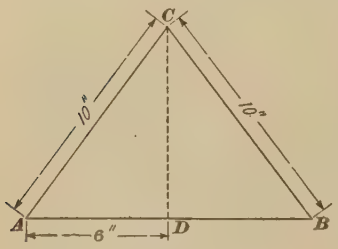

FIG. 50

SOLUTION.—Let ABC, Fig. 50, be the isosceles triangle. Then, AD is one-half of the base; therefore, $AD = 6$ in. In the right triangle, ACD, the side CD is equal to the square root of the remainder obtained by subtracting the square of the side AD from the square of the hypotenuse AC. Hence,

$$CD = \sqrt{10^2 - 6^2} = \sqrt{64} = 8 \text{ in. Ans.}$$

60. The hypotenuse of a right triangle whose sides are of equal length is equal to the square root of twice the square of a side. ABC, Fig. 51, is a right triangle, the side AB being equal to the side BC; then,

$$AC = \sqrt{2 \times \overline{AB}^2} = 1.4142\ AB$$

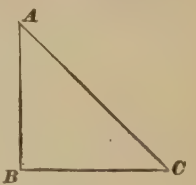

FIG. 51

Rule.—*The hypotenuse of a right triangle having equal sides is equal to 1.4142 times the length of a side.*

NOTE.—In practice, when great accuracy is not required, $1\frac{5}{12}$ is sometimes used instead of 1.4142, as given in the foregoing rule. For example, when each side is 12 inches the hypotenuse is practically $1\frac{5}{12} \times 12 = 17$ inches.

EXAMPLE.—What is the length of a slope of a roof having $\frac{1}{2}$ pitch, the width of the house being 30 feet?

SOLUTION.—Since the roof is $\frac{1}{2}$ pitch, the height of the roof DG, Fig. 52, is equal to one-half of the width of the house: $DG = \frac{1}{2}$ of $EF = EG$. The slope ED is therefore the hypotenuse of a right triangle whose sides are equal.

$EG = \frac{1}{2}$ of 30 = 15 ft.

$ED = 1.4142 \times 15 = 21.21$ ft.

.21 ft. $= .21 \times 12 = 2.52$ in.

$= 2\frac{1}{2}$ in., nearly

Therefore, the length of the slope is 21 ft. $2\frac{1}{2}$ in. Ans.

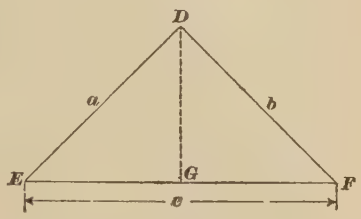

FIG. 52

The diagonal of a square is the hypotenuse of a right triangle having equal sides; hence, the diagonal of a square is equal to 1.4142 times the side of the square. The side of a square can be obtained when the diagonal is known by dividing the diagonal by 1.4142, or by multiplying the diagonal by .7071.

61. A right angle can be laid off by the following method, when the three sides of any right triangle are known. Suppose that AB, Fig. 53, is to be one line of the foundation of a building and it is desired to lay off from A another line that will make a right angle with AB.

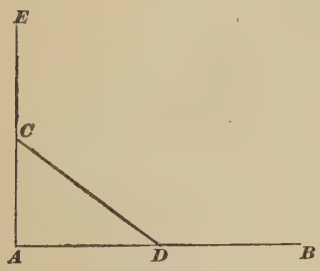

FIG. 53

It is known that a triangle whose sides are respectively 6, 8, and 10 feet is a right triangle, since $6^2 + 8^2 = 10^2$. On AB lay off AD equal to 8 feet. Fasten one end of a tape, or line, at A and draw the other end toward E, so that AE will be as nearly perpendicular to AB as can be estimated by the eye. Make AC on AE 6 feet. Measure the distance CD; if this is more or less than 10 feet, swing the line AE to the right or left, as may be necessary, until DC is exactly 10 feet, when the angle at A will be a right angle.

Any distances that will form three sides of a right triangle may be used in the preceding method; the numbers 3, 4, and 5, or some multiple, as 6, 8, and 10, or 9, 12, and 15, are most convenient.

62. The principle of the right triangle is of very great value in practical work, and the student should become thoroughly familiar with it in all of its variations.

The following is an example showing a double application of the right-triangle principle:

EXAMPLE.—Fig. 54 represents a skylight, which forms the four sides of a pyramid. The length of the base AD is 9 feet and the width CD is 7 feet 6 inches. The point O is 2 feet above the plane of $ABCD$. What is the length of the hip rafter OD at the angle of the skylight?

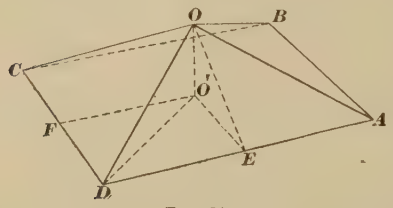

FIG. 54

SOLUTION.—It will be seen that the point O' is directly under O and 2 ft. below it, so that O' is on the same level as $ABCD$. It will also be seen that DO is simply the hypotenuse of a right triangle, whose base is DO' and whose altitude is OO'. But DO' is also the hypotenuse of a right triangle, whose sides are EO' and ED. EO' is one-half of 7 ft. 6 in. or 3.75 ft., and ED is one-half of 9 ft. or 4.5 ft.

Then, $$DO' = \sqrt{3.75^2 + 4.5^2} = 5.86 \text{ ft.}$$

In the triangle $DO'O$, $DO' = 5.86$ ft., and $OO' = 2$ ft.; whence,

$$DO = \sqrt{5.86^2 + 2^2} = 6.19 \text{ ft.} = 6 \text{ ft. } 2\tfrac{1}{4} \text{ in., nearly}$$

One extraction of square root may be dispensed with, thus:

$$DO = \sqrt{(DO')^2 + (OO')^2} = \sqrt{[(EO')^2 + (ED)^2] + (OO')^2}$$

$$= \sqrt{3.75^2 + 4.5^2 + 2^2} = \sqrt{38.31} = 6.19 \text{ ft., as before.} \quad \text{Ans.}$$

Again, if we can find the length of a line along the skylight perpendicular to an edge, as OE to AD, then the hip OD is the hypotenuse of a right triangle of which OE and ED are sides; and

$$OE = \sqrt{(O'E)^2 + (OO')^2}$$

The result will be the same either way.

NOTE.—When decimals occur in solving the examples in *Geometry and Mensuration*, Parts 1 and 2, two decimal places are, as a rule, to be retained (except in constants, as .7854, .5236, etc.), and if the third decimal figure is greater than 5, the second decimal figure is to be increased by 1; thus, 13.537 should be written 13.54, not 13.53. To easily convert feet and inches to feet and decimals of a foot, feet and decimals of a foot to feet and inches, or to change a fraction of an inch to a decimal, or vice versa, Tables I and II, Art. **91**, may be used.

SIMILAR POLYGONS AND TRIANGLES

63. **Similar polygons** are those whose corresponding angles are equal and whose corresponding sides are proportional.

In the quadrilaterals $A\,B\,C\,D$ and $A''\,B''\,C''\,D''$, Fig. 55, the corresponding angles are equal; that is, angle A

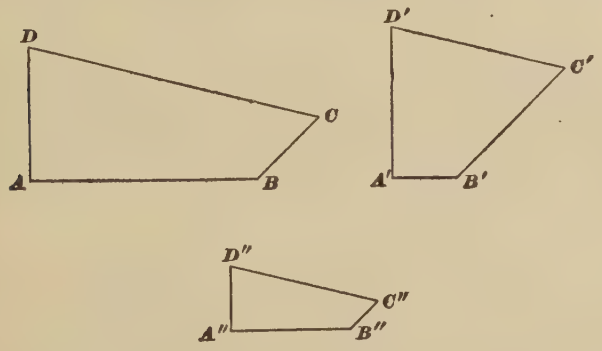

FIG. 55

equals angle A''; angle B equals angle B''; angle C equals angle C''; and angle D equals angle D''. Moreover, the corresponding sides of the figures are proportional; that is, $A''\,B''$ bears the same ratio to $A\,B$ as does $B''\,C''$ to $B\,C$, $C''\,D''$ to $C\,D$, and $D''\,A''$ to $D\,A$. The quadrilaterals $A\,B\,C\,D$ and $A''\,B''\,C''\,D''$ are therefore similar.

The quadrilaterals $A\,B\,C\,D$ and $A'\,B'\,C'\,D'$, Fig. 55, have their corresponding angles equal, but the figures are not similar, since the corresponding sides are not proportional. For example, $A'\,D'$ does not bear the same ratio to $A\,D$ as does $A'\,B'$ to $A\,B$.

64. Two triangles are similar when the angles of one are equal to the angles of the other. In the triangles $A\,B\,C$ and $A'\,B'\,C'$, Fig. 56, angle A equals A', angle B equals angle B', and angle C equals

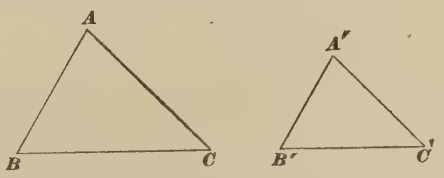

FIG. 56

angle C'; the triangles are therefore similar, and their corresponding sides are proportional.

The following are some of the proportions that may be formed from these similar triangles:

$$AB : A'B' = AC : A'C'$$
$$AB : A'B' = BC : B'C'$$
$$AC : A'C' = BC : B'C'$$
$$AB : BC = A'B' : B'C'$$

EXAMPLE.—In Fig. 56, $AB = 6$ feet, $AC = 7$ feet, $BC = 8$ feet, and $B'C' = 6$ feet; find $A'B'$ and $A'C'$.

SOLUTION.—It will be observed that the lengths of the similar sides BC and $B'C'$ are given, the ratio between their lengths being $8 : 6$, or $\frac{8}{6}$. The ratio between other similar sides will be the same; that is, $\frac{AB}{A'B'} = \frac{8}{6}$, and $\frac{AC}{A'C'} = \frac{8}{6}$. Since

$$AB : A'B' = 8 : 6, \text{ or } 6 : A'B = 8 : 6$$
$$A'B' = \frac{6 \times 6}{8} = 4\tfrac{1}{2} \text{ ft.} = 4 \text{ ft. } 6 \text{ in.} \quad \text{Ans.}$$

Again, $AC : A'C' = BC : B'C'$, or $7 : A'C' = 8 : 6$

Then, $$A'C' = \frac{7 \times 6}{8} = 5\tfrac{1}{4} \text{ ft.} = 5 \text{ ft. } 3 \text{ in.} \quad \text{Ans.}$$

65. Triangles having their corresponding sides parallel are similar.

EXAMPLE.—It is required to find the distance AB, Fig. 57, across a stream.

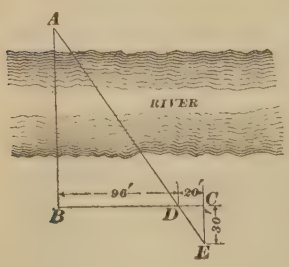

FIG. 57

SOLUTION.—The line BC making any angle with AB is measured, and CE is made parallel with AB, and of any convenient length. The point D is marked where AE intersects BC, and BD and DC are measured. Then, since the triangles ABD and CDE have their corresponding sides parallel, they are similar, and $AB : CE = BD : DC$; or

$$AB = \frac{30 \times 96}{20} = 144 \text{ ft.} \quad \text{Ans.}$$

66. If a straight line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally. Thus, let the line DE be drawn parallel to

the side BC in the triangle ABC, Fig. 58. Then, $AD : AB = AE : AC$, or $AD : DB = AE : EC$. It is to be noticed that the triangles ADE and ABC are similar, and their sides are proportional. Thus $AB : AD = BC : DE$, and $AC : AE = BC : DE$.

If a straight line, as DF, is drawn from D, Fig. 58, parallel to AC, the triangles ADE, ABC, and DBF are all similar, and a number of other proportions may be formed.

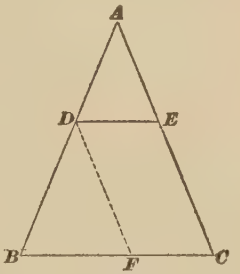

FIG. 58

EXAMPLE 1.—In Fig. 59, DE is parallel to BC, and AD is one-third of AB; find the length of AE.

SOLUTION.—Since DE is parallel to BC, $AD : AB = AE : AC$.

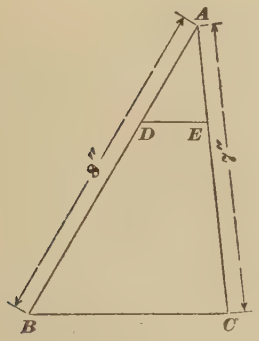

FIG. 59

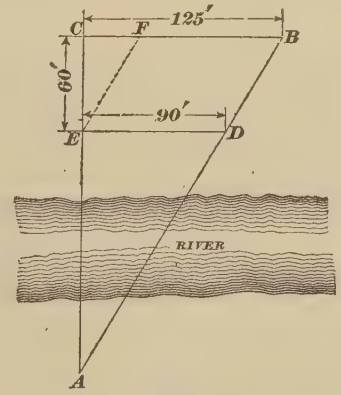

FIG. 60

But, $AD : AB = 1 : 3$; then, $AE : AC = 1 : 3$. Substituting for AC its value, $AE : 7 = 1 : 3$; hence

$$AE = \frac{7 \times 1}{3} = 2\tfrac{1}{3} \text{ in.} \quad \text{Ans.}$$

EXAMPLE 2.—Referring to Fig. 60, it is desired to find the length of the line CA extending across a river. E is on the line AC, D is on the line BA, and DE is parallel to CB; the lengths of CE, ED, and CB are as shown in the figure.

SOLUTION.—Draw EF parallel to AB, then $CF = 125 - 90 = 35$ ft. The triangles CAB and CEF being similar, $CA : CE = CB : CF$, or $CA : 60 = 125 : 35$; whence

$$CA = \frac{60 \times 125}{35} = 214.3 \text{ ft., nearly.} \quad \text{Ans.}$$

EXAMPLE 3.—On a drawing, it is required to divide a line 8 inches long into twelve parts.

SOLUTION.—Let AB, Fig. 61, be the 8-in. line. Through A draw any line AC 12 in. long, and mark the inch points on it. Connect C and B, and through each inch mark on AC, draw a parallel to CB, as DE, etc., cutting AB at E, etc., which will be the points required; for by similar triangles, $AE : AD = AB : AC$; or AE : 1 in. = 8 in. : 12 in.; hence,

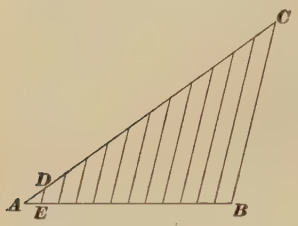

FIG. 61

$$AE = \frac{1 \times 8}{12} = \tfrac{2}{3} \text{ in.} \quad \text{Ans.}$$

NOTE.—This principle is very useful when an exact measurement—as, for example, $\frac{2}{3}$ inch—cannot be obtained by a scale or rule. The equal spaces on AC need not be 1 inch in length. They may be of any length that will be most convenient.

EXAMPLES FOR PRACTICE

1. A ladder 65 feet long reaches to the top of a house when its foot is 25 feet from the house; how high is the house, supposing the ground to be level? Ans. 60 ft.

2. In a triangle ABC, side $AB = 32$ feet, $BC = 34$ feet, and $AC = 48$ feet; if side AB of a similar triangle is 72 feet long, what are the lengths of the other two sides? Ans. $\begin{cases} AC = 108 \text{ ft.} \\ BC = 76.5 \text{ ft.} \end{cases}$

3. The base of a right triangle is 24 inches, and its altitude, 72 inches; at what distance from the top is the triangle 16 inches wide? Ans. 48 in.

4. What length of rafter is required for each side of a gable, the width of which is 24 feet, and whose height is three-fourths the span? Ans. 21.63 ft.

5. $ABCD$, Fig. 62, is a section of a roof. It is desired to extend the sides AB and CD to the ridge E; what are the distances EF and EA?

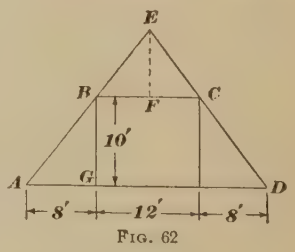

FIG. 62

Ans. $\begin{cases} EF = 7 \text{ ft. } 6 \text{ in.} \\ EA = 22.41 \text{ ft., or } 22 \text{ ft. } 5 \text{ in., nearly} \end{cases}$

6. What is the distance between the 16-inch mark on the body of a steel square and the 12-inch mark on the tongue? Ans. 20 in.

7. The distance EB, Fig. 63, is $8\frac{1}{3}$ feet. If the distance between the pieces parallel to AB, measured from center to center, is 20 inches, and the length of AB is 5 feet 6 inches, what is the length of the piece CD?

Ans. 3 ft. $3\frac{3}{5}$ in.

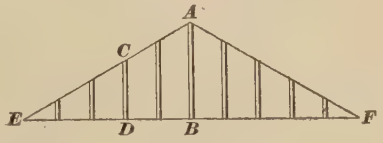

Fig. 63

8. In Fig. 64, $CBAEC$ represents a gable at right angles to the main roof, the line AB, where they meet, being called a *valley*. What is the length of the valley rafter AB, CA being 16 feet, ED, the rise of the rafter, being 8 feet, and $EB = DF$, the distance of B back of ACE, 4 feet?

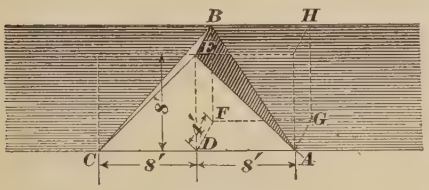

Fig. 64

The principle is the same as explained for hips. Ans. 12 ft.

GEOMETRICAL PRINCIPLES OF THE CIRCLE

67. A **chord** is a straight line joining any two points in a circumference; or, it is a straight line joining the extremities of an arc. Thus, in Fig. 65, AB is the chord of the arc AEB.

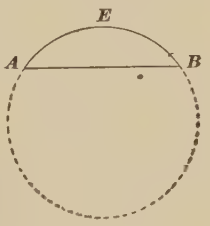

Fig. 65

68. A **segment** of a circle is the space included between an arc and its chord. Thus, in Fig. 65, the space between the arc AEB and the chord AB is a segment.

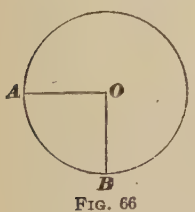

Fig. 66

69. A **sector** of a circle is the space included between an arc and two radii drawn to the extremities of the arc. Thus, in Fig. 66, the space included between the arc AB and the radii OA and OB is a sector of the circle.

70. Two circles are equal when the radius or diameter of one is respectively equal to the radius or diameter of the other.

Two arcs are equal wnen the radius and chord of one are equal to the radius and chord of the other.

71. **An inscribed angle** is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 67, ABC is an inscribed angle, and it is measured by one-half the arc ADC.

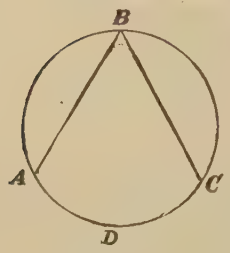

FIG. 67

EXAMPLE.—If in the figure the arc $ADC = 144°$; what is the measure of the inscribed angle ABC?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{1}{2} \times 144° = 72°$. Ans.

72. All angles inscribed in the same segment are equal. Thus, in Fig. 68, the inscribed angles ABE, ACE, and ADE are all equal; for each is measured by one-half of the arc AE.

For example, if arc AE is 86°, each of the angles ABE, ACE, and ADE is equal to one-half of 86°, or 43°. It is thus seen that angles ABE, ACE, and ADE are each

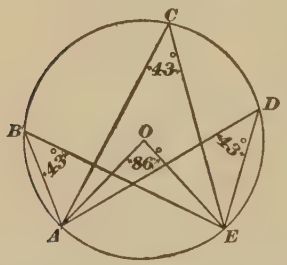

FIG. 68

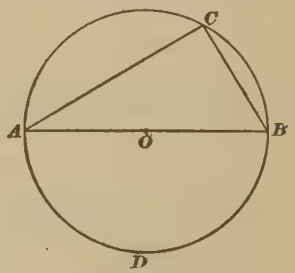

FIG. 69

equal to one-half of the angle AOE, which is formed by drawing radii from the center of the circle O to A and E.

Any angle inscribed in a semicircle is a right angle. Thus, the angle ACB, Fig. 69, is a right angle, for it is measured by one-half of the semi-circumference ADB, and one-half of a semi-circumference is a quadrant.

73. If, in any circle, a radius is drawn perpendicular to any chord, it will bisect (cut in halves) the chord. Thus, if the radius OC, Fig. 70, is perpendicular to the chord AB, $AD = DB$.

A radius that bisects a chord, bisects also the angle included between radii drawn to the ends of the chord. Thus, in Fig. 70, the radius OC bisects the angle AOB.

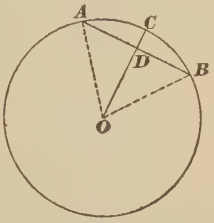

FIG. 70

If a straight line is drawn perpendicular to any chord at its middle point, it must pass through the center of the circle.

74. Through any three points not in the same straight line, a circumference can be drawn. Let A, B, and C, Fig. 71, be any three points. Join A and B also B and C by straight lines. From the middle point of AB draw KH perpendicular to AB; from the middle point of BC draw FE perpendicular to BC. These two perpendiculars intersect at O. With O as a center, and OB, OA, or OC as a radius, describe a circle; it will pass through A, B, and C.

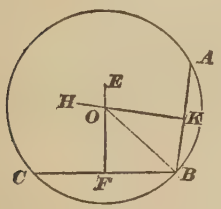

FIG. 71

75. A **tangent** to a circle is a straight line that touches the circle at one point only; it is always perpendicular to a radius drawn to that point. Thus, AB, Fig. 72, is a tangent to the circle; it touches the circle at E and is perpendicular to the radius OE.

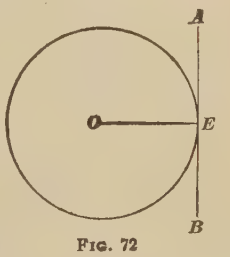

FIG. 72

76. If two circles intersect each other, the line joining their centers bisects at right angles the line joining the two points of intersection. Thus, if the two circles whose centers are O and P, Fig. 73, intersect at A and B, the line OP bisects at right angles the line AB; or $AC = BC$.

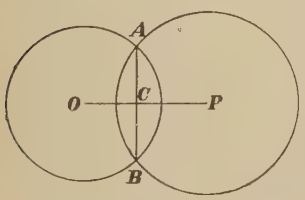

FIG. 73

77. One circle is said to be **tangent** to another circle when they touch each other at one point only, as in

Fig. 74. This point is called the **point of contact,** or the **point of tangency.**

78. When two or more circles are described from the

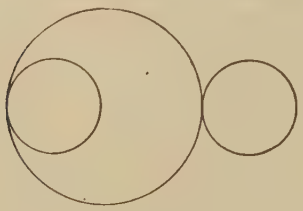

FIG. 74

FIG. 75

same center, as in Fig. 75, they are called **concentric circles.**

79. If two chords of a circle intersect, the product of the segments (parts) thus formed of one of the chords is equal

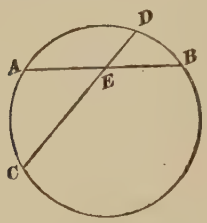

FIG. 76

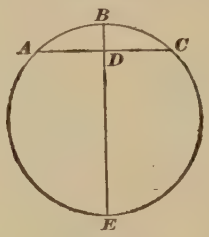

FIG. 77

to the product of the segments of the other. Thus, AB and CD, Fig. 76, are chords that intersect at E. Then,

$$AE \times EB = CE \times ED$$

EXAMPLE.—In Fig. 77, the diameter of the circle BE is 29 feet, and the distance BD, 4 feet; what is the length of the chord AC that passes through D and is perpendicular to BE?

SOLUTION.—Since AC is perpendicular to the diameter BE, BE bisects AC, and $AD = DC$ (Art. **73**).

$$DE = 29 - 4 = 25 \text{ ft.}$$

The products of the segments of the intersecting chords being equal, $AD \times DC = BD \times DE$. But, $AD = DC$; therefore,

$$\overline{AD}^2 = BD \times DE = 4 \times 25 = 100;$$

$$AD = \sqrt{100} = 10 \text{ ft.}$$

and $$AC = 2 \times AD = 2 \times 10 = 20 \text{ ft.} \quad \text{Ans.}$$

80. Since $\overline{AD}^2 = BD \times DE$, in Fig. 77, the following principle may be deduced from the preceding solution: When

a chord is drawn perpendicular to a diameter of a circle, the square of one-half the chord is equal to the product of the parts of the diameter formed by the intersection of the chord.

EXAMPLE.—If, in Fig. 77, BD is 3 feet and DE is 9 feet, find the length of the half-chord AD.

SOLUTION.— $\overline{AD}^2 = BD \times DE = 3 \times 9 = 27$. Since the square of $AD = 27$,

$$AD = \sqrt{27} = 5.20 \text{ ft.} \quad \text{Ans.}$$

81. In the circle $ABCE$, Fig. 78, AC is the chord of the segment ABC. BD is the height of the segment; BC, the chord of one-half the arc; and BE, the diameter perpendicular to the chord. Then,

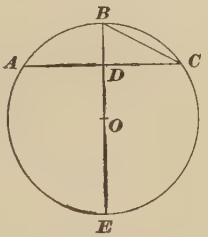

FIG. 78

$$\overline{BC}^2 = BD \times BE, \text{ and } BE = \frac{\overline{BC}^2}{BD},$$

or the radius $$BO = \frac{\overline{BC}^2}{2\,BD}.$$

Rule.—*The radius of a circle is equal to the square of the chord of one-half of a given arc divided by twice the height of the segment.*

EXAMPLE.—Referring to Fig. 78, $BC = 3$ feet 6 inches, and $BD = 1$ foot 7 inches; find the radius BO.

SOLUTION.— $BC = 42$ in.; $BD = 19$ in. According to the rule, the radius

$$BO = \frac{\overline{BC}^2}{2\,BD} = \frac{42^2}{2 \times 19} = 46.42 \text{ in.} \quad \text{Ans.}$$

82. Where the chord of the arc and the height of the segment are known, to find the radius of the circle, add the square of one-half the chord to the square of the height of the segment and divide the sum by twice the height of the segment.

Let r = radius;
c = chord of arc;
h = height of segment.

Then,
$$r = \frac{\left(\frac{c}{2}\right)^2 + h^2}{2h}$$

or,
$$r = \frac{c^2 + 4h^2}{8h}$$

EXAMPLE.—In arches, the *span* is the distance across the opening, as AC, Fig. 79, measured from the ends of the arch, as A and C. The *rise* DB is the perpendicular distance from AC to the highest point B measured on the center line OB.

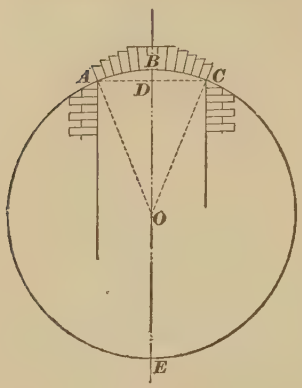

FIG. 79

The span AC, Fig. 79, of an arch is 6 feet, and the rise DB is 8 inches; what is the length of the radius OB?

SOLUTION.—The span AC is a chord of the arc ABC, and BD is the height of the segment. AD, one-half of the chord, is $\frac{1}{2} \times 72$ in. $= 36$ in.

$$OB = \frac{\overline{AD}^2 + \overline{BD}^2}{2BD}$$

$$= \frac{36^2 + 8^2}{2 \times 8} = \frac{1360}{16} = 85 \text{ in.}$$

Therefore, $OB = 85$ in. $= 7$ ft. 1 in. Ans.

INSCRIBED AND CIRCUMSCRIBED POLYGONS

83. An **inscribed polygon** is one whose vertexes lie on the circumference of a circle and whose sides are chords, as $KLMNPQ$, Fig. 80.

84. A **circumscribed polygon** is one whose sides are tangent to a circle, as $ABCDEF$, Fig. 80.

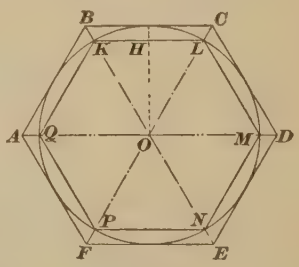

FIG. 80

Circles may be inscribed in and circumscribed about any regular polygon. The radius of the circumscribed circle is the distance from the center to an angle of the inscribed polygon, and of the inscribed circle the perpendicular distance from the center to the side of the circumscribed polygon, as shown in Fig. 80.

85. If lines are drawn from the center to the angles of a regular polygon, they will make equal angles with the sides of the polygon and will form isosceles triangles. As the sum of all the angles formed about a point by lines drawn from it is equal to 360°, the angle at the center of a

regular polygon between two radii equals 360° divided by the number of sides. Thus the angle between two radii drawn to the ends of a side of a regular hexagon is $360° \div 6 = 60°$. The sum of the angles of a triangle is two right angles, or 180°; hence, since the triangles formed by drawing radii to the angles of a regular polygon are isosceles, the angles OQK and OKQ, Fig. 80, are each equal to one-half the difference between 180° and the central angle. Thus, each angle is $\frac{1}{2} \times (180° - 60°) = 60°$, and the triangle OKQ is equilateral. In a regular hexagon, therefore, the sides are equal to the radius of the circumscribed circle.

A line drawn from the center of a regular polygon to an angle bisects the angle; thus OA, Fig. 80, bisects the angle BAF, which, therefore, is equal to $2 \times OAB$, or $2 \times OAF$.

86. The principles of Art. **85** are employed, in practice, to determine the various cuts of material used in the construction of regular polygons.

EXAMPLE 1.—At what angle should a bevel be set to mark the cuts for the moldings that are to be laid around an octagonal room of equal sides, as in Fig. 81?

NOTE.—A *bevel* is an instrument consisting of two straight pieces of wood or metal so jointed that any angle may be formed by the pieces.

SOLUTION.—Draw AO, BO, etc. to the center O, forming triangles AOB, BOC, etc. Then each of the angles at O is one-eighth of $360° = 45°$. The angles OAB and OBA are each

$$\tfrac{1}{2} \times (180° - 45°) = 67\tfrac{1}{2}°$$

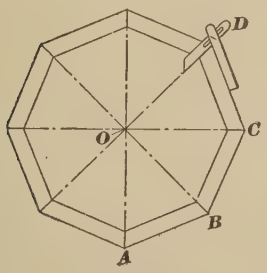

FIG. 81

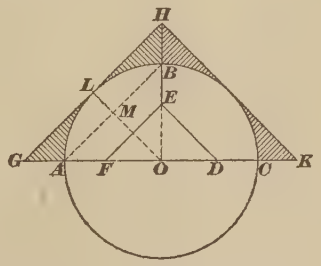

FIG. 82

If the blade of the bevel is set at this angle, as at D, and the ends of the pieces of molding are cut to it, they will fit together properly. Ans.

EXAMPLE 2.—In Fig. 82, $GHEF$ and $HKDE$ represent two boards forming a center for supporting a brick arch during building;

the shaded parts being cut off, leaving a curve, as $A\,L\,B$. What is the length of each board, and the angle of bevel $G\,H\,O$ where they meet? The radius $A\,O$ of the arch is 2 feet.

SOLUTION.—Draw $A\,B$ parallel to $G\,H$ and meeting $O\,G$ and $O\,H$; also draw $O\,L$ perpendicular to $A\,B$; then $A\,M = M\,B$ (Art. 73), and by similar triangles, $L\,G = L\,H$. $O\,L$ bisects the right angle $A\,O\,B$; hence, $A\,O\,M$ is one-half of 90°, or 45°. Then the other acute angle in the right triangle $O\,M\,A$ is $90° - 45° = 45°$ also. The angle $L\,G\,O$ is equal to the angle $M\,A\,O$, or 45°, and the right triangle $O\,L\,G$ having two angles equal, the opposite sides are equal also, and since $O\,L$ = the radius, or 2 ft., $L\,G$ is also 2 ft. and $G\,H$ is $2 \times 2 = 4$ ft., the required length. Ans.

Angle $O\,H\,G$ = angle $O\,G\,H = 45°$, the angle at which to set the bevel. Ans.

The following principle may also be conveniently used in many cases. In any polygon, the sum of all the interior angles equals 180° multiplied by a number that is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a regular polygon, multiply 180° by the number of sides less two, and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle. For example, each interior angle of a regular octagon, as Fig. 81, is $[180 \times (8 - 2)] \div 8 = 135°$.

EXAMPLES FOR PRACTICE

1. The span of an arch is 7 feet, and the rise is 1 inch for each foot of span; what is the radius of the arch?

Ans. 10 ft. $9\frac{1}{2}$ in.

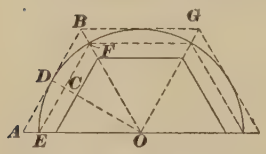

FIG. 83

2. The centering for an arch, Fig. 83, 8 feet in diameter, is made of three pieces of equal length. Knowing that the side of a regular inscribed hexagon is equal to the radius, what should be the length, as $A\,B$, of each piece?

Ans. 4.62 ft., or 4 ft. $7\frac{1}{2}$ in., nearly

SUGGESTION.—Find the length of $O\,C$ in the right triangle $O\,C\,E$; then by the similar triangles $O\,E\,C$ and $O\,A\,D$ find the length of $A\,O$. The proportion will be stated: $O\,C : O\,D = O\,E : O\,A$. $A\,B$ will equal $A\,O$.

3. In the preceding example, what is: (*a*) the bevel angle $A\,B\,O$? (*b*) the angle between any two pieces?

Ans. (*a*) 60°; (*b*) 120°

4. The flooring in a regular octagonal room, Fig. 84, is laid parallel with the side AB, which is 6 feet long; what is the angle of bevel for cutting the ends along AC? Ans. 135°

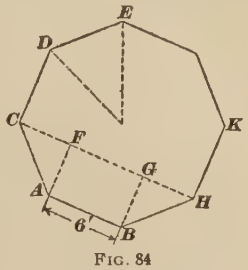

FIG. 84

5. What is the length of the flooring between CD and HK, Fig. 84, knowing that CF and AF are equal and that angle CFA is a right angle? (See Art. **60.**)
Ans. 14.48 ft., or 14 ft. $5\frac{3}{4}$ in., nearly

6. Referring to Fig. 77, if the diameter of the circle is 10 feet and DE is 8 feet 6 inches, find the distance DC. Ans. 3 ft. $6\frac{7}{8}$ in., nearly

READING DRAWINGS

87. The practical application of mensuration is the computation of lengths, areas of surfaces, or volumes of solids. The dimensions that furnish the data required are usually obtained from working drawings, or plans; therefore, before formally taking up the subject of mensuration, it is necessary to explain how drawings and plans are made, and how the dimensions are taken from them.

88. Working drawings are generally made to **scale**; that is, the lines on the drawings have a certain ratio to the corresponding dimensions of the full-size object that the drawing represents; this ratio is called the scale. For example, if the scale of a drawing is $\frac{1}{2}$ inch to 1 foot, it means that each $\frac{1}{2}$ inch on the drawing represents 1 foot on the object, and the drawing is $\frac{1}{2} \div 12 = \frac{1}{24}$ of full size.

89. To make drawings and to obtain measurements from them, divided rules called *scales* are used. These consist of strips or pieces of wood or metal 1 or 2 feet long, having the edges beveled, and on which are engraved the graduations for the different scales, as 1 inch to 1 foot, $\frac{1}{4}$ inch to 1 foot, etc. The marks are numbered in order from the end; the space next to the end being subdivided into twelfths, representing inches, and in the larger scales these spaces are again subdivided for fractions of an inch. There

are usually two scales marked on each edge, and in one the divisions are twice as long as in the other, and are numbered from the other end.

To measure a line on a drawing, place the scale with the proper edge along the line, and move it until one of the foot-marks is opposite one end of the line, as at *b*, Fig. 85, while the other end of the line is at the *0* mark on the scale, or

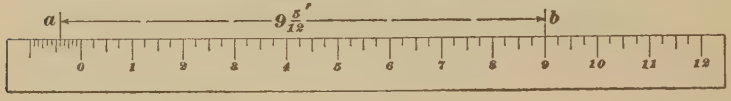

FIG. 85

opposite one of the small graduations, as at *a*. Then read the number of large divisions from the *0* mark to the far end of the line, and also the number of small divisions from *0* to the near end of the line. Thus, in Fig. 85, the line *a b* extends over nine large spaces and five small ones, or five-twelfths of a large one, so that the length of the line represented by *a b* is 9 feet 5 inches.

90. Working drawings usually have the principal dimensions marked on them, as shown in Fig. 86, which represents a side and an end view of a cast-iron lintel. The length is 7 feet 6 inches, as indicated by the broken line *c d* extending between *e c* and *f d*, which are drawn perpendicular to *e f*

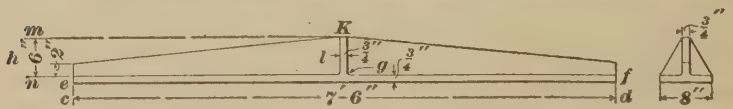

FIG. 86

at the ends *e* and *f*. The arrowheads indicate the ends of the 7′ 6″ space. At *h* is shown that the height from *g* to *K* is 6 inches, being the distance between the parallel lines *m K* and *n g*. At *l* are shown two arrowheads pointed toward each other; these are used where the space is too small to insert the arrowheads and figures in the ordinary manner. In this case, it means that the thickness of the rib is $\frac{3}{4}$ inch, as shown by the near-by figures.

Having read or scaled all the necessary dimensions from the drawing, the proper formulas or rules may be

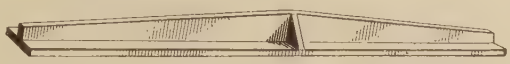

FIG. 87

applied to obtain the required length, area, or contents. Fig. 87 is a perspective view of the lintel just described.

CONVERSION TABLES

91. The **conversion tables** are useful in changing inches or fractions of an inch to decimals of a foot or inch, and vice versa:

TABLE I

INCHES TO DECIMALS OF A FOOT

Inches or Fractions	Decimal of Foot	Approximate Decimal	Inches	Decimal of Foot	Approximate Decimal
$\frac{1}{16}$	.0052	.005	3	.2500	.25
$\frac{1}{8}$	.0104	.010	4	.3333	.33
$\frac{1}{4}$	.0208	.020	5	.4167	.42
$\frac{3}{8}$	.0313	.030	6	.5000	.50
$\frac{1}{2}$	.0417	.040	7	.5833	.58
$\frac{5}{8}$	.0521	.050	8	.6667	.67
$\frac{3}{4}$	.0625	.060	9	.7500	.75
$\frac{7}{8}$	.0729	.070	10	.8333	.83
1	.0833	.080	11	.9167	.92
2	.1667	.170	12	1.0000	1.00

EXAMPLE 1.—Express 7 feet $8\frac{7}{8}$ inches in feet and a decimal of a foot.

SOLUTION.—From Table I, 8 in. = .67 ft., and $\frac{7}{8}$ in. = .07 ft.; hence, 7 ft. $8\frac{7}{8}$ in. = 7 + .67 + .07 = 7.74 ft. Ans.

EXAMPLE 2.—Express 18.19 feet in feet and inches.

SOLUTION.—By consulting the column of approximate decimals, Table I, it will be observed that .19 ft. is greater than .17 ft., or 2 in.,

and is less than .25 ft., or 3 in.; .19 being .02 greater than .17, .19 ft. equals 2 in. and .02 ft. more. According to the table, .02 ft. = $\frac{1}{4}$ in.; hence, .19 ft. = $2\frac{1}{4}$ in., and 18.19 ft. = 18 ft. $2\frac{1}{4}$ in. Ans.

Table II is used in a similar manner. Thus, 9.26 in. = $9\frac{1}{4}$ in., approximately.

TABLE II

FRACTIONS OF AN INCH TO DECIMALS OF AN INCH

Fraction	Exact Decimal	Approximate Decimal	Fraction	Exact Decimal	Approximate Decimal
$\frac{1}{32}$	.03125	.03	$\frac{9}{16}$	.5625	.56
$\frac{1}{16}$	.06250	.06	$\frac{5}{8}$	.6250	.63
$\frac{1}{8}$	.12500	.13	$\frac{11}{16}$	.6875	.69
$\frac{3}{16}$	.18750	.19	$\frac{3}{4}$	.7500	.75
$\frac{1}{4}$	.25000	.25	$\frac{13}{16}$	.8125	.81
$\frac{5}{16}$	.31250	.31	$\frac{7}{8}$	.8750	.88
$\frac{3}{8}$	.37500	.38	$\frac{15}{16}$	.9375	.94
$\frac{7}{16}$	.43750	.44	1	1.0000	1.00
$\frac{1}{2}$	.50000	.50			

MENSURATION OF PLANE SURFACES

92. The **area** of a figure means the extent of the surface included within the bounding lines of the figure, and is expressed by the number of unit squares it will contain.

A **unit square** is the square having the unit for its side. For example, if the unit is 1 inch, the unit square is the square whose side measures 1 inch in length, and the area will be expressed by the number of square inches that the surface contains. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area would be the number of square feet that the surface contains, etc. The square that measures 1 inch on a side is called a **square inch**, and the one that measures 1 foot on a side is called a **square foot**. Square inch and square foot are abbreviated to sq. in. and sq. ft., or are indicated by $\square''$ and $\square'$.

THE QUADRILATERAL

93. A **parallelogram** is a quadrilateral whose opposite sides are parallel. There are four kinds of parallelograms: the *square*, the *rectangle*, the *rhombus*, and the *rhomboid*.

FIG. 88

94. A **rectangle**, Fig. 88, is a parallelogram whose angles are all right angles.

95. A **square**, Fig. 89, is a rectangle, all of whose sides are equal.

FIG. 89

FIG. 90

96. A **rhomboid**, Fig. 90, is a parallelogram whose opposite sides only are equal, and whose angles are not right angles.

97. A **rhombus**, Fig. 91, is a parallelogram having equal sides, and whose angles are not right angles.

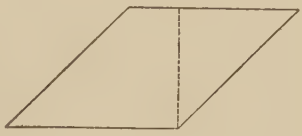
FIG. 91

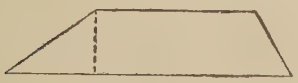
FIG. 92

98. A **trapezoid**, Fig. 92, is a quadrilateral that has only two of its sides parallel.

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99. A **trapezium**, Fig. 93, is a quadrilateral having no two sides parallel.

100. The **altitude** of a parallelogram, or of a trapezoid, is the perpendicular distance between the parallel sides

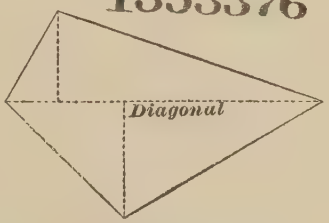


FIG. 93

101. A **diagonal** is a straight line drawn from the vertex of any angle of a quadrilateral to the vertex of the angle opposite; a diagonal divides the quadrilateral into two triangles. A diagonal divides a parallelogram into two *equal* and *similar* triangles.

102. The area of any parallelogram may be found by the following rule:

Rule.—*To find the area of any parallelogram, multiply the base by the altitude.*

Let b = base;
h = altitude;
A = area.

Then, $A = b\,h$

EXAMPLE 1.—What is the area of the rectangle $A\,B\,C\,D$, Fig. 94?

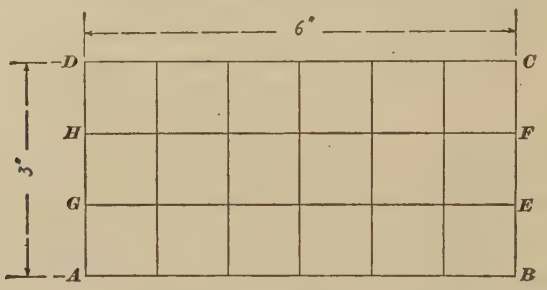


FIG. 94

SOLUTION.—The area of the rectangle equals the product of the base, which is 6 inches, and the height, which is 3 inches. Hence, the area is

$6 \times 3 = 18$ sq. in. Ans.

NOTE.—By dividing Fig. 94 into squares as shown, it is seen that, if the figure is drawn full size, it will require 18 square inches to cover it.

EXAMPLE 2.—A lot is 4 rods wide and 8 rods long: (*a*) How many square feet does it contain? (*b*) What part of an acre is it?

SOLUTION.—(*a*) Reducing rods to feet, 4 rd. = 66 ft., and 8 rd. = 132 ft. Either side may be taken as the base. Applying the formula,

$A = b\,h = 66 \times 132 = 8{,}712$ sq. ft. Ans.

(*b*) As there are 43,560 sq. ft. in an acre, 8,712 sq. ft. = $8{,}712 \div 43{,}560 = .2 = \frac{1}{5}$ A. Ans.

103. The area of a trapezoid may be found as follows:

Rule.—*To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.*

Let a and b = lengths of parallel sides;
h = altitude.

Then, $A = h\left(\frac{a+b}{2}\right)$

EXAMPLE.—How many square feet of floor space are there in a five-story warehouse having the dimensions given in Fig. 95?

SOLUTION.—As either of the parallel sides may be called a, let it be the shorter; then, applying the formula,

$$A = h\left(\frac{a+b}{2}\right) = 27.6\left(\frac{49.5+54.3}{2}\right) = 27.6 \times 51.9 = 1{,}432.44 \text{ sq. ft.}$$

Therefore, for five stories, there are

1,432.44 sq. ft. × 5 = 7,162.2 sq. ft. of floor space. Ans.

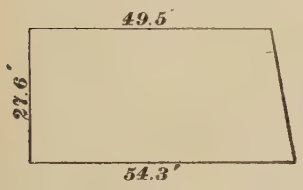
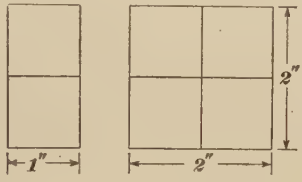


FIG. 95



(a) (b)

FIG. 96

104. It should be observed that *2 square inches* does not mean the same as *2 inches square; 3 square inches* the same as *3 inches square*, etc. Fig. 96 (*a*) shows a rectangle 2 inches by 1 inch, and its area is 2 square inches. Fig. 96 (*b*) shows a square 2 inches by 2 inches; that is, a figure 2 inches square, and its area is 4 square inches.

THE TRIANGLE

105. The following rule may be employed in finding the area of a triangle:

Rule.—*To find the area of a triangle, multiply the base by the altitude and divide the product by 2.*

Let b = base;
h = altitude;
A = area.

Then, $$A = \frac{bh}{2}$$

EXAMPLE.—How many square feet of 1-inch boards will be needed for the two gables of a house having a half-pitch roof and a width of 24 feet?

SOLUTION.—In half-pitch roofs, the rise equals one-half the span; hence, $h = \frac{1}{2}$ of 24 = 12 ft., and b = 24 ft. Applying the formula, $A = \frac{1}{2}bh = \frac{1}{2} \times 24 \times 12 = 144$ sq. ft. for each gable, or 144 × 2 = 288 sq. ft. of boards for the two gables. Ans.

106. If the triangle is a right triangle, one of the short sides may be taken as the base, and the other short side as the altitude; hence, *the area of a right triangle is equal to one-half the product of the two short sides.*

107. The area of any triangle may be found, when the length of each side is known, by means of the following formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}$$

in which a, b, c = lengths of sides;
s = half sum of lengths;
A = area of triangle.

EXAMPLE.—What is the area of a triangular plot of ground, the sides of the plot being 60, 55, and 35 feet, respectively?

SOLUTION.—It does not matter which side is called a, b, or c. Letting $a = 60$, $b = 55$, and $c = 35$,

$$s = \frac{60+55+35}{2} = 75$$
$$s - a = 75 - 60 = 15$$
$$s - b = 75 - 55 = 20$$
$$s - c = 75 - 35 = 40$$

Then,

$$A = \sqrt{75 \times 15 \times 20 \times 40} = \sqrt{900{,}000} = 948.68 \text{ sq. ft.} \quad \textbf{Ans.}$$

POLYGONS

108. To find the area of any figure bounded by straight lines the following rule may be employed:

Rule.—*To find the area of any figure bounded by straight lines, divide the figure into triangles, parallelograms and trapezoids, and find the area of each. The sum of these partial areas will be the area of the figure.*

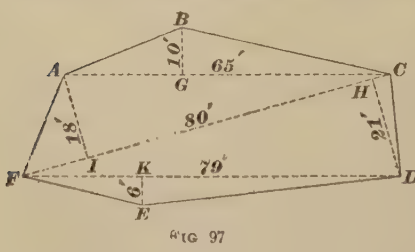


FIG. 97

EXAMPLE.—A lot has the dimensions given in Fig. 97. How many square feet does it contain?

SOLUTION.—The distances marked on Fig. 97 being measured, the partial areas are found to be:

$$ABC = \frac{65 \times 10}{2} = 325 \text{ sq. ft.}$$

$$FAC = \frac{80 \times 18}{2} = 720 \text{ sq. ft.}$$

$$FCD = \frac{80 \times 21}{2} = 840 \text{ sq. ft.}$$

$$FDE = \frac{79 \times 6}{2} = 237 \text{ sq. ft.}$$

The area of the lot is therefore the sum of these, or 2,122 sq. ft. Ans.

109. A regular polygon may be divided into as many equal triangles as the figure has sides by drawing lines from the center of the polygon to its angles. The area of each triangle is equal to one-half of the product of a side and the perpendicular distance from the center to a side. The entire area of the polygon is therefore obtained by multiplying the area of one of these triangles by the number of sides. Or, the area of a regular polygon is found by multiplying one-half of the perimeter of the figure by the perpendicular distance from the center to a side.

EXAMPLE.—What is the area of an octagonal room, each of whose sides is 9 feet, when the distance from the center of the room to a side is 10 feet $10\frac{1}{2}$ inches?

SOLUTION.—The perimeter equals $8 \times 9 = 72$ ft.; 10 ft. $10\frac{1}{2}$ in. = 10.87 ft.

$$\text{Area} = \frac{10.87 \times 72}{2} = 391.32 \text{ sq. ft.} \quad \text{Ans.}$$

THE CIRCLE

110. The ratio of the circumference of a circle to its diameter is an incommensurable number, and is usually denoted by the Greek letter π (pronounced *pi*). The approximate value of the ratio, correct to four decimal places, is 3.1416; hence, approximately, $\pi = 3.1416$. For off-hand calculations, the ratio is frequently taken as $3\frac{1}{7}$; that is, the circumference is $3\frac{1}{7}$ times the diameter.

111. The following rules are used to find the circumference and the diameter of a circle:

Rule I.—*To find the circumference of a circle, multiply the diameter by 3.1416.*

Rule II.—*To find the diameter of a circle, divide the circumference by 3.1416.*

Or,
$$c = \pi d = 2\pi r \qquad (1)$$
$$d = \frac{c}{\pi} \qquad (2)$$
$$r = \frac{c}{2\pi} \qquad (3)$$

in which c = circumference;
d = diameter;
r = radius.

EXAMPLE.—A wheelwright wishes to cut a length of tire iron long enough to go around a $4\frac{1}{2}$-foot wagon wheel; how long should he cut it, allowing $1\frac{1}{2}$ inches for welding?

SOLUTION.—Here $d = 4\frac{1}{2}$ ft. Applying formula **1**,
$c = \pi d = 3.1416 \times 4\frac{1}{2} = 14.14$ ft. $= 14$ ft. $1\frac{3}{4}$ in., nearly
Adding the $1\frac{1}{2}$ in., the length required is 14 ft. $3\frac{1}{4}$ in. Ans.

112. As the circle is divided into 360°, and the length of the circumference is $2\pi r$, the length of 1° is $\frac{2\pi r}{360}$; or, dividing both terms by 2π ($= 6.2832$), this reduces to $\frac{r}{57.3}$, very closely. If the angle is 57.3°, the length of the arc is equal to the radius; this angle is called a **radian.**

113. The length of an arc of a circle may be found by the following rule:

Rule.—*To find the length of an arc of a circle, multiply the number of degrees in the arc by the radius, and divide by 57.3, or by 57.296 if greater precision is required.*

Let l = length of arc;
n = number of degrees in arc;
r = radius of arc.

Then,
$$l = \frac{r \times n}{57.3}$$

If the angle contains degrees, minutes, and seconds, reduce them to degrees and decimals of a degree, thus:

$$37° 30' 15'' = 37° + \tfrac{30}{60}° + \tfrac{15}{3600}°$$
$$= 37° + .5° + .004° = 37.504°$$

EXAMPLE.—In Fig. 98 is shown a segmental stone arch having a radius of 7 feet 1 inch = 85 inches. The angle $A\,O\,B$ is 50°. If there are thirteen ring stones in the arch, what will be the width, as $E\,F$, of each stone?

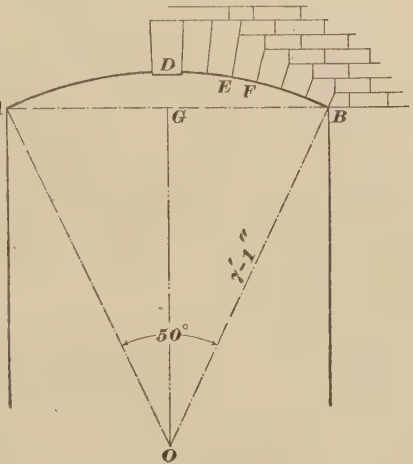


FIG. 98

SOLUTION.—Here $A\,D\,B = l$; $r = 85$ in., and $n = 50°$. Applying the formula,

$$l = \frac{r\,n}{57.3} = \frac{85 \times 50}{57.3} = 74.17 \text{ in.}$$

Dividing by the number of stones,

$74.17 \div 13 = 5.70$ in.
$= 5\frac{11}{16}$ in., nearly. Ans.

114. When only the chord $A\,B$, Fig. 99, of an arc and the height, or rise, $C\,D$ of the segment are known, the following approximate method gives good results. $A\,C$, the chord of half the arc, has the value

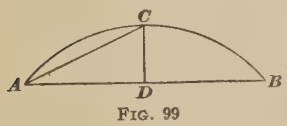


FIG. 99

$$AC = \sqrt{\overline{AD}^2 + \overline{CD}^2} = \sqrt{\left(\frac{AB}{2}\right)^2 + \overline{CD}^2}$$

Then, to find the length of the arc:

Rule.—*From eight times the chord of one-half the arc, subtract the chord of the whole arc and divide the remainder by 3.*

That is, $$\text{arc } ACB = \frac{8 \times AC - AB}{3}$$

Let c = chord of whole arc;
h = height of segment;
l = length of arc.

Then, $$AC = \sqrt{\frac{c^2}{4} + h^2} = \frac{1}{2}\sqrt{c^2 + 4h^2}$$

and $$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}$$

Either the rule or the formula may be used, whichever is the more convenient.

This method gives the length of an arc less than one-sixth of the circumference correct to four figures, and it gives the length of an arc less than one-third of the circumference correct to three figures.

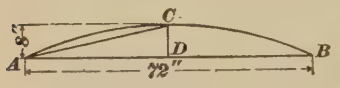


FIG. 100

EXAMPLE.—Find the length of the arc ACB, Fig. 100.

SOLUTION.— AD, which is one-half of AB, $= \frac{1}{2} \times 72 = 36$ in., and $DC = 8$ in.

$$AC = \sqrt{\overline{AD}^2 + \overline{DC}^2} = \sqrt{36^2 + 8^2} = 36.88 \text{ in.}$$

Hence, length of arc ACB is

$$\frac{8 \times AC - AB}{3} = \frac{8 \times 36.88 - 72}{3} = 74.35 \text{ in.} \quad \text{Ans.}$$

115. When the results that may be obtained by the method given in the preceding article are not considered sufficiently accurate, the length of one-half the arc may be found in a similar manner and this multiplied by 2. The following example will fully illustrate the method of procedure:

EXAMPLE.—The segmental arch, Fig. 101, has a span of 60 inches and a rise of 25 inches; what is the length of its intrados ACB?

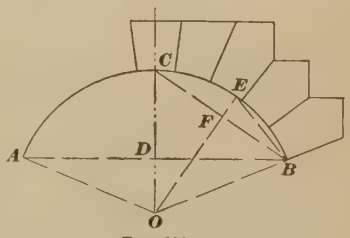


FIG. 101

SOLUTION.—Let O represent the center of the arch, and let F be the middle point of the chord CB.

$$\overline{CB}^2 = \overline{CD}^2 + \overline{DB}^2 = 25^2 + (\tfrac{60}{2})^2 = 1{,}525$$

$$CB = \sqrt{1{,}525} = 39.051 \text{ in.}$$

$$CF \text{ or } FB = CB \div 2 = 39.051 \div 2 = 19.526 \text{ in.}$$

According to Art. **81**, the radius

$$CO = \frac{\overline{CB}^2}{2\,CD} = \frac{1{,}525}{2 \times 25} = 30.5 \text{ in.}$$

In the right triangle CFO,

$$OF = \sqrt{\overline{OC}^2 - \overline{CF}^2} = \sqrt{30.5^2 - 19.526^2} = 23.431 \text{ in.}$$

$$FE = OE - OF = 30.5 - 23.431 = 7.069 \text{ in.}$$

In the right triangle EFB,

$$EB = \sqrt{\overline{EF}^2 + \overline{FB}^2} = \sqrt{7.069^2 + 19.526^2} = 20.766 \text{ in.}$$

According to the rule of Art. **114**,

$$\text{arc } CEB = \frac{8\,EB - CB}{3} = \frac{8 \times 20.766 - 39.051}{3} = 42.359 \text{ in.}$$

The whole arc ACB is therefore

$$2 \times 42.359 = 84.72 \text{ in.} \quad \text{Ans.}$$

Applying the rule in Art. **114** directly,

$$\text{arc } ACB = \frac{8 \times CB - AB}{3} = \frac{8 \times 39.051 - 60}{3} = 84.13 \text{ in.} \quad \text{Ans.}$$

116. The following rule is employed to find the area of a circle:

Rule.—*To find the area of a circle, square the diameter and multiply by .7854; or, square the radius and multiply by 3.1416.*

Let d = diameter of circle;
r = radius of circle;
A = area of circle.

Then,
$$A = \tfrac{1}{4}\pi d^2 = .7854\,d^2 \qquad (1)$$
$$A = \pi r^2 = 3.1416\,r^2 \qquad (2)$$

EXAMPLE 1.—What is the area of a circle whose radius is 14 inches?

SOLUTION.—Applying formula **2**,

$$A = 3.1416 \times 14^2 = 3.1416 \times 196 = 615.75 \text{ sq. in.} \quad \text{Ans.}$$

EXAMPLE 2.—The contract for a brick warehouse provides that all openings over 4 feet wide shall be deducted; what must be deducted for a semicircular arch 4 feet 8 inches in diameter?

SOLUTION.—Here $d = 4$ ft. 8 in. $= 4\frac{2}{3}$ ft. Applying formula **1**,

$$A = .7854\,d^2 = .7854 \times (4\tfrac{2}{3})^2 = .7854 \times 21.78 = 17.11 \text{ sq. ft.}$$

The area of the half circle is

$$17.11 \div 2 = 8.55 \text{ sq. ft., practically } 8\tfrac{1}{2} \text{ sq. ft.} \quad \text{Ans.}$$

117. When the area of a circle is given, the diameter may be found by applying the following rule:

Rule.—*To find the diameter of a circle, the area being given, divide the area by .7854 and extract the square root of the quotient.*

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{4A}{\pi}}$$

EXAMPLE.—To supply a certain quantity of water, it is necessary to have a pipe with an area of 28 square inches; what will be the diameter of the pipe?

SOLUTION.—Applying the formula,

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{28}{.7854}} = \sqrt{35.65} = 5.97 \text{ in.}$$

As this is almost 6 in., a pipe of that diameter would be chosen. Ans.

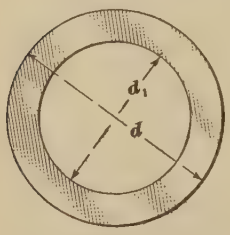


FIG. 102

118. The area of a flat circular ring, Fig. 102, may be found as follows:

Rule.—*To find the area of a flat circular ring, subtract the area of the smaller circle from that of the larger.*

Let d = outside diameter;
d_1 = inside diameter;
A = area of ring.

Then, $A = .7854\,d^2 - .7854\,d_1^2 = .7854\,(d^2 - d_1^2)$

EXAMPLE.—What is the sectional area of a brick stack whose external and internal diameters are 6 feet 6 inches and 4 feet, respectively?

SOLUTION.—Here $d = 6.5$ ft., and $d_1 = 4$ ft. Applying the formula,
$A = .7854 \times (6.5^2 - 4^2) = .7854 \times 26.25 = 20.62$ sq. ft. Ans.

If one diameter and the area of the ring are known, the other diameter may be found as follows: Compute the area of the circle whose diameter is given and add to or subtract from this result the area of the ring. The sum or difference thus found will be the area of the circle whose diameter is required.

EXAMPLES FOR PRACTICE

(*See note at bottom of page 18*)

1. Find the area of a square whose side is 5 feet 9 inches.
Ans. 33.062 sq. ft.

2. Find the area of a rhombus whose side is 12.5 feet, and whose height is 9.25 feet. Ans. 115.62 sq. ft.

3. How many square feet are there in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end?
Ans. 15 sq. ft.

4. Find the distance around the outside of a waterwheel whose outside diameter is 22 feet 8 inches. **Ans. 71.21 ft.**

5. The wheel of a carriage is observed to turn 375 times in going from a certain place to another; the diameter of the wheel is 3.5 feet; what is the distance between the two places? Ans. 4,123.4 ft.

6. A circular column measures 45.5 inches around the outside; what is its diameter? Ans. 14.48 in.

7. A belt covers an arc of 50° on a pulley whose diameter is 5 feet; what length of the belt is in contact with the pulley? Ans. 2.18 ft.

8. Find the area of a triangle whose three sides are 13, 14, and 15 feet. Ans. 84 sq. ft.

9. Find the area of a right triangle whose hypotenuse is 50 feet and one of whose sides is 40 feet. Ans. 600 sq. ft.

10. The parallel sides of a trapezoid are 321.51 and 214.24 feet, and the perpendicular distance between them is 171.16 feet; what is the area of the trapezoid? Ans. 45,849 sq. ft.

11. One side of a room is 16 feet long; if the floor contains 240 square feet, what is the width of the room? Ans. 15 ft.

12. (*a*) How many squares (100 sq. ft.) of shingling, and (*b*) how many bundles of shingles, 250 to the bundle, each shingle being 6 inches wide and laid 5 inches to the weather, will be required for the gable shown in Fig. 103, adding 5 per cent. for waste?

Ans. { (*a*) $2\frac{1}{4}$ squares
(*b*) $4\frac{1}{2}+$ bundles

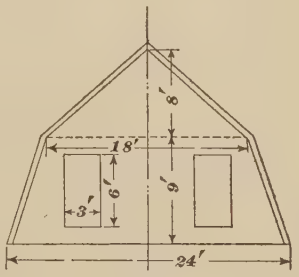


FIG. 103

SUGGESTION.—Gable consists of trapezoid and triangle. Find exposed area of each shingle and number required per square of 14,400 square inches.

13. The cable of a suspension bridge measures 40 inches in circumference; find: (*a*) the diameter of the cable; (*b*) the area of the cross-section.

Ans. { (*a*) 12.73 in.
(*b*) 127.32 sq. in.

14. Referring to Fig. 99, if $AB = 40$ inches and $CD = 7\frac{1}{2}$ inches, find: (*a*) the length of the arc ACB; (*b*) the radius of the arc.

Ans. { (*a*) 43.63 in.
(*b*) 30.42 in.

GEOMETRY AND MENSURATION
(PART 2)

PLANE SURFACES AND AREAS
(Continued)

SECTORS AND SEGMENTS OF CIRCLES

1. The area of a sector of a circle may be found by the following rule:

Rule.—*To find the area of a sector of a circle, divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

Let n = number of degrees in arc;
A = area of circle;
r = radius of circle;
a = area of sector.

Then, $$a = \frac{nA}{360} = .0087267\ nr^2$$

EXAMPLE.—A circular window 4 feet in diameter is divided by radial ribs into twelve equal sections: what is the area of each space?

SOLUTION.—Here $n = \frac{360^\circ}{12} = 30^\circ$. Applying the formula,

$$a = \frac{nA}{360} = \frac{30 \times .7854 \times 4^2}{360} = 1.05 \text{ sq. ft. Ans.}$$

Or. $a = .0087267\ nr^2 = .0087267 \times 30 \times 4 = 1.05$ sq. ft. Ans.

2. The area may also be found as follows:

Rule.—*To find the area of a sector, multiply one-half of the length of the arc by the radius.*

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Let l = length of arc;
r = radius of arc;
a = area of sector.

Then, $$a = \tfrac{1}{2} l r$$

EXAMPLE.—In the sector $OACB$, Fig. 1, the radius ($OA = OC$) of the arc is 6 inches, and the length of the chord AB is 7 inches; what is the area of the sector $OACB$?

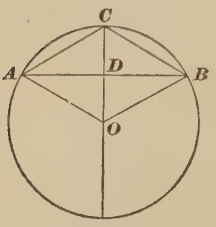


FIG. 1

SOLUTION.—In order to find the area, it will be necessary to first find the length of the arc, by applying the rule given in *Geometry and Mensuration*, Part 1; but before applying this rule, the height CD of the segment will have to be found. Since $AD = \frac{1}{2} AB = \frac{1}{2} \times 7 = 3.5$,

$$OD = \sqrt{\overline{OA}^2 - \overline{AD}^2} = \sqrt{6^2 - 3.5^2} = 4.87 \text{ in.}$$

Then, $$CD = OC - OD = 6 - 4.87 = 1.13 \text{ in.}$$

$$CB = \sqrt{\overline{CD}^2 + \overline{DB}^2} = \sqrt{1.13^2 + 3.5^2} = 3.68 \text{ in.;}$$

and $$\text{arc } ACB = \frac{8 \times CB - AB}{3} = \frac{8 \times 3.68 - 7}{3} = 7.48 \text{ in.}$$

Now applying the formula just given,

$$a = \tfrac{1}{2} l r = \tfrac{1}{2} \times 7.48 \times 6 = 22.44 \text{ sq. in.} \quad \text{Ans.}$$

3. The area of a segment of a circle may be found, approximately, as follows:

RULE.—*To two-thirds of the product of the height of the segment and the chord add the quotient obtained by dividing the cube of the height of the segment by twice the chord.*

Let c = chord;
h = height of segment;
A = area of segment.

Then, $$A = \tfrac{2}{3} c h + \frac{h^3}{2c}$$

This rule will give results correct to three figures when the height of segment does not exceed the length of one-fourth of the chord, and the error will be less than 1 per cent. in finding the area of a semicircle. The rule is therefore sufficiently accurate for most ordinary calculations, as in estimating quantities and costs.

EXAMPLE.—What is the area of the upper pane of glass in the window shown in Fig. 2?

SOLUTION.—The glass may be divided into a rectangle 36 in. × 3 ft. 6 in. and a segment of a circle, the length of its chord being 3 ft. 6 in. and the height of segment 6.75 in. 3 ft. 6 in. = 42 in. The area of the rectangle is, therefore, 42 × 36 = 1,512 sq. in. Referring to the segment, c = 3 ft. 6 in. = 42 in., h = 6.75 in. Substituting in the formula,

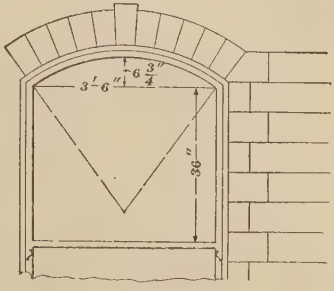


FIG. 2

$$A = \tfrac{2}{3} \times 42 \times 6.75 + \frac{6.75^3}{2 \times 42}$$

$$= 189 + \frac{307.54}{84} = 189 + 3.66$$

$$= 192.66 \text{ sq. in.}$$

The entire area of the glass is

$$1{,}512 + 192.66 = 1{,}704.66 \text{ sq. in.} \quad \text{Ans.}$$

4. The exact area of a segment of a circle may be found by the following rule:

Rule.—*To find the area of a segment of a circle, find the area of the sector of which the segment is a part, and from this area subtract the area of the triangle formed by drawing radii to the extremities of the chord of the segment. The result is the area of the segment.*

EXAMPLE.—What is the area of the segment ACB, Fig. 1?

SOLUTION.—The area of the segment is equal to the difference between the areas of the sector $OACB$ and the triangle OAB. The area of $OACB$ has been found, in the solution to the example in Art. 2, to be 22.44 sq. in. Area of triangle OAB is

$$\tfrac{1}{2} \times OD \times AB = \tfrac{1}{2} \times 4.87 \times 7 = 17.05 \text{ sq. in.}$$

Area of segment ACB is, therefore,

$$22.44 - 17.05 = 5.39 \text{ sq. in.} \quad \text{Ans.}$$

THE ELLIPSE

5. An **ellipse** is a plane figure bounded by a curved line, to any point of which the sum of the distances from two fixed points within, called the *foci*, is equal to the sum of the distances from the foci to any other point on the curve.

In Fig. 3, let A and B be the foci, and let C and D be any two points on the perimeter. Then, according to the definition just stated,

$$AC + CB = AD + DB$$

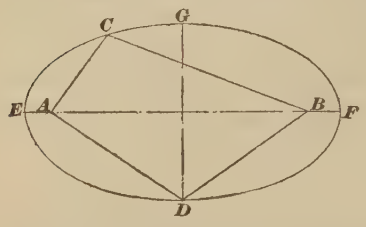


FIG. 3

Each of these sums is also equal to the major axis EF. The long diameter EF is called the *major axis;* the short diameter GD, the *minor axis.* The foci may be located from D or G as a center, and radius $DA = DB = \frac{1}{2}EF$.

6. There is no simple and exact method of finding the periphery (perimeter) of an ellipse. The following formula gives values very nearly exact:

Let C = periphery;
a = half the major axis;
b = half the minor axis;

$$D = \frac{a - b}{a + b}.$$

Then,
$$C = \pi (a + b) \frac{64 - 3D^4}{64 - 16D^2}$$

EXAMPLE.—What is the periphery of an ellipse whose axes are 12 inches and 8 inches long?

SOLUTION.—In this example, $a = 6, b = 4, D = \frac{6 - 4}{6 + 4} = \frac{2}{10} = .2.$

Then,
$$C = \pi (6 + 4) \frac{64 - 3(.2)^4}{64 - 16(.2)^2}$$

The only difficulty in simplifying this expression will be in reducing the fraction $\frac{64 - 3(.2)^4}{64 - 16(.2)^2}$ to its simplest form.

The numerator is simplified by raising .2 to the fourth power and subtracting three times the power from 64. Thus,

$$.2^4 = .2 \times .2 \times .2 \times .2 = .0016;\ 3 \times .0016 = .0048$$

Then, $64 - .0048 = 63.9952$. The denominator is simplified by multiplying the second power of .2 by 16 and subtracting the product from 64. Thus,

$$.2^2 = .2 \times .2 = .04;\ 16 \times .04 = .64;\ 64 - .64 = 63.36$$

The fraction therefore equals $\frac{63.9952}{63.36} = 1.010$. Consequently,

$$C = 3.1416 \times 10 \times 1.010 = 31.73 \text{ in.} \quad \text{Ans.}$$

7. The following rule gives the method of finding the area of an ellipse:

Rule.—*To find the area of an ellipse, multiply the product of its two diameters by .7854.*

Let A = area of ellipse;
D = long diameter;
d = short diameter.

Then, $A = \frac{1}{4}\pi d D = .7854\, d D$

EXAMPLE.—What is the area of the elliptic top of a table whose long and short diameters are 4 and 3 feet, respectively?

SOLUTION.—Here $D = 4$ ft. and $d = 3$ ft. Applying the formula,

$A = .7854\, D d = .7854 \times 4 \times 3$
$= 9.42$ sq. ft. Ans.

AREA OF ANY PLANE FIGURE

8. The area of any plane figure may be found as follows:

Rule.—*To find the area of any plane figure, divide it into triangles, quadrilaterals, circles, or parts of circles, and ellipses; find the area of each part separately and add the partial areas.*

FIG. 4

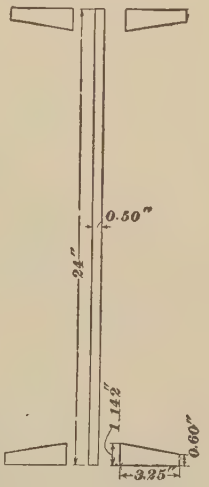


FIG. 5

EXAMPLE 1.—Fig. 4 shows the cross-section of an **I** beam; find the area of this section, neglecting the rounding of the corners.

SOLUTION.—This section can be divided into a rectangle and four equal trapezoids, as shown in Fig. 5. The area of the rectangle is $24 \times .50 = 12$ sq. in. The area of each of the trapezoids is

$$\tfrac{1}{2}(1.142 + .60) \times 3.25 = (\tfrac{1}{2} \times 1.742 \times 3.25) \text{ sq. in.}$$

Therefore, the sum of the areas of the four trapezoids is

$$4 \times \tfrac{1}{2} \times 1.742 \times 3.25 = 1.742 \times 6.5 = 11.323 \text{ sq. in.}$$

Hence, the area of the whole section is

$$(12 + 11.323) \text{ sq. in.} = 23.323 \text{ sq. in.} \quad \text{Ans.}$$

EXAMPLE 2.—Find the sectional area of the half arch and abutment shown in Fig. 6.

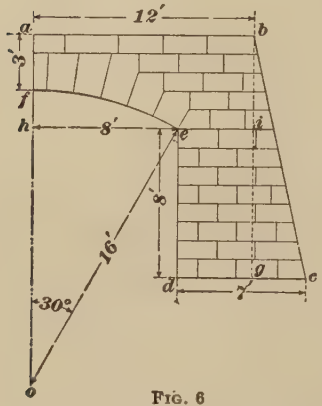


FIG. 6

SOLUTION.—Divide the section into parts, as $abih$, $eigd$, and bgc. Find

$$fh = fo - ho = 16 - \sqrt{16^2 - 8^2} = 2.14 \text{ ft.};$$

then, area of $abih$ is

$$12 \times 5.14 = 61.68 \text{ sq. ft.};$$

area of $eigd$ is

$$8 \times 4 = 32 \text{ sq. ft.};$$

area of bgc is

$$\frac{3 \times 13.14}{2} = 19.71 \text{ sq. ft.}$$

Adding these partial areas, the sum is 113.39 sq. ft. The area of sector foe is

$$.0087267\, nr^2 = .0087267 \times 30 \times 16^2 = 67.02 \text{ sq. ft.}$$

The area of triangle heo is $\frac{1}{2}(ho \times eh)$;

$$ho = 16 - 2.14 = 13.86 \text{ ft.}$$

The area heo is $\dfrac{13.86 \times 8}{2} = 55.44$ sq. ft.

The area of segment feh is $67.02 - 55.44 = 11.58$ sq. ft. Deducting 11.58 sq. ft. from 113.39 sq. ft., the net area is 101.81 sq. ft. Ans.

AREAS OF IRREGULAR FIGURES

9. If the figure is so irregular in shape that the ordinary rules cannot be easily applied, the area may be obtained as follows: Let Fig. 7 represent a drawing of such an area. Suppose it to be divided by equidistant parallel lines into strips, as ab, bc, etc.; then each of these small areas may be considered a trapezoid, and the

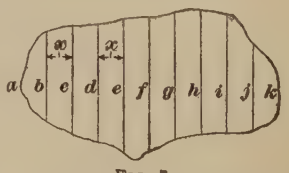


FIG. 7

whole area is the sum of the partial areas. From these considerations the following rule is obtained:

Rule.—*To find the area of any irregular figure, divide the figure into any number of strips by equidistant parallel lines. Measure the length of the lines; add together one-half the lengths of the end lines and the lengths of the remaining lines, and multiply this sum by the distance between the lines.*

Let a = length of one end line;
k = length of other end line;
b, c, etc. = lengths of intermediate lines;
x = distance between parallels;
A = area.

Then, $$A = \left(\frac{a+k}{2} + b + c + \text{etc.} + j\right)x$$

The shorter the distance x is, the more accurate will be the result.

EXAMPLE.—Fig. 8 is a map of a small island whose area is to be determined. The parallel lines are 10 feet apart, and are of the lengths marked thereon. Required, the area of the island.

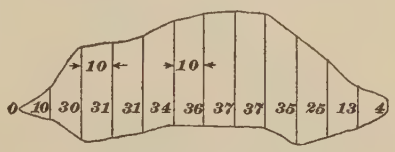


FIG. 8

SOLUTION.—The extreme left end of the island is a point; hence, its length, or a, is 0; k is 4 ft. Applying the formula,

$$A = \left(\frac{a+k}{2} + b + \text{etc.}\right)x$$
$$= \left(\frac{0+4}{2} + 10 + 30 + 31 + 31 + 34 + 36 + 37 + 37 + 35 + 25 + 13\right) \times 10$$
$$= 321 \times 10 = 3{,}210 \text{ sq. ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE*

1. How many laths 4 feet long and $1\frac{1}{2}$ inches wide, spaced $\frac{1}{4}$ inch apart, transversely, will be required for the walls of a room 14 ft. 6 in. × 10 ft. 9 in. × 8 ft. high, deducting two doors 36 in. × 7 ft. and two windows 2 ft. 6 in. × 5 ft. 6 in.? Add 5 per cent. for waste.

Ans. 601 laths

SUGGESTION.—Number of laths required for each square foot is
144 ÷ [(width of 1 lath + 1 space) × length of lath in inches]

*As stated in *Geometry and Mensuration*, Part 1, two decimal places, as a general rule, are to be retained when decimal quantities occur in the solutions of examples.

2. A hollow circular cast-iron column, having an external diameter of 8 inches, is to carry a load of 47,120 pounds, and must not be loaded more than 4,000 pounds per square inch; what will be the thickness of the metal ring? Ans. $\frac{1}{2}$ in.

SUGGESTION.—Area of ring is total load ÷ load per square inch. Area of smaller circle is area of 8-inch circle − area of ring; from this find smaller diameter.

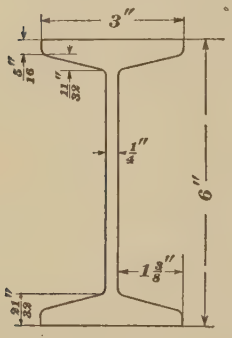


FIG. 9

3. If brick masonry will safely carry 10 tons per square foot, what should be the size of the square base of the column given in the previous example? (Tons of 2,000 lb.)

Ans. 1.54 ft., or 18½ in., square

SUGGESTION.—Side of square is square root of area required. The weight of column is neglected.

4. A bar of iron 1 yard long and 1 inch square weighs 10 pounds; what will be the weight per yard of the I beam shown in Fig. 9? Disregard curved edges and corners. All work in fractions. Ans. 41⅔ lb. per yd.

SUGGESTION.—Note solution of example 1, Art. 8.

5. What is the length of the intrados $ABECD$ of the three-centered arch shown in Fig. 10? Ans. 16.76 ft.

SUGGESTION.—Find lengths of arcs BEC, AB, and CD separately. $AB = CD$.

6. The space above the line AD, in Fig. 10, is to be covered with grille-work, costing $1.50 per square foot: what will be the total cost? Ans. $65.02

SUGGESTION.—Area required = area of three sectors less the area of triangle GOF. $GF = GO = FO$, because angles are all equal; then lengths of three sides are known, to find the area of triangle.

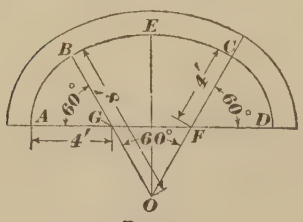


FIG. 10

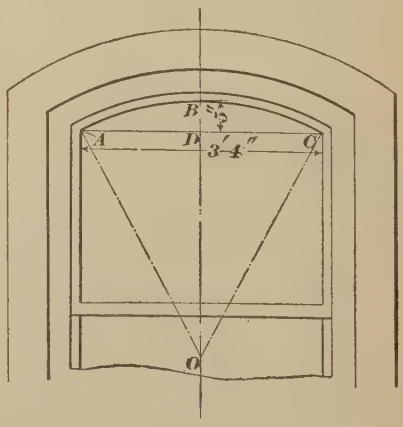


FIG. 11

7. The segment ABC of the window shown in Fig. 11 is to be filled with ornamental glass; what is the cost at $6 per square foot? Ans. $5.64

8. What is the area of an elliptic window whose axes are 7 feet 4 inches and 4 feet? Ans. 23.03 sq. ft.

9. Pressed brick are estimated at seven per square foot of external area. How many will be required for a building 20 ft. × 40 ft. and 20 feet high; with two chimneys 20 in. × 28 in. and 8 feet high; and one chimney 20 in. × 28 in. and 12 feet high? Deduct six window openings, 3 ft. × 6 ft. 6 in.; one double window, two openings, each 2 ft. 8 in. × 9 ft.; one front door, 4 ft. 8 in. × 9 ft.; one rear door, 3 ft. × 9 ft.; six window openings, 3 ft. × 6 ft.; and one double window, two openings, 2 ft. 8 in. × 6 ft. Ans. 15,750 bricks

10. (*a*) How many squares (100 square feet) of slating are there on a house 25 ft. × 40 ft. with a half-pitch roof, supposing the roof to extend beyond the walls 1 foot (along the roof) at eaves and ends? (*b*) How many slates, 14 inches wide and exposed $8\frac{1}{2}$ inches, will be required? Ans. { (*a*) 15.69 squares; (*b*) 1,899 slates

SUGGESTION.—Rise of roof is one-half width. Find exposed area of each slate and number required per square of 14,400 square inches.

USE OF THE TABLE OF POLYGONS

10. The areas of regular polygons whose names are given in column *B*, of Table I, and whose sides are each 1 or unity are found in column *C* opposite the corresponding name.

For example, the area of a regular decagon each of whose sides is 1 foot is 7.6942 square feet. Since the areas of similar figures are proportional to the squares of their corresponding sides, the areas of regular polygons mentioned in the table may be obtained as follows:

Rule I.—*To find the area of a regular polygon, multiply the square of the side by the number in column C standing opposite the name of the polygon in column B.*

EXAMPLE 1.—What is the area of a regular twelve-sided figure, each of its sides being 9 feet long?

SOLUTION.—The figure is a dodecagon, and the corresponding number in column *C* is 11.1962.

$$\text{Area} = 9 \times 9 \times 11.1962 = 906.89 \text{ sq. ft.} \quad \text{Ans.}$$

Rule II.—*To find the side of a regular polygon when the area is known, divide the area by the number in column C corresponding to the name of the polygon and extract the square root of the result.*

TABLE I

TABLE OF REGULAR POLYGONS

Number of Sides	Name	Area, When Side = 1	Radius of Circumscribed Circle, When Side = 1	Side, When Radius of Circumscribed Circle = 1	Radius of Inscribed Circle, When Side = 1	Side, When Radius of Inscribed Circle = 1	Radius of Circumscribed Circle, When Radius of Inscribed Circle = 1	Angle at Each Vertex	One-Half of Angle at Vertex
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
3	Triangle	.4330	.5773	1.7321	.2887	3.4641	2	60°	30°
4	Square	1	.7071	1.4142	.5	2	1.4142	90°	45°
5	Pentagon	1.7205	.8507	1.1756	.6882	1.4531	1.2361	108°	54°
6	Hexagon	2.5981	1	1	.8660	1.1547	1.1547	120°	60°
7	Heptagon	3.6339	1.1524	.8678	1.0383	.9632	1.1099	128° 34′	64° 17′
8	Octagon	4.8284	1.3066	.7654	1.2071	.8284	1.0824	135°	67° 30′
9	Nonagon	6.1818	1.4619	.6840	1.3737	.7279	1.0642	140°	70°
10	Decagon	7.6942	1.6180	.6180	1.5388	.6498	1.0515	144°	72°
11	Undecagon	9.3656	1.7747	.5635	1.7028	.5873	1.0422	147° 16′	73° 38′
12	Dodecagon	11.1962	1.9319	.5176	1.8660	.5359	1.0353	150°	75°

EXAMPLE 2.—What is the side of a regular octagon that contains an area of 2,000 square feet?

SOLUTION.—The number in column *C* to be used for an octagon is 4.8284. $2{,}000 \div 4.8284 = 414.22$. The square root of 414.22 is 20.35. Hence, 20.35 ft. is the length of the side. Ans.

11. In column *D* are given the radii of the circles that may be described about the regular polygons named, when each side is of unit length. In column *E* are given the lengths of sides of inscribed polygons when the radius of the circle is 1, etc. The use of the various columns will be made clear by the following rules and illustrative examples:

Rule I.—*To find the radius of the circumscribed circle when the side of a regular polygon is known, multiply the side by the number in column D opposite the name of the polygon.*

EXAMPLE 1.—It is required to lay out a regular seven-sided figure, each side being 2 feet; what is the radius of the circle whose circumference will pass through each corner of the figure?

SOLUTION.—The number in column *D* opposite heptagon is 1.1524. Radius of circumscribed circle is

$$2 \times 1.1524 = 2.3048, \text{ or, } 2.30 \text{ ft.} \quad \text{Ans.}$$

Rule II.—*To find the length of the side of a regular polygon that may be inscribed in a circle of known radius, multiply the known radius by the number in column E opposite the name of the polygon.*

EXAMPLE 2.—What is the length of the side of an octagon that may be inscribed in a circle 12 feet in diameter?

SOLUTION.—Radius is $12 \div 2 = 6$ ft. Number in column *E* opposite octagon is .7654. Length of side is

$$6 \times .7654 = 4.5924, \text{ or } 4.59 \text{ ft.} \quad \text{Ans.}$$

Rule III.—*To find the radius of the inscribed circle when the side of a regular polygon is known, multiply the side by the number in column F opposite the name of the polygon.*

EXAMPLE 3.—It is desired to determine the perpendicular distance from the center of a regular nonagon to each of its sides, when the length of each side is 5 feet.

SOLUTION.—The perpendicular distance from the center of the polygon to its sides is equal to the radius of the circle that may be inscribed in the polygon. The number in column *F* opposite nonagon is 1.3737; hence, perpendicular distance is

$$5 \times 1.3737 = 6.8685, \text{ or } 6.87 \text{ ft.} \quad \text{Ans.}$$

Rule IV.—*To find the side of a regular polygon when the radius of the inscribed circle is known, multiply the radius by the corresponding number in column G.*

Rule V.—*To find the radius of the circle circumscribed about a regular polygon when the radius of the inscribed circle is known, multiply the given radius by the corresponding number in column H.*

12. In column *I* are found the angles at each vertex of a regular polygon formed by the adjacent sides.

In column *J* are found the values of one-half the angle at each vertex, or the angles that should be used in making the bevel cuts for moldings or material laid parallel to the sides, and jointed.

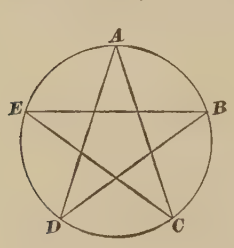


FIG. 12

EXAMPLES FOR PRACTICE

1. A five-pointed star is inscribed in the circle *A B C D E*, Fig. 12. The diameter of the circle is 4 feet 6 inches. What is the distance between consecutive points of the star measured on a straight line? Ans. 2.65 ft.

2. Each side of an octagonal-shaped room is 12 feet 3 inches; find the number of square feet of flooring required for the room, making no allowance for waste. Ans. 724.56 sq. ft.

3. What is the weight for each foot in length of a hexagonal wrought-iron rod, the distance between parallel sides of the rod being 2 inches and the weight of a cubic inch of wrought iron being .28 pound? Ans. 11.64 lb.

SUGGESTION.—To find volume, multiply area of end, in square inches, by 12. See rule in Art. 25.

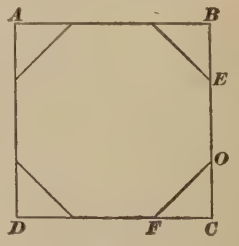


FIG. 13

4. In Fig. 13, *A B C D* represents the end section of a piece of timber 6 inches square. (*a*) What will be the side of a regular octagon that may be formed from the stick, if the corners are cut off as shown? (*b*) What is the distance *B E*?

Ans. { (*a*) 2.49 in. (*b*) 1.76 in.

MENSURATION OF SOLIDS

DEFINITIONS

13. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides that enclose it are called the **faces,** and their intersections are called **edges.**

14. The **entire surface** of a solid is the area of the whole outside of the solid, including the ends.

The **convex,** or **lateral, surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

15. The **volume** of a solid is the space included between its bounding surfaces, and is expressed by the number of times it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubical contents** is frequently used.

THE PRISM AND CYLINDER

16. A **prism** is a solid whose ends are equal parallel polygons and whose sides are parallelograms. Prisms take their names from their bases. Thus, a triangular prism is one whose bases are triangles; a pentagonal prism is one whose bases are pentagons, etc.

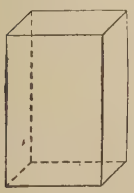


FIG. 14

17. A **parallelopiped** is a prism whose bases (ends) are parallelograms. See Fig. 14.

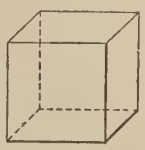


FIG. 15

18. A **cube** is a parallelopiped whose faces and ends are squares. See Fig. 15.

19. The cube whose edges are equal to the unit of length is taken as the unit of volume when finding the

volume of a solid. Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges measure 1 inch, or 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

20. A **cylinder** is a solid whose ends are parallel and equal *curved* figures, and whose cross-section is uniform throughout its length. A *circular cylinder* is one any section of which, perpendicular to the axis, is a circle. See Fig. 16. Unless otherwise expressed, the word *cylinder* always means a *circular cylinder.*

FIG. 16

21. A *right prism*, or *right cylinder*, is one whose center line (axis) is perpendicular to its base.

22. The *altitude* of a prism or cylinder is the perpendicular distance between its two ends.

23. The area of the convex surface of any right prism or right cylinder may be found according to the following rule:

Rule.—*To find the area of the convex surface of any right prism or right cylinder, multiply the perimeter of the base by the altitude.*

Let p = perimeter of base;
h = altitude;
S = convex surface.

Then, $S = ph$

To find the entire area, add the areas of the two ends to the convex area.

EXAMPLE 1.—How many square feet of 1-inch sheathing will be needed for a house 20 ft. × 28 ft. × 18 ft. high, including gables under a half-pitch roof, making no allowance for openings?

SOLUTION.—The perimeter p of the house is

$$20 \times 2 + 28 \times 2 = 96 \text{ ft.}$$

$h = 18$ ft.; hence, $S = 96 \times 18 = 1{,}728$ sq. ft. Adding two gables, 20 ft. wide at base and (since the roof is half pitch) 10 ft. high, or an area of 200 sq. ft., the total is 1,928 sq. ft. Ans.

EXAMPLE 2.—A circular iron chimney, 7 feet in diameter and 85 feet high, is to be covered outside with a preservative paint; what will it cost to paint the chimney at 20 cents per square yard?

SOLUTION.—The perimeter p of the base is $\pi d = 21.99$ ft., and $h = 85$ ft. The surface of the chimney is $21.99 \times 85 = 1{,}869.2$ sq. ft., or 207.7 sq. yd. The cost of painting at 20 ct. per sq. yd. is therefore

$$207.7 \times \$.20 = \$41.54. \quad \text{Ans.}$$

EXAMPLE 3.—How many square feet of zinc will be required to line a refrigerator having interior dimensions of 4 ft. × 6 ft. × 8 ft. high?

SOLUTION.—The area of the sides is

$$(4 \times 2 + 6 \times 2) \times 8 = 160 \text{ sq. ft.}$$

Adding for floor and ceiling

$$(4 \times 6) \times 2 = 48 \text{ sq. ft.; the total is } 208 \text{ sq. ft.} \quad \textbf{Ans.}$$

EXAMPLE 4.—What will be the cost, at 35 cents per square yard, of plastering the walls and ceiling of an octagonal room, having sides 5 feet long and 11 feet high, the distance between the parallel sides being 12 feet?

SOLUTION.—The area of the sides is $8\ (5 \times 11) = 440$ sq. ft., the area of the ceiling is one-half the perimeter times one-half the distance between the parallel sides, or $\frac{8 \times 5 \times 6}{2} = 120$ sq. ft.; the total is 560 sq. ft., or 62.22 sq. yd. At 35 ct. per sq. yd., the cost is \$21.78. Ans.

24. The volume of a rectangular solid may be found as follows:

Rule.—*To find the volume of a rectangular solid, multiply together the length, breadth, and height.*

Let l = length;
b = breadth;
h = height;
V = volume.

Then, $V = lbh$

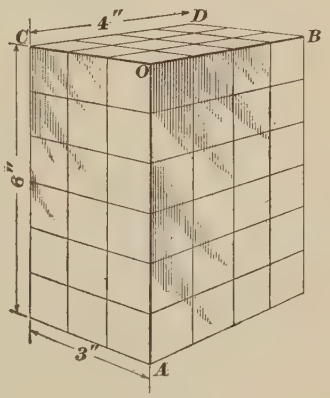


FIG. 17

EXAMPLE.—Find the volume of the rectangular block shown in Fig. 17.

SOLUTION.—The length is 4 in., the breadth is 3 in., and the height is 6 in. The volume therefore equals

$$4 \times 3 \times 6 = 72 \text{ cu. in.} \quad \text{Ans.}$$

The block may be divided into six pieces, each 4 in. long, 3 in. wide, and 1 in. thick. Each of these pieces may be cut up into twelve 1-in. cubes. There will, therefore, be $6 \times 12 = 72$ one-inch cubes, or 72 cu. in. in the entire block.

25. The volume of a right prism, or cylinder, may be found by the following rule:

Rule.—*To find the volume of a right prism, or cylinder, multiply the area of the base by the altitude.*

Let a = area of base;
h = altitude;
V = volume.

Then, $V = a\,h$

If the given prism is a cube, the three dimensions are all equal, and the volume is equal to the cube of one of the edges.

If the volume and area of the base are given, the altitude is $\frac{V}{a}$. If the cylinder, or prism, is hollow, the base is equal to the area of the cross-section of the material.

EXAMPLE 1.—If brickwork averages 21 bricks per cubic foot, how many bricks will there be in a pier 18 inches square and 6 feet high?

SOLUTION.—The sectional area or a is $1.5 \times 1.5 = 2.25$ sq. ft.; $h = 6$ ft.; hence,

$$V = a\,h = 2.25 \times 6 = 13.5 \text{ cu. ft.}$$

At 21 bricks per cubic foot, the number of bricks in the pier is

$$13.5 \times 21 = 283.5. \quad \text{Ans.}$$

EXAMPLE 2.—If cast iron weighs .26 pound per cubic inch, what length of sash weight will be necessary to balance a window weighing 8 pounds, the diameter of the cylindrical weight being $1\frac{1}{2}$ inches?

SOLUTION.—As there are two weights, the weight of each is 4 lb. The number of cubic inches required, or V, is $4 \div .26 = 15.38$. The area a of a $1\frac{1}{2}$-in. circle is 1.77 sq. in.; hence, h, the length, is

$$\frac{V}{a} = \frac{15.38}{1.77} = 8.69 \text{ in., practically } 8\tfrac{3}{4} \text{ in.} \quad \text{Ans.}$$

BOARD MEASURE

26. In measuring sawed lumber, the unit of measure is the **board foot,** and it is equal to the contents of a board 1 foot square and 1 inch thick. A board foot is, therefore, equal to one-twelfth of a cubic foot. Boards less than 1 inch thick are usually reckoned as though the thickness were 1 inch.

Rule I.—*To find the number of board feet in a piece of lumber, multiply together the length, in feet, the width, in feet, and the thickness, in inches.* Or,

Rule II.—*Multiply together the length, in feet, the width, in inches, and the thickness, in inches, and divide the product by 12.*

EXAMPLE 1.—How many board feet are there in a plank 10 feet 6 inches long, 15 inches wide, and 3 inches thick?

SOLUTION.—Length = 10 ft. 6 in. = $10\frac{6}{12}$ ft. = $10\frac{1}{2}$ ft. = 10.5 ft., width = $\frac{15}{12}$ ft. = 1.25 ft. The number of feet equals

$$10.5 \times 1.25 \times 3 = 39.375, \text{ or } 39.4$$

The result is, therefore, 39.4 ft. B. M. (board measure). Ans.

EXAMPLE 2.—Find the number of board feet in twenty pieces of siding $\frac{1}{2}$ inch thick, $5\frac{1}{2}$ inches wide, and 10 feet 9 inches long.

SOLUTION.—Since the thickness is less than 1 in., the boards are considered to be 1 in. thick when finding the amount of material in them. The calculation can be more easily performed in this case by using the second rule. Length = $10\frac{9}{12}$ ft. = 10.75 ft., width = $5\frac{1}{2}$ in. = 5.5 in.

$$\frac{20 \times 10.75 \times 5.5 \times 1}{12} = \frac{1{,}182.5}{12} = 98.54 \text{ ft. B. M.}$$

Ans.

THE PYRAMID AND CONE

27. A **pyramid** is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex.** See Fig. 18.

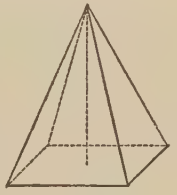


FIG. 18

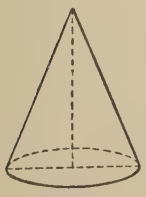


FIG. 19

28. A **cone** is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the *vertex.* See Fig. 19.

29. A **right pyramid** or **cone** is one whose axis is perpendicular to the base.

30. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

31. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

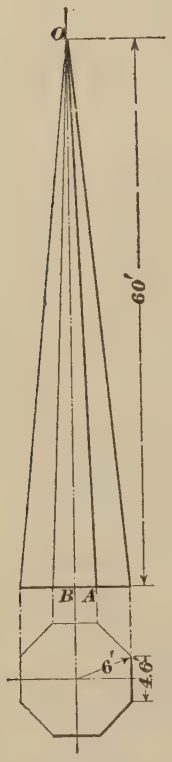


FIG. 20

32. The convex area of a right pyramid or cone may be found as follows:

Rule.—*To find the convex area of a right pyramid or cone, multiply the perimeter of the base by one-half of the slant height.*

Let p = perimeter of base;
s = slant height;
A = convex area.

Then, $$A = \frac{p\,s}{2}.$$

EXAMPLE.—An octagonal church steeple is 60 feet high. The outer radius of the base (see Fig. 20) is 6 feet, and the length of one of the sides of the base is 4.6 feet. Compute the convex surface.

SOLUTION.—To find the slant height, the length OA along the hips is first found. This is the hypotenuse of a right triangle of which the altitude is the height of the tower, or 60 ft.; and the base is the radius, or 6 ft. Hence, OA is $\sqrt{60^2 + 6^2} = 60.3$ ft. Now, in the triangle OAB, OA is 60.3 ft. and AB is $4.6 \div 2 = 2.3$ ft. The slant height OB is, therefore, $\sqrt{60.3^2 - 2.3^2} = 60.3$ ft., nearly. The perimeter of the base is $4.6 \times 8 = 36.8$ ft. Applying the formula,

$$A = \frac{p\,s}{2} = \frac{36.8 \times 60.3}{2} = 1{,}109.5 \text{ sq. ft.} \quad \text{Ans.}$$

33. The following rule states how to find the volume of a pyramid or cone:

Rule.—*To find the volume of a pyramid or cone, multiply the area of the base by one-third of the altitude.*

Let a = area of base;
h = altitude;
V = volume.

Then, $$V = \frac{a\,h}{3}$$

EXAMPLE 1.—The granite capstone of a monument is a square pyramid having a base 4 feet square and an altitude of 6 feet. If granite weighs 170 pounds per cubic foot, what will be the freight charges on the piece at 20 cents per 100 pounds?

SOLUTION.—The area a of the base is $4 \times 4 = 16$ sq. ft., and the altitude h is 6 ft. Applying the formula,

$$V = \frac{a\,h}{3} = \frac{16 \times 6}{3} = 32 \text{ cu. ft.}$$

At 170 lb. per cu. ft., the weight is $32 \times 170 = 5{,}440$ lb. $= 54.4$ cwt. The freight is, therefore,

$$54.4 \times \$.20 = \$10.88. \quad \text{Ans.}$$

EXAMPLE 2.—If in the preceding example the capstone were conical, having a base 4 feet in diameter and a height of 6 feet, how much less would it weigh than the pyramidal stone?

SOLUTION.—The area of the base is $.7854\,d^2 = 12.57$ sq. ft. Using the formula, the volume is

$$\frac{12.57 \times 6}{3} = 25.14 \text{ cu. ft.}$$

and the weight at 170 lb. per cu. ft. is 4,274 lb., nearly. As the pyramidal capstone weighs 5,440 lb., the difference in weight is

$$5{,}440 - 4{,}274 = 1{,}166 \text{ lb.} \quad \text{Ans.}$$

THE FRUSTUM OF A PYRAMID OR CONE

34. If a pyramid or cone is cut by a plane parallel to the base, so as to form two parts, the lower part is called the **frustum** of the pyramid or cone. See Figs. 21 and 22.

The upper end of the frustum of a pyramid or cone is called the *upper base*, and the lower end the *lower base*. The altitude of a frustum is the perpendicular distance between the bases.

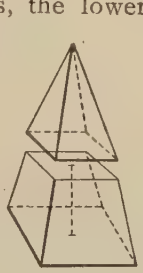


FIG. 21

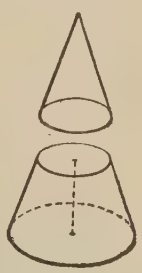


FIG. 22

35. The convex area of a frustum of a right pyramid or right cone is found as follows:

Rule.—*To find the convex area of a frustum of a right pyramid or right cone, multiply one-half the sum of the perimeters of the bases by the slant height.*

Let P = perimeter of lower base;
p = perimeter of upper base;
s = slant height;
A = convex area.

Then, $$A = \frac{(P+p)s}{2}$$

To find the entire area, add to the convex area the areas of the two bases.

EXAMPLE.—A square mansard-tower roof has the dimensions shown in Fig. 23. To compute the slate required to cover the tower the convex area is required. What is this convex area?

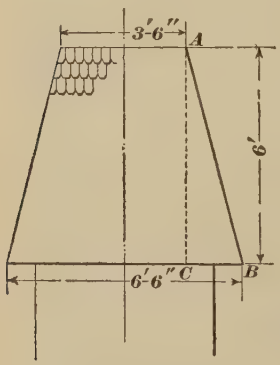


FIG. 23

SOLUTION.—The tower has the form of a frustum of a pyramid, 6 ft. 6 in. square at the bottom and 3 ft. 6 in. square at the top, 6 ft. being the altitude. 6 ft. 6 in. $= 6\frac{1}{2}$ ft. and 3 ft. 6 in. $= 3\frac{1}{2}$ ft. The perimeter of the lower base is $4 \times 6\frac{1}{2} = 26$ ft., and that of the upper base is $4 \times 3\frac{1}{2} = 14$ ft. The slant height of the frustum is the hypotenuse of the right triangle whose height is equal to the altitude of the frustum AC, or 6 ft., and whose base is equal to BC, or

$$\tfrac{1}{2}(6\tfrac{1}{2} - 3\tfrac{1}{2}) = 1\tfrac{1}{2} \text{ ft.}$$

The slant height is, therefore,

$$\sqrt{(1\tfrac{1}{2})^2 + 6^2} = 6.18 \text{ ft.}$$

Applying the formula,

$$A = \frac{(P+p)s}{2} = \frac{(26+14) \times 6.18}{2} = 123.6 \text{ sq. ft.} \quad \text{Ans.}$$

36. The following rule gives a method of finding the volume of the frustum of a pyramid or cone:

Rule.—*To find the volume of the frustum of a pyramid or cone, add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.*

Let A = area of lower base;
a = area of upper base;
h = altitude;
V = volume.

Then, $$V = \frac{h}{3}(A + a + \sqrt{A \times a})$$

EXAMPLE.—A marble monument is $2\frac{1}{2}$ feet square at the base, 1 foot square at the top, and 16 feet high; if marble weighs 160 pounds per cubic foot, what is the weight of the stone?

SOLUTION.—The area of the lower base is $2\frac{1}{2} \times 2\frac{1}{2} = 6.25$ sq. ft.; that of the upper base is 1 sq. ft. Applying the formula,

$$V = \frac{h}{3}(A + a + \sqrt{A \times a}) = \frac{16}{3}(6.25 + 1 + \sqrt{6.25 \times 1}) = 51.97 \text{ cu. ft.}$$

At 160 lb. per cu. ft., the weight is $51.97 \times 160 = 8{,}315$ lb. Ans.

37. When the bases of the frustum are square, let d and D represent the sides of the bases, respectively, and V the volume of the frustum; then,

$$V = \frac{h}{3}(d^2 + d\,D + D^2)$$

When the bases are circular, let d and D represent the respective diameters; then,

$$V = \frac{h}{3}(d^2 + d\,D + D^2) \times \frac{\pi}{4}$$

EXAMPLE.—Find the volume of a pail having the form of a frustum of a cone, the upper and lower diameters being 10 and 8 inches, respectively, and the height of pail $9\frac{1}{2}$ inches.

SOLUTION.—

$$V = \frac{9.5}{3}(8^2 + 8 \times 10 + 10^2) \times .7854$$

$$= \frac{9.5}{3} \times 244 \times .7854 = 606.85 \text{ cu. in.} \quad \text{Ans.}$$

THE WEDGE

38. A **wedge** is a solid having plane surfaces, of which the base is a parallelogram, the ends are triangles, and the sides are quadrilaterals meeting in a line parallel to the sides of the base. See Fig. 24.

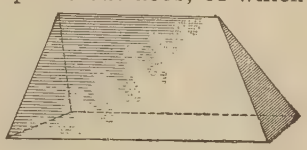


FIG. 24

39. The rule for finding the volume of a wedge is as follows:

Rule.—*To find the volume of a wedge, multiply together the width of the base, the perpendicular distance from the base to the edge, and the sum of the lengths of the three parallel edges; divide the product by 6.*

Let w = width of base;
h = perpendicular distance from base to edge;
s = sum of lengths of the three parallel edges;
V = volume.

Then,
$$V = \frac{whs}{6}$$

If the base is a rectangle, and the triangular ends are parallel, the wedge becomes a triangular prism, and the rule for prisms may be used.

EXAMPLE.—Steel weighs .28 pound per cubic inch; how heavy is a steel wedge 8 inches long, and $1\frac{1}{2}$ in. $\times$ 3 in. at the head?

SOLUTION.—Here, w is $1\frac{1}{2}$ in., h is 8 in., s is $3 + 3 + 3 = 9$ in. Then,
$$V = \frac{1.5 \times 8 \times 9}{6} = 18 \text{ cu. in.}$$

Or, as this wedge is a prism, the volume = area of end $\times$ length of edge; area of triangle is
$$\frac{1.5 \times 8}{2} = 6 \text{ sq. in.}$$
Multiplying by the length of the edge, 3 in., the volume is 18 cu. in. The weight at .28 lb. per cu. in. is
$$.28 \times 18 = 5.04 \text{ lb.} \quad \text{Ans.}$$

THE PRISMOIDAL FORMULA

40. If any solid is cut into two parts so that the surfaces thus formed are flat, such a surface is called a **plane section** of the solid.

Plane sections are divided into three classes: longitudinal sections, cross-sections, and right sections. A **longitudinal section** is any plane section taken lengthwise through the solid; any other plane section is called a **cross-section.** If a plane section of a solid is perpendicular to the center line

of the solid, the section is called a **right section.** Any longitudinal section of a cylinder is a rectangle. A right section of a cube is a square; of a cylinder or cone, a circle; an oblique cross-section of a cylinder is an ellipse.

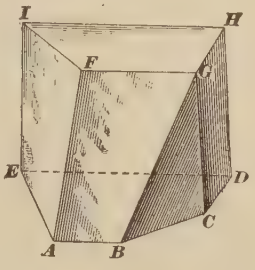


FIG. 25

41. A **prismoid** is a solid whose bases are parallel and whose faces are triangles or quadrilaterals joining the bases. Thus, the solid shown in Fig. 25 is a prismoid; its bases are the pentagon $ABCDE$ and the quadrilateral $FGHI$, which are parallel; and its faces are the triangle GBC and the quadrilaterals $GCDH$, $HDEI$, $IEAF$, and $FABG$.

42. The *altitude* of a prismoid is the perpendicular distance between the bases or parallel faces.

43. The parallel faces or bases of a prismoid are commonly called its **end sections.**

A prismoid is also defined as a solid having two parallel end faces, and composed of any combination of prisms, wedges, and pyramids, whose common altitude is the perpendicular distance between the parallel faces.

44. The **middle section** of a prismoid is the polygon formed by a plane parallel to the bases, and cutting the prismoid at equal distances from the two bases or end sections. Thus, polygon $PQRS$ is the middle section of the prismoid shown in Fig. 26.

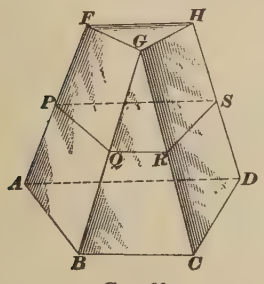


FIG. 26

45. Any dimension of the middle section of a prismoid may be taken equal to one-half the sum of the corresponding dimensions of the two end sections or bases. Thus, in Fig. 26, $PQ = \frac{1}{2}(AB + FG)$, $QR = \frac{1}{2}BC$, $RS = \frac{1}{2}(GH + CD)$, and $SP = \frac{1}{2}(HF + DA)$.

46. The area of the middle section of a prismoid may be measured directly, or calculated from its dimensions as determined from the dimensions of the end sections. It is not, in general, equal to one-half the sum of the areas of the bases.

The area of the middle section of a prism is the same as the area of either base; the area of the middle section of a wedge, when the three parallel edges are of the same length, is equal to one-half the area of the base; the area of the middle section of a pyramid is equal to one-fourth the area of the base.

47. The volume of a prismoid may be found by the following rule:

Rule.—*To find the volume of a prismoid, add together the areas of the two parallel ends, and four times the area of a parallel section midway between them; multiply the sum by one-sixth of the perpendicular distance between the parallel ends.*

Let A = area of one end;
a = area of other end;
M = area of middle section;
h = distance between ends;
V = volume.

Then, $$V = \frac{h}{6}(A + a + 4M)$$

This formula, called the *prismoidal formula*, is of very extended application and may be used to calculate the volume of a prism, cylinder, pyramid, cone, frustum of pyramid or cone, wedge, sphere, or segment of a sphere, in addition to the irregularly shaped bodies to which it is usually applied. One end area of a pyramid, cone, or wedge is 0; and the end areas of a sphere are both 0.

EXAMPLE 1.—What is the volume, in cubic yards, of a masonry pier 30 feet high, the upper and lower rectangular bases being, respectively, 7 ft. × 24 ft. and 11 ft. × 30 ft.?

SOLUTION.—The area of the upper base is $7 \times 24 = 168$ sq. ft. The area of the lower base is $11 \times 30 = 330$ sq. ft. The dimensions

of the middle section are the half sums of the corresponding dimensions of the bases; therefore, the length is

$$\frac{24+30}{2} = 27 \text{ ft.},$$

and the width is

$$\frac{7+11}{2} = 9 \text{ ft.}$$

The area of the middle section is $27 \times 9 = 243$ sq. ft. Applying the prismoidal formula, the volume is

$$V = \tfrac{30}{6}(330 + 168 + 4 \times 243) = 7{,}350 \text{ cu. ft.} = (7{,}350 \div 27) \text{ cu. yd.}$$
$$= 272\tfrac{2}{9} \text{ cu. yd.} \quad \text{Ans.}$$

EXAMPLE 2.—The upper and lower bases of the frustum of a right cone are respectively 30 inches and 14 inches in diameter, and the perpendicular distance between them is 36 inches; required, the volume of the frustum.

SOLUTION.—The diameter of the middle section is the mean, or average, of the diameters of the ends, or

$$\frac{30+14}{2} = 22 \text{ in.}$$

Area of lower base is

$$A = .7854 \times 30^2 = 706.86 \text{ sq. in.}$$

Area of upper base is

$$a = .7854 \times 14^2 = 153.94 \text{ sq. in.}$$

Area of middle section is

$$M = .7854 \times 22^2 = 380.13 \text{ sq. in.}$$

Using the prismoidal formula,

$$V = \tfrac{36}{6}(706.86 + 153.94 + 4 \times 380.13) = 14{,}287.92 \text{ cu. in.} = 8.27 \text{ cu. ft.}$$

Ans.

THE SPHERE

48. A **sphere** is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within called the center. See Fig. 27. The word *ball* is commonly used instead of sphere.

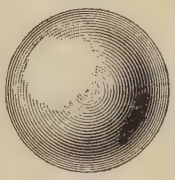


FIG. 27

49. The following rule states the method of finding the area of the surface of a sphere:

Rule.—*To find the area of the surface of a sphere, multiply the square of the diameter by 3.1416.*

Let d = diameter of sphere;
S = surface.

Then, $S = \pi d^2 = 3.1416 d^2$

EXAMPLE.—A ball on a flagstaff is 10 inches in diameter and is to be gilded; deducting 20 square inches for space covered by attachment to pole, how many books of gold leaf, each containing 25 leaves $3\frac{3}{8}$ inches square, will be required for gilding it?

SOLUTION.—Applying the formula, $S = \pi d^2 = 3.1416 \times 10 \times 10 = 314.16$ sq. in. Deducting 20 sq. in., the net surface is 294.16 sq. in. Each gold leaf has an area of $3\frac{3}{8}$ in. $\times$ $3\frac{3}{8}$ in., or 11.4 sq. in., nearly; hence, for 294.16 sq. in., there will be needed $294.16 \div 11.4 = 25.8$ leaves, or 1 book and 1 leaf. Ans.

50. The volume of a sphere may be found when the diameter is known by applying the following rule:

Rule.—*To find the volume of a sphere, multiply the cube of the diameter by .5236.*

Let d = diameter;
V = volume.

Then, $$V = \frac{\pi}{6} d^3 = .5236\, d^3$$

Since $\frac{\pi d^3}{6} = \frac{1}{6} \times \pi d^2 \times d$, the volume of a sphere is equal to one-sixth of the surface multiplied by the diameter.

EXAMPLE.—What is the weight of a cast-iron ball 6 inches in diameter, if the metal weighs .26 pound per cubic inch?

SOLUTION.—Applying the formula,

$$V = .5236\, d^3 = .5236 \times 216 = 113.1 \text{ cu. in.}$$

The weight is therefore $113.1 \times .26 = 29.41$ lb. Ans.

51. The volume of a spherical shell is equal to the difference in volume between two spheres having the outer and inner diameters of the shell.

52. The diameter of a sphere of known volume may be found by the following rule:

Rule.—*To find the diameter of a sphere of known volume, divide the volume by .5236 and extract the cube root of the quotient.*

$$d = \sqrt[3]{\frac{V}{.5236}}$$

EXAMPLE.—A cast-iron ball weighing 25 pounds is required for a certain purpose; if cast iron weighs .26 pound per cubic inch, what will be the diameter of the ball?

SOLUTION.—The volume of the ball will be $25 \div .26 = 96.1$ cu. in. Using the formula,

$$d = \sqrt[3]{\frac{V}{.5236}} = \sqrt[3]{\frac{96.1}{.5236}} = \sqrt[3]{183.5} = 5.68 \text{ in.} = 5\tfrac{11}{16} \text{ in., nearly.} \quad \text{Ans.}$$

SYMMETRICAL AND SIMILAR FIGURES

53. An **axis of symmetry** is any line so drawn that, if the part of the figure on one side of the line is folded over on this line, it will coincide exactly with the other part, point for point and line for line. Thus, in Fig. 28, if the upper semicircle is folded over on the diameter CD, it will coincide exactly with the lower semicircle; also, if the part on the right of the diameter AB is folded over on AB, it will coincide exactly with the part on the left of this line.

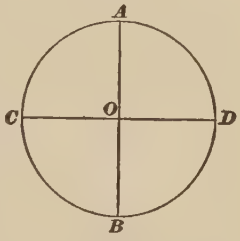


FIG. 28

It is evident from what has just been stated that a circle may have any number of axes of symmetry. In certain cases, however, a figure may be symmetrical with regard to only one axis. Thus, the isosceles triangle ABC, Fig. 29, is symmetrical with regard to the axis BD, because the part BCD would coincide with the part BAD if folded over on the line BD; but no other axis of symmetry could be drawn. A rectangle has two axes of symmetry at right angles to each other. A regular hexagon has six axes of symmetry.

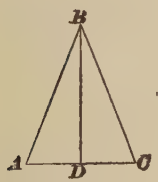


FIG. 29

54. **Similar figures** are those that are alike in form. As in the case of polygons, which have been considered, two figures, to be similar, must have their corresponding sides in proportion, and the angles of one equal to the corresponding angles of the other. Circles are, of course, similar figures.

55. *The areas of two similar figures are to each other as the squares of any one dimension.* Thus, a regular octagon whose sides are 1 inch long contains 4.828 square inches; another with sides 4 inches long contains 4^2, or 16, times 4.828 square inches = 77.25 square inches; for, let A = required area, then $A : 4.828 \text{ sq. in.} = 4^2 : 1^2$, or $A = 4.828 \times 16$ square inches.

The use of this principle often saves considerable labor in determining the areas of similar figures, as, for instance, the end areas of frustums of pyramids, cones, etc.

EXAMPLE.—Required, the volume of the frustum of a pyramid, the bases of which are equilateral triangles, one with sides 6 feet in length, the other with sides 2 feet in length. The altitude of the frustum is 8 feet.

SOLUTION.—The area of the lower triangular base may be found from the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{6+6+6}{2} = 9$$

and the factors $(s-a)$, etc. are each $9-6=3$. Hence,

$$A = \sqrt{9 \times 3 \times 3 \times 3} = 15.59 \text{ sq. ft.}$$

Since the ends are similar triangles, their areas are proportional to the squares of the sides; or, denoting the area of the upper base by a,

$$a : 15.59 = 2^2 : 6^2$$

Hence,

$$a = 15.59 \times \frac{2^2}{6^2} = 15.59 \times \tfrac{1}{9} = 1.73 \text{ sq. ft.}$$

A section midway between the bases is evidently also an equilateral triangle with sides $\frac{6+2}{2} = 4$ ft. in length; hence, its area may be obtained from the proportion

$$M : 15.59 = 4^2 : 6^2$$

or

$$M = 15.59 \times \frac{4^2}{6^2} = 6.93 \text{ sq. ft.}$$

Now, using the prismoidal formula,

$$V = \tfrac{8}{6}\,(15.59 + 1.73 + 4 \times 6.93) = 60.05 \text{ cu. ft.}$$ Ans.

56. The cubical contents (and weights) of similar solids are to each other as the cubes of any one dimension.

EXAMPLE.—If a cast-iron ball 9 inches in diameter weighs 100 pounds, what would a ball 15 inches in diameter weigh?

SOLUTION.— $100 : x = 9^3 : 15^3$, or

$$x = \frac{100 \times 3{,}375}{729} = 462.96 \text{ lb.}$$

The weight of the larger ball is, therefore, 462.96 lb. Ans.

EXAMPLES FOR PRACTICE

1. What is the weight of the cast-iron lintel shown in Fig. 30,

FIG. 30

estimating iron at .26 pound per cubic inch? Ans. 201.08 lb.

SUGGESTION.—The lintel consists of a rectangular base, a longitudinal rib, and two triangular cross-ribs.

2. Figure: (*a*) the number of feet B. M. (board measure) in the following bill of material; (*b*) the cost at $16.50 per M. (thousand) feet.

23 pieces 3″ × 10″; 18′ 6″ long
3 pieces 3″ × 10″; 15′ 0″ long
3 pieces 3″ × 10″; 12′ 0″ long
34 pieces 3″ × 6″; 14′ 0″ long
4 pieces 3″ × 6″; 6′ 0″ long
38 pieces 3″ × 4″; 5′ 6″ long
2 pieces 3″ × 6″; 10′ 6″ long
5 pieces 3″ × 8″; 18′ 6″ long
56 pieces 2″ × 4″; 9′ 9″ long
81 pieces 2″ × 4″; 9′ 3″ long
42 pieces 2″ × 4″; 1′ 2″ long

Ans. { (*a*) 3,338 ft. B. M., nearly
(*b*) $55.08

3. What is the weight of the boiler front shown in Fig. 31 if the metal is $1\frac{1}{4}$ inches thick, and cast iron weighs .26 pound per cubic inch? Ans. 1,638 lb.

SUGGESTION.—First find net area of surface; door openings consist of rectangular portion + circular segment whose chord and height are known.

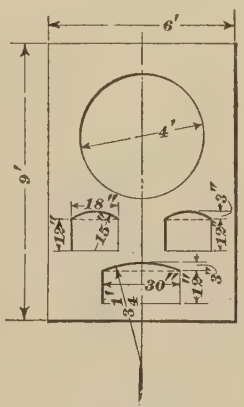


FIG. 31

4. Figure the cost of digging and lining a well 24 feet deep and $3\frac{1}{2}$ feet clear diameter, the lining to be one brick thick, laid without mortar; allow eighteen bricks per square foot of surface of interior of lining. The excavation is to be $5\frac{1}{4}$ feet in diameter. The digging is to cost $1.25 per cubic yard, and the brickwork $12 per thousand, furnished and laid. Ans. $81.04

5. Find the diameter of the balls of a 10-pound dumbbell, the cylindrical handle of which is $4\frac{1}{2}$ inches long and $1\frac{1}{4}$ inches in diameter. Cast iron weighs

.26 pound per cubic inch. Disregard the small volumes at ends of handle, which are figured twice. Ans. 3.16 in. = $3\frac{1}{8}+$ in.

SUGGESTION.—Find contents of handle; one-half of remainder is volume of each ball.

6. A window weighing 10 pounds is to be balanced by two $1\frac{1}{2}$-inch (inside diameter) lead-pipe weights. If the metal is $\frac{1}{4}$ inch thick, and lead weighs .41 pound per cubic inch, what will be the length of each piece? Ans. 8.9 in., say 9 in.

SUGGESTION.—Find number of cubic inches in each weight. Length required = volume ÷ area of ring.

7. A shop 28 ft. × 40 ft. and 18 feet high, with gables 8 feet 8 inches high, is to be built of brick. The lower walls are 12 inches, and the gables 8 inches, in thickness. Deduct twelve windows 3 ft. × 6 ft. 4 in., and four doors 4 ft. × 7 ft., all in 12-inch wall. Estimating seven bricks to each square foot of wall 4 inches thick and counting corners twice, how many thousand bricks will be required, with 5 per cent. allowance for breakage, etc.? Ans. 50.05 thousand

SUGGESTION.—Find areas and not volumes.

8. (*a*) What will be the weight of the square granite shaft shown in Fig. 32, estimating the stone at 170 pounds per cubic foot? (*b*) Allowing 4,000 pounds per square foot on the soil, what will be the size of foundation required?

Ans. { (*a*) 168,640 lb. = 84.3 T.
(*b*) 6.5 ft. square, nearly

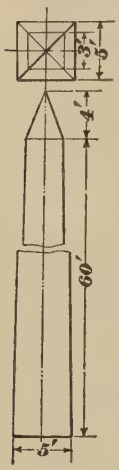


FIG. 32

QUERY.—What two solids make up the monument?

9. What is the weight per foot of the Z-bar column shown in Fig. 33, figuring steel at .283 pound per cubic inch? Neglect rounding of edges of Z bars, but figure the fillets, as at *a*. Check result by multiplying area of section by 3.4, which is the weight per

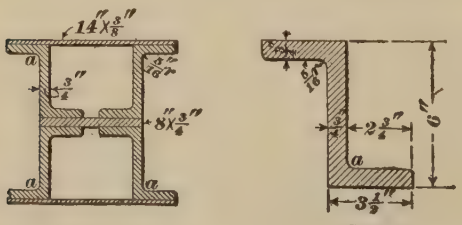
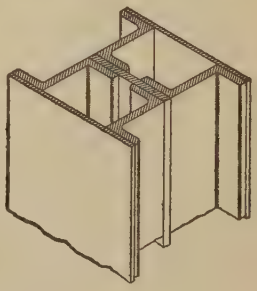


FIG. 33



foot of steel for each square inch of section. The cut on the right shows how the column is formed.

Ans. { 173.67 lb.
173.88 lb.

SUGGESTION.—Area of a fillet is the square of the radius minus one-fourth the area of a circle having the same radius.

10. Find the number of cubic yards of masonry in the foundation walls shown in Fig. 34, the wall being 18 inches wide and 9 feet high,

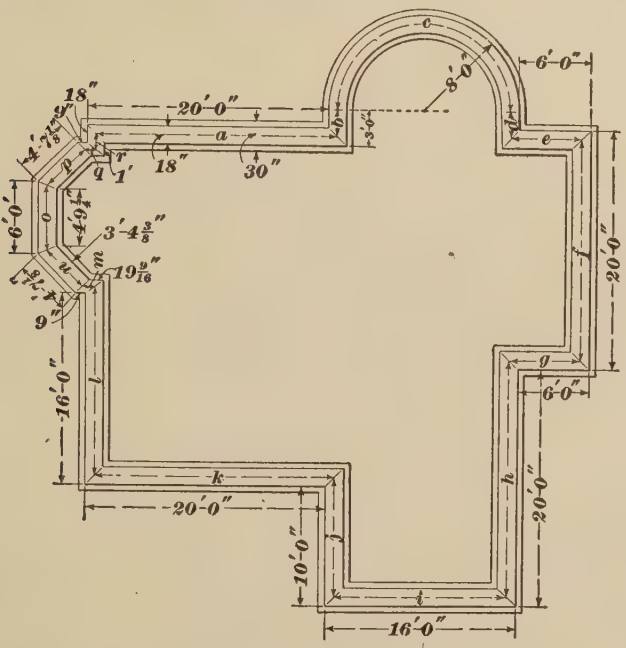


FIG. 34

and the footing course 2 feet 6 inches wide and 6 inches deep, projecting 6 inches each side of the foundation wall.

Ans. 95.73 cu. yd., nearly

SUGGESTION.—Find length of center line of wall, reducing all dimensions to feet and decimals of a foot; thus, a = 20 feet, b = 3 feet less one-half width of wall = 3 feet − 9 inches = 2 feet 3 inches, or 2.25 feet. c is a semi-circumference whose radius is 8 feet − 9 inches = 7.25 feet. f = 20 feet − width of wall. m is one-half of the sum of inside and outside measurements. $19\frac{9}{16}$ inches = 1.63 feet; 9 inches = .75 inch = (1.63 + .75) ÷ 2 = 1.19 feet. m = one-half sum of 4 feet $7\frac{1}{2}$ inches and 3 feet $4\frac{3}{8}$ inches, which is 3.98 feet. $p = n$, etc. The length of center line will be found to be 175.26 feet with a possible difference in the last figure owing to the different manner in which the quantities may be computed. The same center line applies to both footing and wall.

ELECTRIC WIRING AND BELLWORK

(PART 1)

ELEMENTS OF ELECTRICITY AND MAGNETISM

INTRODUCTION

1. **Electricity** is the name given to the cause of all electric phenomena; though the exact nature of electricity is not known, many of its effects are well known, and methods for controlling and utilizing it are well understood. Electricity is neither a form of matter, as matter is generally understood, nor is it a form of energy, although energy is required to move it, and when in motion it is capable of doing work.

Electricity can manifest its presence in various ways. For example, it may cause attractions or repulsions of some kinds of matter; it may decompose into elements some kinds of matter through which it passes, as the decomposition of water into two gases, hydrogen and oxygen; it may cause a magnetic needle to deflect; it may cause physiological effects, or shocks, to the nervous systems of all animals; or it may heat substances through which it passes.

2. Electricity may appear either to reside on the surface of bodies as a **charge,** or to flow through the substance or on the surface of a body as a **current.**

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That branch of the science which treats of charges on the surface of bodies is termed *electrostatics*, and the charges are said to be *electrostatic*, or simply *static*, *charges* of electricity.

Electrodynamics is that branch of science which treats of the action of *electric currents*.

Very little will be said in this Section about electrostatics, as an architect's work in connection with electricity will be almost wholly with electrodynamics. One or two manifestations of static electricity will, however, serve to illustrate an important point.

ELECTROSTATICS

3. Static Charges.—When a glass rod or a piece of amber is rubbed with silk or fur, the parts rubbed have the property of attracting light particles of matter, such as pieces of silk, wool, feathers, gold leaf, pith, etc., which, after momentary contact, are repelled. These attractions and repulsions are caused by a charge of static electricity residing on the rubbed surfaces. A body in this condition is said to be **electrified.**

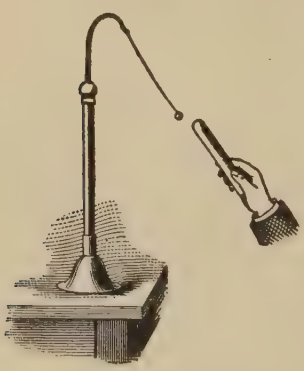

FIG. 1

An experiment for demonstrating this action is to suspend a small pith ball by a silk thread from a support or bracket, as shown in Fig. 1. This apparatus is sometimes called an *electric pendulum*. If a charge of static electricity is developed on a glass rod by rubbing it with silk, and the rod is brought near the pith ball, the ball will be attracted to the rod, but, after momentary contact, will be repelled. By this contact, the ball receives a charge of the same nature as that on the glass rod, and as long as the two bodies retain their charges, mutual repulsion will take place whenever they are brought near each other. If a stick of sealing wax, electrified by being rubbed with fur, is brought near another pith ball, the ball will fly toward the sealing wax and after

contact will be repelled. But the charges developed in these two cases are not of exactly the same character. For, the pith ball that was charged and repelled by the glass rod will be attracted by the sealing wax, and the ball that was charged and repelled by the sealing wax will be attracted by the glass rod.

4. These experiments illustrate the fact that there are two kinds of electrification. They are distinguished as positive and negative; the charge developed on glass by rubbing it with silk is called a **positive (+) charge,** and that developed on resinous bodies by rubbing them with flannel or fur is called a **negative (−) charge.** These different charges are always developed separately in equal amounts on the two bodies between which rubbing takes place. For example, when a positive charge was developed on the glass rod, an equal negative charge was developed on the silk with which the glass was rubbed; also, when a negative charge was developed on the sealing wax, an equal positive charge was developed on the fur or flannel.

MAGNETISM

5. Near the town of Magnesia, in Asia Minor, the ancients found an ore that possessed a remarkably attractive power for iron. This attractive power they named **magnetism,** and a piece of ore having this power was termed a **magnet.** The ore itself has since been named *magnetite,* and has been found to be a chemical combination of about 72 parts of iron and 28 parts of oxygen, by weight.

6. A still more remarkable discovery was made concerning this ore. It was found that when a piece of the ore was hung from a thread, it invariably swung around to such a position that one of its ends pointed north and the other south. It was also observed that the same end always pointed north. Due to this fact, small pieces of the ore so suspended were used in navigation. Ships could be steered in any direction by its aid, because the direction of the north was always shown by one end of the stone. From this fact

the name *lodestone*, meaning "leading stone," was given to the natural ore.

7. When a bar or needle of hardened steel is rubbed with a piece of lodestone, it acquires magnetic properties similar to those of the lodestone. Such bars are called **artificial magnets.**

Artificial magnets that retain their magnetism for a long time are called **permanent magnets.** The common form of artificial magnet is a bar of steel bent in the shape of a horseshoe and then hardened and magnetized. A piece of soft iron, called an **armature,** or **keeper,** is placed across the two free ends, which helps to prevent the magnet from losing its magnetism.

8. If a bar magnet is dipped into iron filings, the filings are attracted toward the two ends and adhere there in tufts, as shown in Fig. 2, while toward the center of the bar, half

FIG. 2

way between the ends, there is no such tendency. Those parts of the magnet where the attractions for iron filings are greatest are called the **poles.**

9. The **magnetic compass** consists of a magnetized steel needle, Fig. 3, resting on a fine point, so as to turn freely in a horizontal plane. When not in the vicinity of other magnets or magnetized iron, the needle will always come to rest with one end pointing toward the north and the other toward the south. The end pointing northwards, or the north-seeking pole, is invariably called the **north pole;** the opposite end is called the **south pole.** This polarity applies as well to all magnets.

FIG. 3

10. If the north pole of one magnet is brought near the south pole of another magnet, *attraction* will take place; but

if two north poles or two south poles are brought together, they will *repel* each other. In general, *like magnetic poles repel one another; unlike poles attract.*

11. The earth is a great magnet whose magnetic poles coincide nearly, but not quite, with the true geographical north and south poles. By the laws of attraction and repulsion just given, it is seen why a freely suspended magnet, therefore, will always point in a north-south direction. The north-seeking pole of a magnetic needle, or other magnet, although invariably known as a north pole, is, of course, of opposite polarity to the north pole of the earth; nevertheless, the north-seeking pole is never called a south pole. If it were only customary to do so, it would be more correct to call the north-seeking pole a south pole, or to call the earth's north pole a south pole.

12. Magnetic Field.—All magnets will attract iron or other magnetic substances through short distances. The space through which these forces act around a magnet is

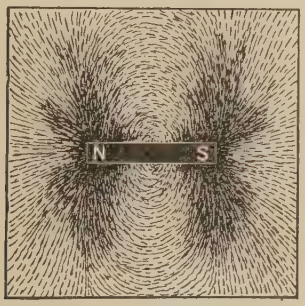

FIG. 4

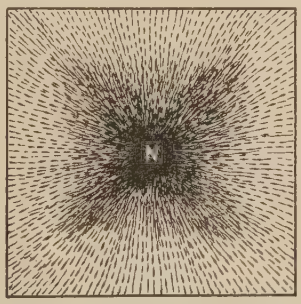

FIG. 5

called **the magnetic field.** If a sheet of paper is placed over a magnet when in a horizontal position, and iron filings are sprinkled on the paper, they will assume positions as indicated in Fig. 4. Fig. 5 shows the positions the filings would assume if the magnet were vertical and the paper were placed over one end. The filings arrange themselves in lines and indicate the directions in which the forces are acting. The letters *N*, *S* indicate north poles and south poles, respectively.

13. Electromagnets.—If a wire is wound around an iron *core*, as shown in Fig. 6, and an electric current flows through the wire around the core as indicated by the arrowheads, the core will become a magnet and will attract magnetic substances. A magnet so formed is called an **electromagnet.** Electromagnets are frequently made in the form of a horseshoe, or **U** shape, as shown in Fig. 7, where the magnet consists of two cores *M*, *M*, magnetizing coils *c*, an iron yoke *b*, and an iron armature *a*. In the illustration, one-half of the electromagnet is shown in section. The yoke is rigidly fastened to one end of each core, and in some cases the cores and yoke are made in one solid piece. When a current is flowing through the coils, the armature is held firmly to the cores by magnetic attraction; when no current is flowing, the attraction ceases.

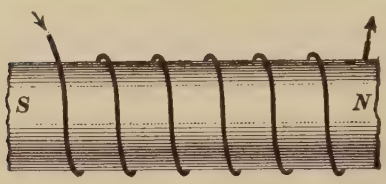

FIG. 6

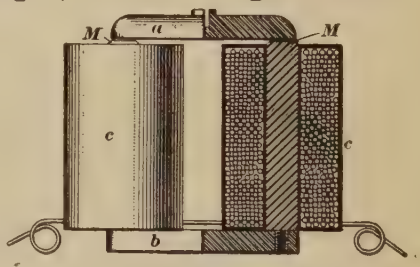

FIG. 7

ELECTRODYNAMICS

CONDUCTORS AND INSULATORS

14. In the experiment described in Art. **3,** only that part of a dry glass rod that had been rubbed was electrified; the charge did not pass to other parts of the rod. This may be proved by noting that the unrubbed parts produce neither attraction nor repulsion when brought near a pith ball. The same is true of a piece of sealing wax or resin. These bodies do not readily conduct electricity; that is, they oppose, or resist, the passage of electricity through them. Therefore, electricity can reside only as a charge on that part of their

surfaces where it is developed. Experiments show that when a metal receives a charge at any point, the electricity immediately passes, or flows, through its substance to all parts. Metals, therefore, are said to be *good conductors* of electricity. Bodies have accordingly been divided into two classes: **non-conductors,** or **insulators,** those bodies that offer a high resistance to the passage of electricity; and **conductors,** those that offer a comparatively low resistance to its passage. This distinction is not absolute, for all bodies conduct electricity to some extent, while there is no known substance that does not offer some resistance to its flow.

POTENTIAL AND ELECTROMOTIVE FORCE

15. In order to cause electricity to move, there must be created an electric pressure, just the same as to cause water to move there must be a water pressure. The study of *Electrodynamics* naturally includes that of the creation of electric pressures and the movements of electricity.

16. Electric Potential.—The term **electric potential** signifies an electrical condition of a body. Electric potential is analogous to *pressure* in gases, *head* in liquids, and *temperature* in heat.

When an electrified body *positively* charged is connected to the earth by a conductor, electricity is said to *flow from* the body *to* the earth; and, conversely, when an electrified body *negatively* charged is connected to the earth, electricity is said to *flow from* the earth *to* that body. That which determines the *direction of flow* is the relative electric *potential* of the two charges in regard to the earth.

17. It is impossible to say with certainty in which direction electricity really flows, or, in other words, to declare which of two points has the higher and which the lower electric potential, or pressure. All that can be said with certainty is that, when there is a *difference of electric potential*, or a *pressure*, an electric current tends to flow *from* one point, which is said to be at the higher potential, *to* another point, which is said to be at the lower potential.

For convenience, it has been arbitrarily assumed and universally adopted that that electric condition called *positive* is at a *higher potential* than that called *negative*, and that an electric charge, or current, flows *from* a positively *to* a negatively electrified body.

18. The zero or normal level of water is taken as that of the surface of the sea, and the normal pressure of air as that of the atmosphere at the sea level; similarly, the earth may be regarded as a reservoir of electricity of infinite quantity, and its potential may be taken as zero. All electric charges, or currents, have the tendency to reach this zero level, exactly as water on a mountain top tends to flow to the sea level; hence, it becomes necessary to insulate most electrical apparatus, otherwise the electric charge, or current, will leak away to the earth. The electric condition that is called positive is assumed to be at a higher potential than the earth, and that called negative is assumed to be at a lower potential than the earth.

19. Electromotive Force.—The term **electromotive force,** usually written E. M. F., or E., is employed to denote that which moves or tends to move electricity from one place to another. A *difference of electric potential* has practically the same meaning as electromotive force.

20. The Volt.—In order to express the value of quantities, there must be units. For example, the pressure of a liquid on the bottom of a vessel is expressed in units called pounds; the temperature of a body is expressed in units called degrees; likewise, an electric pressure, or difference of potential, or electromotive force, as it is variously called, is expressed by means of a unit called the **volt.** Thus, an electromotive force of a number of volts is spoken of in a manner somewhat analogous to the expression of a water pressure of a number of pounds. There does not need to be a movement of electricity in order to have an electromotive force any more than there needs to be a current of water through the vessel in order to have a water pressure. The volt is the practical unit of electromotive force.

METHODS OF PRODUCING ELECTROMOTIVE FORCE

21. An electromotive force may be produced, or generated, in a number of different ways, among which are the following:

1. By friction and electrostatic induction.

2. By dipping the ends of two strips of dissimilar materials into a liquid that has a greater tendency to chemically act on one material than on the other. The electromotive force is due to chemical action, or affinity, between the strips and the liquid.

3. By moving a conductor across a magnetic field.

4. By the contact of two dissimilar materials, especially when the junction *d*, Fig. 8, is at a different temperature than the two ends *a* and *b*, which are supposed to be at the same temperature. An electromotive force produced in this manner may be called a *thermoelectromotive force*. This electromotive force is seldom used and will not be referred to again.

FIG. 8

The experiments described in Art. 3 illustrate the production of an electromotive force by friction. The charges accumulated on large moving belts when driving machinery are another illustration of frictional electricity. These are classified under the heading Electrostatics.

22. The Voltaic Cell.—If two dissimilar materials, as zinc and copper or zinc and carbon, are partially or wholly immersed in saline or acidulated water and are brought into electric contact outside the liquid, chemical action will occur between the two materials and the liquid, and an electric current will flow through the liquid and the contact. Fig. 9 shows such an arrangement, known as a **voltaic**, or **galvanic, cell** from its discoverers, two Italian physicists, Volta and Galvani. A vessel *A* contains the liquid, which may be a dilute solution of sulphuric acid; a copper plate *C* and a zinc plate *Z* are partially immersed in the liquid and are connected

outside by twisting their wire terminals together. The arrows show the direction of the current flow. The liquid is called the **electrolyte** and it must be such that it will act chemically more readily on one of the two materials than on the other, in order to produce any difference in potential or electromotive force between the two.

The two dissimilar materials, when spoken of separately, are called **voltaic elements;** when taken collectively, they are known as a **voltaic couple.** Voltaic cells are frequently called primary cells.

In any voltaic cell, the element on which the chemical action is the more vigorous is known as the *positive element*, and the other as the *negative element*. While the submerged portion of the zinc plate *Z* in Fig. 9 is the positive element, as indicated by the plus sign (+), the exposed end, where the connecting wire is attached to it, is always known as the **negative terminal,** or **electrode,** as indicated by the minus sign (−). In a corresponding manner, the submerged portion of the copper plate *C* is the negative element, and the exposed end, where the connecting wire is attached, the **positive terminal,** or **electrode,** as indicated by the minus (−) and plus (+) signs, respectively.

The element by which the current leaves any electrolytic cell is known as the **cathode,** and the element by which the current enters the electrolyte is known as the **anode.** Thus, in the cell already considered, the copper plate is the cathode and the zinc plate the anode.

23. Chemical Action in a Voltaic Cell.—The electric current passing through the electrolyte decomposes it, or breaks it up into its chemical elements, which then recombine with one another and with some of the positive voltaic elements, thus forming other substances. For example, if the electrolyte is sulphuric acid and water and the voltaic elements are copper and zinc, the current in passing causes electrochemical action, which gradually eats away the zinc and converts portions of the electrolyte into another chemical compound, *zinc sulphate* and a gas, *hydrogen*. The zinc sulphate immediately

dissolves and so makes no change in the appearance of the liquid; the hydrogen attaches itself to the cathode, which soon becomes covered with clearly visible bubbles of gas. In a simple cell as shown in Fig. 9, these bubbles on the negative element soon become so thick as to greatly decrease the flow of current, and a cell in such a condition is said to be **polarized.** Both the decomposition of the electrolyte and the polarization of the cell decrease the electromotive force. If a substance with which the free hydrogen can unite is placed near the cathode, the accumulation of gas bubbles on the cathode will be prevented and the electromotive force of the cell will be increased. Such a substance is known as a **depolarizer.**

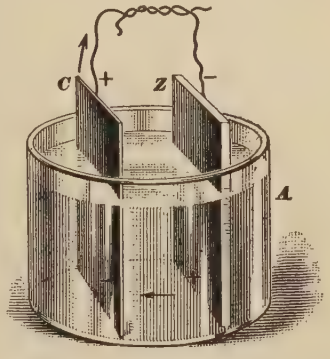

FIG. 9

24. A **voltaic battery** is a number of simple voltaic cells properly joined together. The electrodes of a cell are used to connect it to any exterior conductor or to another cell. In electric diagrams, cells are represented as drawn in Fig. 10. *M* and *N* each represent a single cell; the two cells joined together in this manner constitute a battery, *a* and *d* representing the terminals of the battery, as well as the positive electrode of *M* and the negative electrode of *N*, respectively. The negative electrode of *M* and the positive electrode of *N* are *b* and *c*, respectively. If there is only one cell in a circuit, it may be called a battery; in fact, a single cell is often called a battery. The arrows indicate the direction in which the current flows.

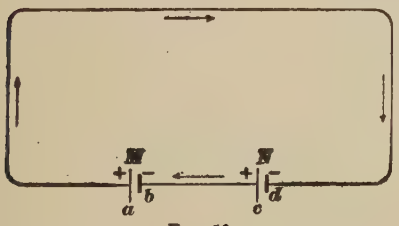

FIG. 10

25. The Dynamo.—If a conductor is moved across a magnetic field, an electromotive force will be generated between the ends of the conductor. The amount of the

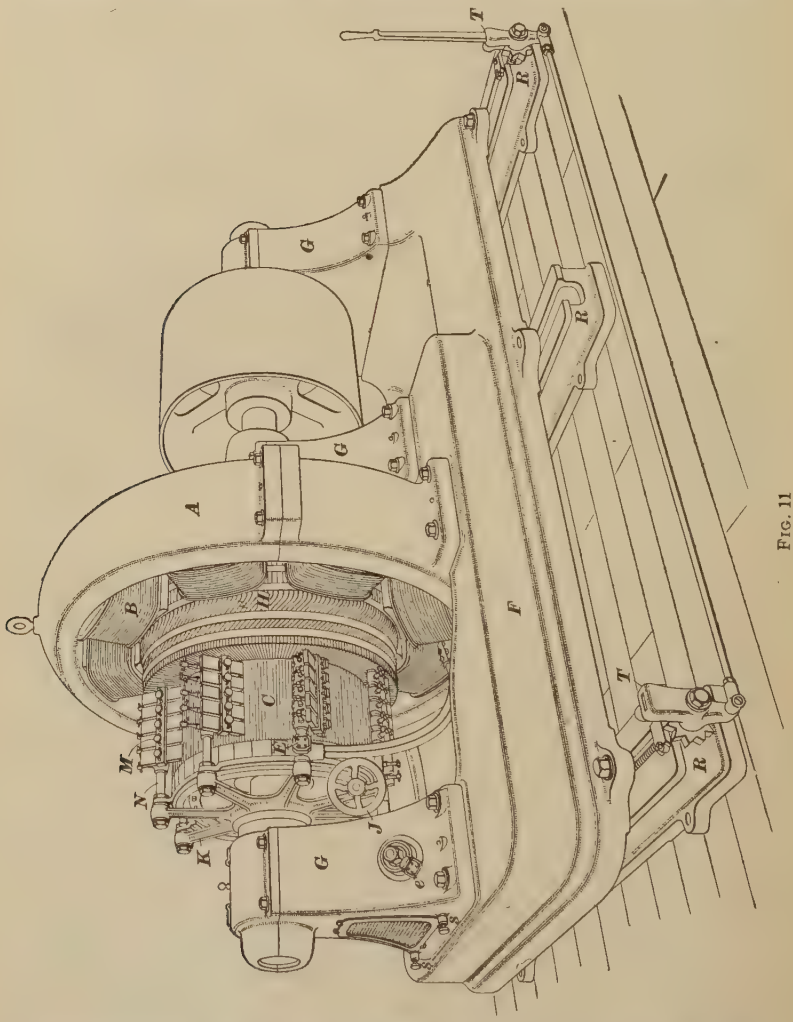

FIG. 11

electromotive force will depend on the strength of the magnetic field and the speed with which the conductor is moved. A **dynamo** is a machine by which conductors are

moved across magnetic fields. The conductors are usually arranged on a cylinder, or core, which is rotated, thereby moving the conductors past the face of magnets, or pole pieces. The core with the conductors assembled thereon is called the *armature*. It is immaterial which moves, the conductors or the magnets, and some dynamos are arranged so that the magnets rotate and the conductors remain stationary; the rotating field is then arranged inside the stationary armature.

26. Fig. 11 shows a modern belt-driven dynamo with stationary field and rotating armature. *A* is the circular yoke or field magnet, inside which are numerous inwardly projecting pole pieces, each covered by a coil of wire *B* called a field coil; *C* is the commutator, a cylinder built up of copper bars, each bar being connected to certain conductors on the armature. The stationary brushes *M*, held in holders supported on insulated studs *N*, rub on the rotating commutator and collect the current therefrom. The handwheel *J* serves to rotate the rocker-arm *K* through a small arc and thus adjust the position of the brushes on the commutator. The three bearing pedestals *G* rest on the bedplate *F*, which is supported on the slide rails *R*. The ratchet *T* is attached to a screw by means of which it is possible to move the dynamo on the rails so as to adjust the belt tension. The armature terminals are at *E*, *e*, one on a brush-holder stud and one on a pedestal. All the positive brushes are connected to one terminal, and all the negative brushes to the other. Field-coil terminals are shown at *s*, *s*. It will be necessary in the following pages to make frequent reference to a dynamo and this very brief explanation is inserted to make those references clear. Although all dynamos have the same essential parts, it must not be inferred that all are constructed exactly like the one shown.

RESISTANCE AND CONDUCTANCE

27. **Electric resistance** is a general property of matter, varying with different substances, by virtue of which matter *opposes*, or *resists*, the passage of electricity. Electric

resistance may be compared to the friction of water against the sides of a pipe through which it is passing, or to the resistance of a substance to the passage of heat.

28. Conductance, or **conductivity,** is the facility with which a body transmits electricity, and is the reciprocal, or opposite, of resistance. For instance, copper is of low resistance and high conductance; wood is of high resistance and low conductance.

Table I gives a list of a few conducting and non-conducting substances arranged in the order of their conductance, beginning with silver, which is the best conductor known. The division is made only as a guide. There are many

TABLE I

CONDUCTORS AND INSULATORS IN ORDER OF THEIR VALUE

Conductors	Insulators (Non-Conductors)	
Silver	Dry air	Glass
Copper	Shellac	Mica
Other metals	Paraffin	Ebonite
Charcoal	Amber	India rubber
Plumbago	Resin	Silk
Moist earth	Sulphur	Paper
Water	Porcelain	Oils

other substances that might be included among either the conductors or the insulators.

A general idea of these values may be obtained from the fact that water has about 6,754 million times greater resistance than copper.

29. The Ohm.—Electric resistance enters so much into the calculation and expression of electric quantities that a unit is necessary to express its value. The practical unit is the **ohm** and the resistance of a conductor is expressed in ohms. The international ohm is the resistance of a column of mercury 106.3 centimeters high and 1 square millimeter in cross-section, at a temperature of zero centigrade. Expressed

in English units, it is the resistance of a mercury column 3.4875 feet long and .0155 square inch cross-section, at a temperature of 32° F.

The ohm is the only unit in electrical measurements for which a material standard can be adopted. The one just mentioned is now generally used, although other standards have been employed.

The **microhm** equals one-millionth ($\frac{1}{1000000}$) ohm and is sometimes used to express very small resistances. The **megohm** equals 1,000,000 ohms and is sometimes used to express large resistances.

ELECTRIC CIRCUITS AND CONNECTIONS

30. An **electric circuit** is a path composed of a conductor, or of several conductors joined together, through which an electric current may flow from a given point around the conducting path back again to its starting point.

A circuit is **broken**, or **opened**, when its conducting elements are disconnected in such a manner as to prevent the current from flowing.

A circuit is **closed**, or **completed**, when its conducting elements are so connected as to allow the current to pass.

A circuit in which the conductors have come in contact with the ground, or with some electric conductor leading to the ground, is said to be a **grounded circuit**, and the contact is called an **earth**, or **ground**.

A path of low resistance across the terminals of a voltaic cell or of a dynamo or of any source of electromotive force is called a **short circuit**; for example, in Fig. 9, the cell is short-circuited by the contact of its wire terminals. Two grounds, one on each terminal of a cell or of a dynamo, may produce a short circuit.

The **external circuit** is that part of a circuit which is outside, or external, to the source of electricity.

The **internal circuit** is that part of a circuit which is included within the source of electricity.

In the case of the simple cell, as shown in Fig. 9, the internal circuit consists of the two metallic plates, or elements,

and the liquid, or electrolyte; the external circuit is the wire connecting the plates together outside the electrolyte.

31. Series-Connections.—Conductors are said to be connected in **series** when they are joined so that the entire current must pass through each conductor successively. For instance, in Fig. 12, all the current generated in the voltaic cell B must flow successively through the conductors r_1 and r_2. Hence, r_1 and r_2 are connected in series with each other. In this case, moreover, the battery B and the conductors r_1 and r_2 are all connected in one series-circuit; c, e, and d represent connecting wires, and the arrows show the direction in which the current is flowing.

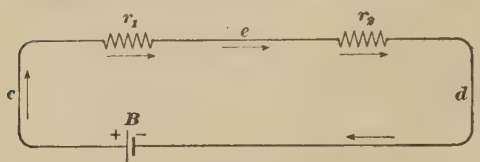

FIG. 12

32. Parallel, or Multiple, Connections.—A circuit divided into two or more branches, each branch transmitting part of the current, is a **divided circuit**; the conductors forming these branches are said to be connected in **parallel**, or **multiple**. Each branch taken separately is called a **shunt**. Such a circuit is shown in Fig. 13. The conductors r_1 and r_2 are the two branches into which the portion $a\,b$ of the circuit is divided. The conductors r_1 and r_2 are said to be connected in parallel or multiple, and r_1 is said to shunt r_2, or r_2 is said to shunt r_1. The total current I divides between the two branches, the portion I_1 flowing through r_1, and I_2 through r_2.

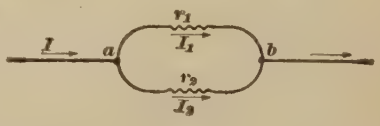

FIG. 13

NOTE.—The letter I will be used in this Section to represent a current, because it has been adopted for that purpose by an international convention of electrical engineers. However, the letter C has been used to represent current in the past, and is still sometimes so employed.

33. A battery of voltaic cells is said to be connected in series when the cells are arranged in one circuit by joining the positive electrode of one cell to the negative electrode

of the adjacent cell, so that the entire current passes successively through each, as shown in Fig. 14.

When so connected, the total electromotive force of the group of cells is equal to the sum of the electromotive forces of all; or if they are all of the same kind and condition, as is usually the case, the electromotive force of the group is equal to that of one cell multiplied by the number of cells. The current must necessarily be the same through all the cells. In connecting cells in series, whether they are near together or some distance apart, care must be taken to connect the positive electrode, or pole, of one cell to the negative pole of the next cell, etc.; that is, like poles must not be connected together.

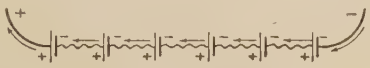

FIG. 14

The addition of electromotive forces with series-connection as explained for voltaic cells, applies to all sources of electromotive force. For example, dynamos are sometimes connected in series and the total electromotive force of the series is then the sum of the electromotive forces of all the dynamos so connected.

34. A battery of voltaic cells is said to be connected in multiple, or parallel, when the positive electrodes of all the cells are connected to one main positive conductor, and all the negative electrodes are connected to one main negative conductor, as shown in Fig. 15.

When so connected, all the positive electrodes have the same potential and all the negative electrodes have the same potential; hence, joining cells together in parallel does not increase the electromotive force above that of one cell.

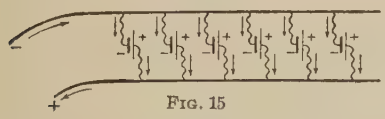

FIG. 15

A number of cells that are joined in parallel is really the equivalent of one large cell that has exposed to the electrolyte a plate-surface area equal to the sum of the exposed areas of all the plates of the group. There is no increase of electromotive force, but the current capacity increases with the increase of exposed plate surface because the resistance of cells is less.

Dynamos are also connected in parallel to increase the current output; the current output of the machines so connected is the sum of the outputs of all the machines, but the electromotive force is not increased.

35. Parallel-Series, or Multiple-Series, Connections.—When the series and multiple connections are combined, the battery is said to be connected in **multiple series,** or **parallel series.** This is accomplished by joining several groups in multiple, or parallel, the cells in each group being connected in series, as shown in Fig. 16. When so connected, the total electromotive force of the group is equal to the electromotive force of one series-group.

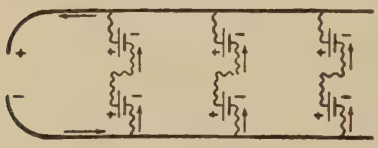

FIG. 16

EXAMPLE.—If the electromotive force of each cell is 1.8 volts, what is the total electromotive force of the groups of cells shown in Figs. 14, 15, and 16?

SOLUTION.—In Fig. 14, the cells are all connected in series; hence, the total E. M. F. of the group is $6 \times 1.8 = 10.8$ volts. Ans.

In Fig. 15, the cells are all joined in parallel; hence, the total E. M. F. of the group is that of one cell, or 1.8 volts. Ans.

In Fig. 16, there are three parallel groups, each group consisting of two cells joined in series; hence, the total E. M. F. of the group is equal to the E. M. F. of one series-row, which is $2 \times 1.8 = 3.6$ volts. Ans.

ELECTRIC CURRENT

36. It must be understood that electricity is a condition of matter and not matter itself, for it possesses neither weight nor extension. Consequently, the statement that electricity is flowing through a conductor must not be taken too literally; it must not be supposed that any material substance, such as a liquid, is actually passing through the conductor in the same sense as water flows through a pipe. The statement that electricity is flowing through a conductor is only another way of expressing the fact that the conductor and the space surrounding it are in different conditions than usual, and that they possess unusual properties.

The action of electricity, however, is quite similar in many respects to the flow of liquids, and the study of electric currents is much simplified by the analogy. The flow of electricity along a conductor has also been compared to the flow of heat in solids. If one end of an iron rod is heated, the heat is said to flow to the other end, which soon becomes warm though nothing material has passed along the rod.

37. There are two kinds of electric current; namely, **direct,** or **continuous, current,** which always flows in the same direction, and **alternating current,** which regularly reverses its direction of flow a definite number of times per second. There is a slight distinction between the meanings of the terms direct current and continuous current, but for present purposes they may be considered to mean the same thing. Direct current, generated by voltaic cells or batteries, is much used for bell ringing. Both direct current, generated by direct-current dynamos, and alternating current, generated by alternating-current generators, or alternators, have many uses, in particular in connection with electric lighting. A current that is maintained at a steady, or unchanging, value is known as a **constant current,** and may be either direct or alternating.

38. The Ampere.—The practical unit of electric current is the **ampere.** If an electric current is made to pass through water, the water will be broken up, or decomposed, into two gases, hydrogen and oxygen. The rate of decomposition depends on the strength of the current and the time. For example, 2 amperes in 1 second will decompose twice as much water as 1 ampere in 1 second, and 2 amperes in 2 seconds will decompose as much as 4 amperes in 1 second, etc. It has been experimentally determined and universally accepted that 1 ampere is that strength of current which will decompose .00009349 gram, or .0014427 grain, of water in 1 second. The ampere may also be defined as the current caused by an electromotive force of 1 volt acting through a resistance of 1 ohm.

OHM'S LAW

39. In every electric circuit in which direct current is flowing, there are, particularly, three factors, the true relation of which should be clearly understood. These three factors are:

1. The force tending to move the electricity, termed *electromotive force* (written E. M. F., or E.), of which the practical unit is the *volt*.

2. The rate of flow of the electricity, termed *current* (written I), of which the practical unit is the *ampere*.

3. The opposition to the flow of electricity, termed *resistance* (written R), of which the practical unit is the *ohm*.

40. In general, it may be stated that a result is equal to the effort put forth divided by the opposition, or,

$$\text{result} = \frac{\text{effort}}{\text{opposition}}$$

For example, the rate of flow of water through a pipe depends directly on the pressure applied to the water and inversely on the resistance offered by the pipe to the passage of water. The rate of flow may be increased either by increasing the pressure, or by enlarging the pipe, or by making the interior of the pipe smoother and thus decreasing its resistance.

A German electrician, Dr. G. S. Ohm, was the first to discover that the general law just stated applies to electric circuits; hence, the ratio

$$\text{strength of current} = \frac{\text{electromotive force}}{\text{resistance}}$$

is known as **Ohm's law.**

41. Ohm's Law.—*The strength of a continuous current of electricity in a circuit is directly proportional to the electromotive force, and inversely proportional to the resistance of the circuit, and is equal to the quotient arising from dividing the electromotive force by the resistance.*

42. The law may be stated in practical units by the following rules and formulas:

Rule I.—*The strength, in amperes, of a continuous current* (I) *flowing in a closed circuit, when the electromotive force* (E) *and the total resistance* (R) *are known, is found by dividing the electromotive force, in volts, by the total resistance, in ohms;* that is,

$$\text{current in amperes} = \frac{\text{electromotive force in volts}}{\text{resistance in ohms}}$$

or
$$I = \frac{E}{R} \qquad (1)$$

Rule II.—*The total resistance* (R), *in ohms, of a closed circuit, when the electromotive force* (E) *and the continuous current* (I) *are known, is found by dividing the electromotive force, in volts, by the current, in amperes;* that is,

$$\text{resistance in ohms} = \frac{\text{electromotive force in volts}}{\text{current in amperes}}$$

or
$$R = \frac{E}{I} \qquad (2)$$

Rule III.—*The total electromotive force* (E), *in volts, developed in a closed circuit, when the continuous current* (I) *and the total resistance* (R) *are known, is found by multiplying the current, in amperes, by the total resistance in ohms;* that is,

$$\text{electromotive force in volts} = \text{currents in amperes} \times \text{resistance in ohms}$$

or
$$E = I \times R \qquad (3)$$

43. The following examples show the application of Ohm's law:

Formula **1**, Art. **42**, determines the strength of current that will flow in a conductor of a given resistance, when the pressure in volts is known.

EXAMPLE 1.—A circuit has a resistance of 50 ohms and an available pressure of 100 volts; what is the strength of the current in amperes?

SOLUTION.—Applying formula **1**,

$$I = 100 \div 50 = 2 \text{ amperes.} \quad \text{Ans.}$$

EXAMPLE 2.—If the pressure in a circuit is 3 volts and the resistance is 15 ohms, how many amperes will flow?

SOLUTION.—Applying formula **1**,

$$I = 3 \div 15 = \tfrac{1}{5} \text{ ampere.} \quad \text{Ans.}$$

EXAMPLE 3.—What current can be made to flow through a circuit having a resistance of 10 ohms, if an electromotive force of 100 volts is applied?

SOLUTION.— $E = 100$; $R = 10$; hence, by formula **1**, the required current

$$I = 100 \div 10 = 10 \text{ amperes.} \quad \text{Ans.}$$

44. In case the electromotive force, or difference of potential, E is known, formula **2**, Art. **42**, must be used to calculate the resistance of the circuit that will allow a given current I to flow through it.

EXAMPLE 1.—The electromotive force of a circuit is 500 volts; it is desirable to have a current of .5 ampere flowing in it; what should be the resistance of the circuit?

SOLUTION.—According to formula **2**,

$$R = 500 \div .5 = 1{,}000 \text{ ohms.} \quad \text{Ans.}$$

EXAMPLE 2.—Through what resistance can a current of 50 amperes flow, if the electromotive force is 500 volts?

SOLUTION.— $I = 50$; $E = 500$; hence, by formula **2**, the required resistance

$$R = 500 \div 50 = 10 \text{ ohms.} \quad \text{Ans.}$$

45. To find how much pressure will be required to force a given current through a given resistance, it will be necessary to use formula **3**, Art. **42**.

EXAMPLE 1.—How much pressure will it take to force a current of 18 amperes through a resistance of 5 ohms?

SOLUTION.—Applying formula **3**,

$$E = 18 \times 5 = 90 \text{ volts.} \quad \text{Ans.}$$

EXAMPLE 2.—What voltage is required to send a current of 25 amperes through a resistance of 4 ohms?

SOLUTION.— $I = 25$; $R = 4$; hence, by formula **3**, the required voltage

$$E = 25 \times 4 = 100 \text{ volts.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The total resistance of a closed circuit is 49.3 ohms; if the current is 2.73 amperes, what is the total electromotive force, in volts?

Ans. 134.589 volts

2. A difference of potential of 110 volts exists between the terminals of a conductor whose resistance is 20 ohms; find the current flowing through the conductor. Ans. 5.5 amperes

3. A circuit has an available pressure of 220 volts; what is its resistance if a current of 50 amperes can flow through it?

Ans. 4.4 ohms

APPLICATION OF OHM'S LAW

46. When applying Ohm's law, the following facts should be carefully noted:

1. *The strength of a current is the same in all parts of a closed circuit, except in the case of divided circuits.*

2. *In the case of a divided circuit, the sum of the currents in the separate branches is always equal to the current in the main or undivided circuit.*

3. *The total resistance of a circuit is the sum of the resistances of the internal circuit and of the external circuit, or its equivalent.*

4. *When several electromotive forces act in the same circuit, the electromotive force that is effective to produce current is the difference between all those acting in one direction and all those acting in the other.*

47. Fig. 17 represents a closed circuit having connected in series in it a battery B of four cells, an incandescent

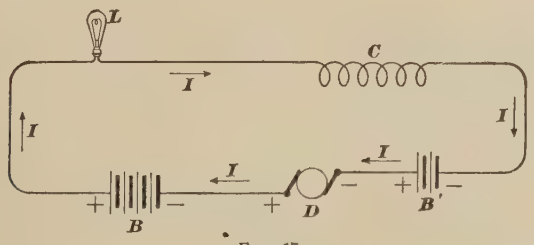

FIG. 17

lamp L, a coil of wire C, a second battery B' of two cells, and a dynamo D. The current I has the same strength in all parts of the circuit; that is, the strength of the current flowing through B is exactly the same as that flowing through L, C, B', and D.

48. If, as shown in Fig. 18, a main circuit a is split up into two or more branches, as r_1, r_2, and r_3, that again reunite at b, the current I in the main circuit subdivides through the various branches and the current I in the main circuit is equal to the sum of the currents in all the branches; that is, $I = I_1 + I_2 + I_3$.

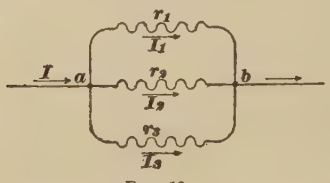

FIG. 18

EXAMPLE 1.—If the currents in the three branches r_1, r_2, and r_3, of a divided circuit are 5, 7, and 9 amperes, respectively, what will be the current I in the main circuit?

SOLUTION.—The current I in the main circuit is equal to the sum of the currents in the three branches; hence,

$$I = 7 + 5 + 9 = 21 \text{ amperes.} \quad \text{Ans.}$$

EXAMPLE 2.—If a main circuit in which a current of 18 amperes is flowing is subdivided into four parallel branches, the currents in three of the branches being 2, 4, and 5 amperes, respectively, what is the current in the remaining fourth branch?

SOLUTION.—The current in the main circuit is 18 amperes, and the sum of the currents in three branches is $2 + 4 + 5 = 11$ amperes; hence, the current in the fourth branch is

$$18 - 11 = 7 \text{ amperes.} \quad \text{Ans.}$$

49. When Ohm's law is applied to the whole of a closed circuit, R must represent the entire resistance of the circuit,

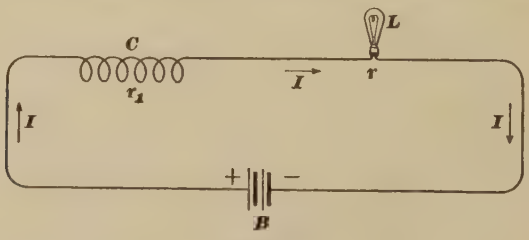

FIG. 19

which includes the internal resistance of the battery or other source of current and all resistances connected in series in the external circuit.

In Fig. 19, suppose that the total internal resistance of the battery is 2 ohms; the resistance of the lamp, 220 ohms:

and the resistance of the coil and all connecting wires, 8 ohms; then the total resistance of the circuit will be $2 + 220 + 8 = 230$ ohms.

The total resistance of the circuit shown in Fig. 17 is equal to the resistance of the lamp L + the resistance of the coil C + the internal resistance of the battery B' + the internal resistance of the dynamo D + the internal resistance of the battery B + the resistance of all the connecting wires. The resistance of connecting wires is very often, but not always, very small compared with the resistance of the apparatus and devices connected in the circuit, and in such cases the resistance of the connecting wires is not mentioned or considered in any way.

EXAMPLE 1.—The two electrodes of a simple voltaic cell are connected together by a copper wire, the resistance of which is 1 ohm. If the internal resistance of the cell is 4 ohms and the electromotive force developed is 2 volts, what is the strength of the current in all parts of the circuit?

SOLUTION.—The total resistance of the circuit, $R = 4 + 1 = 5$ ohms, and, by formula **1**, Art. **42**,

$$I = 2 \div 5 = .4 \text{ ampere.} \quad \text{Ans.}$$

EXAMPLE 2.—The total electromotive force developed in a closed circuit is 1.2 volts and the strength of the current flowing is .3 ampere; find the total resistance of the circuit.

SOLUTION.—By formula **2**, Art. **42**,

$$R = 1.2 \div .3 = 4 \text{ ohms.} \quad \text{Ans.}$$

EXAMPLE 3.—The internal resistance of a dynamo is 10.9 ohms and the external resistance of the circuit of which the dynamo is a part is 73 ohms; the electromotive force of the machine is 250 volts. Find the strength of the current flowing in the circuit.

SOLUTION.—The total resistance of the circuit, $R = 10.9 + 73 = 83.9$. By formula **1**, Art. **42**,

$$I = 250 \div 83.9 = 2.98 \text{ amperes.} \quad \text{Ans.}$$

50. In Fig. 20, cell B, having an electromotive force of 1.4 volts, is connected in a circuit with a dynamo D generating an electromotive force of 6 volts. Moreover, the electromotive forces, as indicated by the + and − signs, act in the same direction and hence tend to help each other in producing the current I that flows through the circuit.

Consequently, the effective electromotive force acting in the circuit is equal to $6 + 1.4 = 7.4$ volts.

If the cell B is reversed in the circuit, as shown in Fig. 21, the electromotive forces of B and D oppose each other and

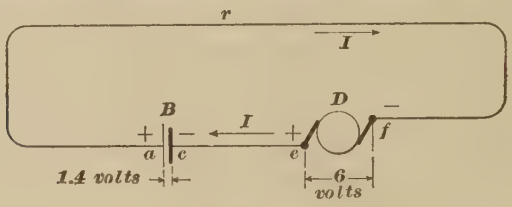

FIG. 20

the effective electromotive force acting in the circuit is $6 - 1.4 = 4.6$ volts. This electromotive force acts in the direction of the greater of the two electromotive forces, and, hence, whatever current is produced flows in the direction of the arrows I_1. Storage cells (which are explained later),

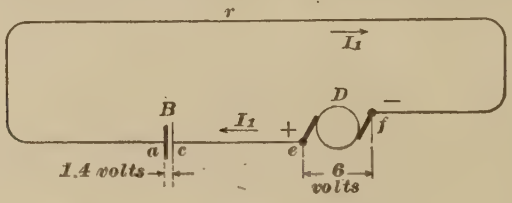

FIG. 21

while charging, are connected with the dynamo in this manner. The current flowing in either case is obtained by dividing the effective electromotive force by the total resistance in the circuit.

EXAMPLE.—If the internal resistance of the battery B is 2 ohms, the internal resistance of the dynamo D, 1 ohm, and the resistance of the external circuit, 7 ohms; what will be the currents I and I_1, flowing in the circuits shown in Figs. 20 and 21?

SOLUTION.—In Fig. 20, the effective E. M. F. acting in the circuit in this case is $6 + 1.4 = 7.4$ volts. The total resistance of the circuit is $2 + 1 + 7 = 10$ ohms. Hence, the current

$$I = 7.4 \div 10 = .74 \text{ ampere.} \quad \text{Ans.}$$

In Fig. 21, the effective E. M. F. $= 6 - 1.4 = 4.6$ volts, and the total resistance, as in preceding solution, is 10 ohms. Hence, the current

$$I = 4.6 \div 10 = .46 \text{ ampere.} \quad \text{Ans.}$$

TABLE II

ANALOGIES BETWEEN THE FLOW OF WATER AND ELECTRICITY

	Water in Pipes	Electricity in Conductors
I	Difference of level tends to make water fall from the upper level to the lower level.	Difference of potential tends to make electric current flow from point of high potential to point of lower potential.
II	Difference of level, hence acts as a pressure tending to cause a flow.	Difference of potential or electromotive force hence acts as a pressure tending to cause a flow of current.
III	If not entirely obstructed, this pressure actually produces a flow of water.	If not entirely obstructed this pressure or electromotive force actually produces a flow of current.
IV	Some of this pressure is lost by friction of the water against inside walls of pipe.	Some of this pressure is lost by the electrical resistance of the conductor; the loss is called *drop of potential.*
V	This loss by friction is directly proportional to the length of the pipe, and inversely proportional to the diameter of the pipe.	This loss or drop of potential due to resistance is directly proportional to the length of the conductor, and inversely proportional to its area of cross-section.
VI	No quantity of water can flow through a pipe without suffering some loss in this manner; in other words, there is no such thing as an absolutely frictionless pipe.	No quantity of electricity can flow through a conductor without suffering some loss in this manner; in other words, there is no such thing as an absolutely resistanceless conductor.

EXAMPLES FOR PRACTICE

1. If an electric-bell circuit has a total resistance of 12 ohms, how many voltaic cells good for 1.5 volts each will be required to furnish .5-ampere ringing current, and how must the cells be connected?

Ans. 4 cells connected in series

2. A battery of ten cells connected in series generates an electromotive force of 1.2 volts per cell. If the total resistance of the ten cells is 40 ohms and the resistance of the external circuit is 8 ohms, what is the strength of current flowing in the circuit? Ans. .25 ampere

3. Given an internal resistance of 4 ohms, an electromotive force of 1.5 volts, and a current of .2 ampere; find the external resistance.

Ans. 3.5 ohms

4. Given an electromotive force of 24 volts and a current of .6 ampere; if the external resistance is three times the internal, what is the resistance of each?

Ans. { External, 30 ohms
Internal, 10 ohms

DROP OF POTENTIAL

51. Referring to the flow of water in pipes, the analogies may be tabulated as given in Table II. A careful study of this table will assist very much in understanding what is to follow.

52. The fourth analogy of the table states that the loss of pressure, or electromotive force, due to the resistance of

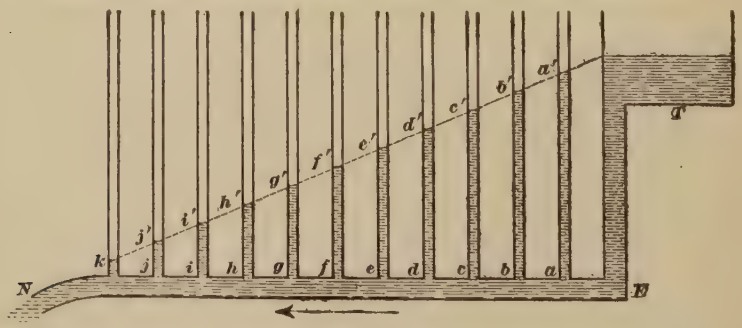

FIG. 22

conductor, is termed **drop of potential.** This drop may be made clearer by the following:

Let Fig. 22 represent a tank T of water with a horizontal discharge pipe EN, which is provided with open vertical

tubes at a, b, c, etc. If the outlet at N is closed, the water in the vertical tubes will rise to the height of the water in the tank; but if the water is allowed to flow freely from the outlet at N, the height of the water in the tubes will be represented by the inclined line at a', b', c', etc. The pressure, or head, of the water, which is measured by the height of the water in the tubes, decreases in the direction in which the water is flowing, so that the water that leaves the discharge outlet at N is under considerably less pressure than the water entering at E.

53. The same action takes place in a current of electricity flowing along a conductor, and can also be graphically shown. In Fig. 23, B represents a voltaic battery with the negative electrode connected directly to the earth at E, and the positive electrode to a long conductor AL, which is also connected to the earth at E'. The battery may be regarded as a machine that raises the pressure or potential of elec-

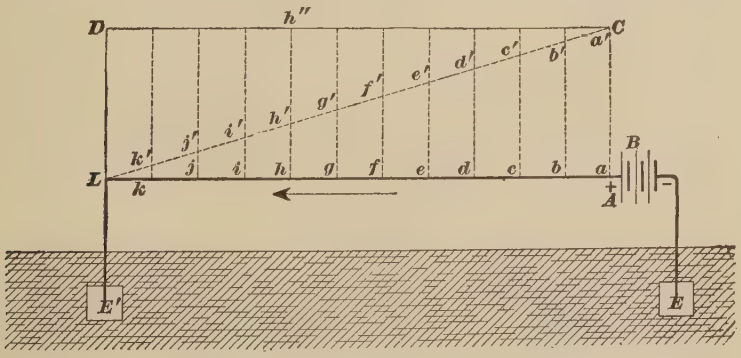

FIG. 23

tricity from zero (or that of the earth) to a height represented by the distance $a\,a'$; that is, the distance $a\,a'$ represents the total difference of potential across the terminals of the battery. If the circuit is open between L and E' so that no current flows, the difference of potential between the conductor and the earth is the same at all points along the conductor, and is represented by the distances between the line CD and the conductor AL; but when a current is

allowed to flow along the conductor, the difference of potential between the conductor and the earth decreases in the direction in which the current is flowing.

The vertical distances $b\,b'$, $c\,c'$, $d\,d'$, etc. represent the difference of potential at the points b, c, d, etc. along the conductor. The loss, or drop, of potential is represented by the vertical distances between the inclined line CL and the horizontal line CD. This loss, or drop, also represents the difference of potential between the point a and any other point along the conductor. For example, at h the difference of potential between the conductor at that point and the earth is represented by the distance $h\,h'$; the loss or drop of potential is represented by the vertical distance $h'\,h''$, which distance also represents the difference of potential existing between the points a and h.

54. The graphic method of determining the difference of potential, as shown in Fig. 23, is seldom used. The difference of potential (E), in volts, between any two points along a circuit is equal to the product of the strength of the current (I), in amperes, and the resistance (R), in ohms, of that part of the circuit included between those two points; or $E = IR$, which is an example of the use of Ohm's law. E, in this case, represents the loss or drop of potential, in volts, between the two points.

EXAMPLE.—Fig. 24 represents part of a circuit in which a current of 2.5 amperes is flowing. The resistance from a to b is 10 ohms; from b to c, 15 ohms; and from c to d, 20 ohms. Find the difference of potential between a and b, b and c, c and d, and a and d.

FIG. 24

SOLUTION.—Since, by Ohm's law, $E = IR$, the difference of potential between

a and b is $2.5 \times 10 = 25$ volts
b and c is $2.5 \times 15 = 37.5$ volts
c and d is $2.5 \times 20 = 50$ volts
a and d is $25 + 37.5 + 50 = 112.5$ volts

or, the drop in potential between a and d is 112.5 volts.

55. In a great many cases, it is desirable to have the current flow from the source a long distance to some electric

receptive device, such as an incandescent lamp or an electric motor, and return without causing an excessive drop of potential in the conductors leading to and from the two places. In such circuits, the greater part of the total generated electromotive force is expended in the receptive device itself, and only a small fraction of it is lost in the rest of the circuit. Under these conditions, it is customary to decide on a certain drop of potential beforehand, and from that and the current calculate the resistance of the two conductors.

EXAMPLE.—It is desired to transmit a current of 10 amperes to an electric device situated 1,000 feet from the source; the total generated electromotive force is 110 volts, and only 5 per cent. of this is to be lost in the conductors leading to and from the two points. Find (*a*) the total resistance of the two conductors, and (*b*) the resistance per foot of the conductors, assuming each to be 1,000 feet long, and that the resistance of 1,000 feet is 1,000 times the resistance of 1 foot.

SOLUTION.—(*a*) 5 per cent. of 110 volts is $110 \times .05 = 5.5$ volts, which represents the total drop of potential in the two conductors. Then, $E = 5.5$ volts, $I = 10$ amperes, and $R =$ the total resistance of the two conductors, and by Ohm's law,

$$R = 5.5 \div 10 = .55 \text{ ohm.} \quad \text{Ans.}$$

(*b*) The resistance of the conductor is directly proportional to its length, and hence the resistance of 1 ft. is $\frac{1}{2000}$ of the resistance of 2,000 ft. But the resistance of 2,000 ft. is .55 ohm; hence, the resistance per foot is

$$.55 \div 2{,}000 = .000275 \text{ ohm.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In a part of a closed circuit, the drop of potential caused by the resistance of the conductor is 10 volts; if the current flowing is 4 amperes, what is the resistance of that part of the circuit?

Ans. 2.5 ohms

2. The total generated electromotive force in a circuit is 220 volts. A current of 10 amperes is transmitted to and from a receptive device situated some distance from the source, with a drop of potential of 10 per cent. Find the total resistance of the two conductors leading to and from the device. Ans. 2.2 ohms

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56. The total drop of potential in a circuit containing a number of resistances connected in series is equal to the sum of the resistances multiplied by the current. For

instance, the total drop from a to d, Fig. 24, if the resistance of $a\,b$ is 10 ohms, $b\,c$ is 15 ohms, $c\,d$ is 20 ohms, and a current of 2.5 amperes is flowing through them, is equal to

$$(10 + 15 + 20) \times 2.5 = 45 \times 2.5 = 112.5 \text{ volts}$$

This is exactly the same total drop as in the solution to the example in Art. **54**, but is computed in a slightly different manner. In that solution, the total drop was computed by adding together the drops across the individual resistances. Hence, the total drop may be calculated either way—whichever happens to be the more convenient.

In a closed circuit, the current is computed by dividing the total or effective electromotive force acting in the circuit by the total resistance of the circuit.

EXAMPLE.—In Fig. 17, suppose that the electromotive forces of the batteries B and B' and the dynamo D are 8, 4, and 13 volts, respectively, and their internal resistances are 12, 6, and 2 ohms, respectively, and that the resistances of the lamp L and coil C, including the connecting wires, are 50 and 30 ohms, respectively. What will be the strength of the current flowing in all parts of this circuit?

SOLUTION.—The total or resultant E. M. F. is $8 + 4 + 13 = 25$ volts, and the total resistance of the circuit is $12 + 50 + 30 + 6 + 2 = 100$ ohms. Then, by Ohm's law, the current flowing through the entire circuit is

$$I = 25 \div 100 = .25 \text{ ampere.} \quad \text{Ans.}$$

ELECTRIC POWER

57. Work, or energy, is required to force a current through a conductor; this energy is expended in heating the conductor. If the current is large enough, the conductor will be melted or burned off. Similarly, if a stream of water is forced through a pipe, some heat will be developed, due to the friction of the water against the interior of the pipe. The heat will be dissipated before the pipe heats sensibly, to be sure, but close measurements will show a slight rise in the temperature of the water and the pipe. If the pipe opposed no resistance to the passage of the water, there would be no friction and consequently no heat development. Likewise, if an electric conductor did not resist the flow of electricity, it would not be heated.

The power expended in heating an electric conductor is proportional to the square of the current flowing in the conductor and to the resistance of the conductor.

Let I = current;
R = resistance;
P = power;

then $P = I^2 R$

58. The Watt.—If the current is in amperes and the resistance in ohms, the energy is in units called *watts*. The **watt** is the practical unit of electric power, and *1 watt is expended when a direct current of 1 ampere is forced through a resistance of 1 ohm.*

The formula stated in the preceding article may be expressed by the following rule:

Rule.—*In an electric circuit through which a direct current is flowing, the power, in watts, is equal to the product of the square of the current, in amperes, times the resistance of the circuit, in ohms.*

EXAMPLE 1.—How much power is expended in a circuit whose resistance is 2.5 ohms when a current of 5 amperes is flowing?

SOLUTION.—By substitution in the formula of Art. **57**,

$P = 5 \times 5 \times 2.5 = 62.5$ watts. Ans.

EXAMPLE 2.—If the resistance of a 16-candlepower, 110-volt, incandescent lamp is 220 ohms: (*a*) how much current does such a lamp take? (*b*) how much power is expended in it?

SOLUTION.—(*a*) By Ohm's law,

$I = 110 \div 220 = .5$ ampere. Ans.

(*b*) By the formula of Art. **57**,

$P = .5 \times .5 \times 220 = 55$ watts. Ans.

59. By formula **2**, Art. **42**, $R = \frac{E}{I}$ and by the formula of Art. **57**, $P = I^2 R$. If in place of R in the formula of Art. **57**, its value $\frac{E}{I}$ is substituted, a new formula $P = \frac{I^2 E}{I}$ will be obtained, and by cancelation this becomes

$$P = IE$$

This formula may be expressed by the following rule:

Rule.—*In an electric circuit through which a direct current is flowing, the power, in watts, is equal to the product of the current, in amperes, times the electromotive force, in volts.*

EXAMPLE 1.—If the electromotive force at the terminals of an electric device is 220 volts and the device takes a current of 5 amperes, how many watts are consumed?

SOLUTION.—By the formula just stated,

$$P = 5 \times 220 = 1{,}100 \text{ watts.} \quad \text{Ans.}$$

EXAMPLE 2.—How many watts are consumed by a 32-candlepower, 110-volt lamp if it takes 1.2 amperes current?

SOLUTION.—By the formula just given,

$$P = 1.2 \times 110 = 132 \text{ watts.} \quad \text{Ans.}$$

60. The Kilowatt.—The watt is too small to express conveniently the large values of electric power frequently used; hence, the **kilowatt**, equivalent in value to 1,000 watts, is a convenient unit much in use. Kilowatt or kilowatts is very frequently written K. W. For use in formulas, it is nearly always necessary to reduce kilowatts to watts. For example, 10 kilowatts divided by 50 amperes gives $\frac{10 \times 1{,}000}{50}$ = 200 volts and not $10 \div 50 = .2$ volt.

EXAMPLES FOR PRACTICE

1. Given an electromotive force of 80 volts and a resistance of 2 ohms, find the power in watts and kilowatts.

Ans. 3,200 watts, or 3.2 K. W.

2. Given a resistance of 11.8 ohms and a strength of current of 20 amperes, find the power, in watts and kilowatts.

Ans. 4,720 watts, or 4.72 K. W.

3. Given an electromotive force of 112 volts and a strength of current of 12 amperes, find the power in watts and kilowatts.

Ans. 1,344 watts, or 1.344 K. W.

---

61. The Kilowatt-Hour.—Electrical energy is usually measured in kilowatt-hours. A **kilowatt-hour** is equivalent to the work done when 1 kilowatt has been expended for 1 hour. The kilowatt-hours used in a circuit are then equivalent to the product of the kilowatts and the hours. For

example, if the electric lamps in a store require 1.5 kilowatts and the lamps are kept lighted 5 hours a day, there will be consumed each day $1.5 \times 5 = 7.5$ kilowatt-hours.

The **watt-hour** is the work done when 1 watt has been expended for 1 hour. To change watt-hours to kilowatt-hours divide the number of watt-hours by 1,000.

EXAMPLE.—A large hall is lighted by fifty 32-candlepower, 110-volt incandescent lamps, each lamp requiring 1 ampere. If the average drop is 3 per cent. of the pressure at the lamps, find: (*a*) how much power in kilowatts is expended in the wires; (*b*) how much is used in the lamps; (*c*) how much energy in kilowatt-hours is used in the wires in 5 hours; (*d*) how much energy is used in the lamps in 5 hours.

SOLUTION.—(*a*) The lamps require a total current I of 50 amperes. The drop e in the wires is 3 per cent. of 110 or

$$e = \frac{3 \times 110}{100} = 3.3 \text{ volts}$$

Applying the formula, Art. **59**,

$$P = 3.3 \times 50 = 165 \text{ watts.} \quad \text{Ans.}$$

(*b*) The lamps consume 50 amperes at 110 volts and by formula,

$$P = 110 \times 50 = 5{,}500 \text{ watts.} \quad \text{Ans.}$$

(*c*) The power lost in the wires is 165 watts or .165 K. W. and in 5 hr. the energy loss is

$$5 \times .165 = .825 \text{ K. W.-hr.} \quad \text{Ans.}$$

(*d*) The power used in the lamps is 5,500 watts, or 5.5 K. W. and in 5 hr. the energy consumption is

$$5 \times 5.5 = 27.5 \text{ K. W.-hr.} \quad \text{Ans.}$$

62. Electric Horsepower.—The unit most used to express mechanical power is the *horsepower*. One horsepower equals the work done when a weight of 550 pounds is lifted vertically 1 foot in 1 second, or when a weight of 33,000 pounds is lifted vertically 1 foot in 1 minute; that is, one mechanical horsepower is the equivalent of 550 foot-pounds per second, or 33,000 foot-pounds per minute. One mechanical horsepower expressed in electric units equals 746 watts; hence, 746 watts is termed an **electric horsepower.**

EXAMPLE.—How many electric horsepower is consumed by a device that takes 68 amperes at 220 volts?

SOLUTION.—The watts consumed are $68 \times 220 = 14{,}960$. One electric horsepower is 746 watts; therefore,

$$14{,}960 \div 746 = 20 \text{ H. P., about.} \quad \text{Ans.}$$

PRACTICAL APPLICATION OF ELECTRICITY AND MAGNETISM

INCANDESCENT LAMPS

63. A very brief description of some of the electric devices to which interior wires may be led will now be given. Fig. 25 shows the outline of an incandescent lamp and a section of an Edison key socket. The name of each part is given and the general appearance and many of the details are probably familiar to every one. An incandescent lamp consists essentially of a long, thin, high-resistance conductor looped, or coiled, inside of a glass bulb from which the air is then exhausted. This conductor is called the lamp **filament**, and an electric current heats this filament to incandescence, that is, until it glows with heat. If the filament were so heated in open air, it would almost immediately burn up; but in the absence of air, that is, in a *vacuum*, the lamp will glow, or burn, a long time without losing much of its light-giving power. The length of time that a lamp will burn before losing over 20 per cent. of its light-giving power is commonly called its *life*, and the average life of ordinary incandescent lamps is from 500 to 700 hours.

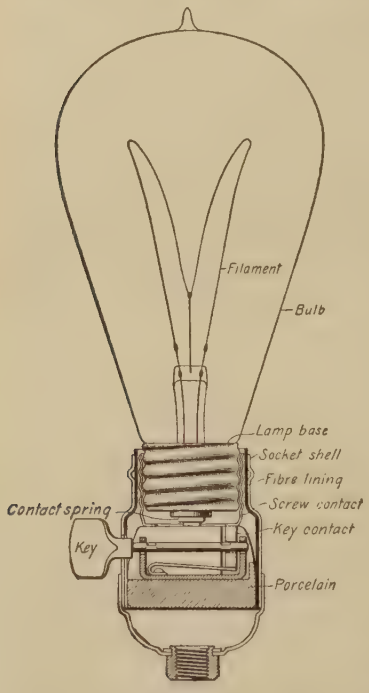

FIG. 25

64. **Lamp Bases.**—Each lamp is provided with a **base** by means of which it can be attached to a socket to which

the electric wires are connected. There are several types of lamp bases. Fig. 26 shows three types; at (*a*) is the Edison; (*b*), the Thompson-Houston, or T.-H.; and (*c*), the Westinghouse, or Sawyer-Man.

The Edison base is superseding all the others and will probably ultimately be the only one in use. It consists of

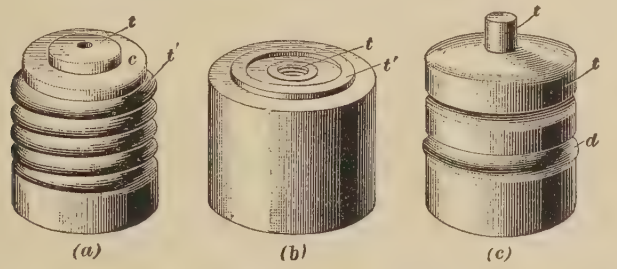

FIG. 26

an outer brass shell t' in the form of a coarse screw thread, to which one terminal of the lamp filament is connected, and an inner brass centerpiece t to which the other filament terminal is connected. The two brass pieces are separated by porcelain, or cement, c. When the lamp is screwed into the socket, as shown in Fig. 25, the screw shell makes one connection through its contact with the socket shell, and the centerpiece makes the other through its contact with a spring or a centerpiece in the socket. Fig. 25 shows a key socket, but keyless sockets are also in very common use.

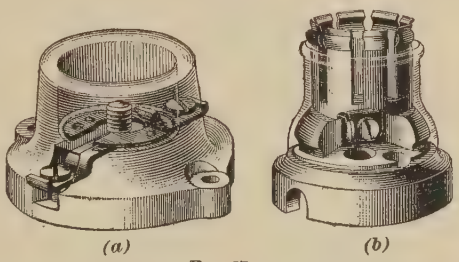

FIG. 27

When the T.-H. base is used, the filament terminals are connected to a threaded brass centerpiece t, Fig. 26 (*b*), and to a brass ring t'. The lamp screws into a socket as shown in Fig. 27 (*a*), the projecting stud in the socket screwing into the centerpiece in the lamp base, and the ring t' coming in contact with a metal ring in the socket.

With the Westinghouse, or Sawyer-Man, base, the connections are made to the outer shell t', Fig. 26 (c), and to a projecting pin t. The base is provided with a rib d, and when the base is pushed into a socket, Fig. 27 (b), this rib slips into an annular groove in the split bushing of the socket, thus holding the lamp in place.

Neither the T.-H. nor the Westinghouse bases are held in the socket so firmly as the Edison base.

65. Lamp Ratings and Efficiency.—Electric lamps are rated in **candlepower;** that is, a lamp that will give as much light as 16 spermaceti candles of standard dimensions is called a 16-candlepower lamp. Incandescent lamps for illuminating purposes are made in sizes of 4, 8, 10, 16, 20, 32, 50, and 100 candlepower; for decorative purposes, $\frac{1}{2}$-, 1-, and 2-candlepower sizes are made. The sizes most used for general illumination are probably 10, 16, and 32 candlepower. For all sizes, the bulbs may be clear, frosted, or colored, as desired. Clear bulbs give the most light, but frosted bulbs break up the light rays so that the disagreeable glare of a bare lamp with a clear bulb is avoided. Colored bulbs are mostly used for decoration.

66. Each lamp, when burning, consumes electric energy and gives off light. The ratio of the quantity of light given off to the quantity of energy consumed is, properly speaking, the *efficiency* of the lamp. The practical way of stating lamp efficiency is in watts consumed per candlepower given off. For example, if a lamp consumes 56 watts and gives off 16 candlepower, the efficiency is said to be $56 \div 16 = 3.5$ watts per candlepower, and the lamp will be commonly spoken of as a 3.5-watt lamp. The lamps now in common use have efficiencies, or energy consumptions, of from 3 to 4 watts per candlepower, but lamps of higher efficiencies (1.5 to 2.5 watts per candlepower) are made and will doubtless soon come into general use. It is to be noted that with this method of stating efficiency, the lower the watts per candlepower the higher is the efficiency; that is, a 2.5-watt lamp is more efficient than a 3.1-watt lamp.

67. Lamp Connections.—Incandescent lamps are nearly always connected in *parallel*, or *multiple*, as shown in Fig. 28. In this case, the pressure between the two lines must be kept constant and the system is called **constant potential.** A wire leading to a lamp may be provided with a switch, that is, a device for opening the circuit through the lamp, as shown at *a*; or key sockets may be used, as

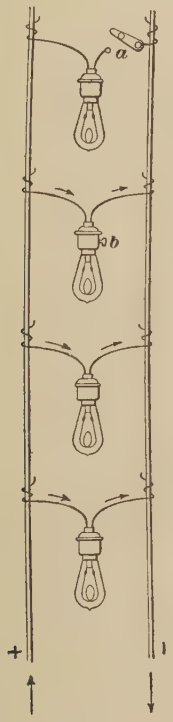

FIG. 28

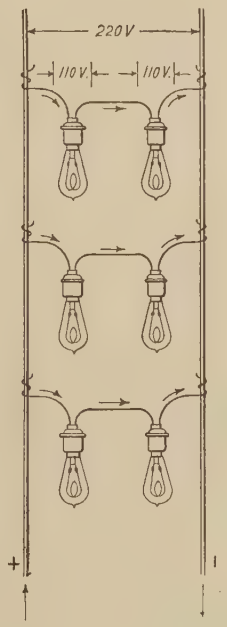

FIG. 29

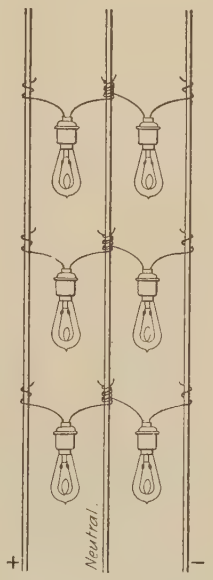

FIG. 30

shown at *b*. If it is unnecessary to turn lamps on or off independently of others on the same circuit, neither switches nor key sockets are provided, but the lamps remain burning as long as the circuit to which they are connected is alive.

Lamps are sometimes connected in *multiple series* as shown in Fig. 29. In the system illustrated, each lamp is made for 110 volts and they are connected two in series across a

220-volt circuit. This method is now seldom used; but by running a middle, or neutral, wire as shown in Fig. 30, the three-wire system, which is in very extensive use, is obtained.

ARC LAMPS

68. If an electric circuit in which a current is flowing is broken, and the broken parts are separated a short distance, the current will continue to flow, and, in jumping across the gap between the broken parts, will make a very bright light. This is an **electric arc** and it is always accompanied by intense heat, so that the terminals between which it flows are rapidly burned away unless they are made of a substance having a very high melting point. Carbon is most used for terminals, or *electrodes*, of arc lamps, and the lamp consists of a mechanism that automatically maintains the proper distance between the carbon electrodes, or the proper length of arc. Even carbon is burned away gradually by the intense heat of the arc, and the lamp frequently readjusts the length of the arc to keep it burning.

There are many varieties of arc lamps, all of which may be classified as *open-arc lamps* and *enclosed-arc lamps*. In an open-arc lamp, the air has free access to the arc; but in an enclosed-arc lamp, the access of air to the arc is greatly restricted by a small enclosing globe, and the carbons last much longer than when burning in open air.

69. **Open-arc lamps** are nearly always connected in series, a constant current passing through all the lamps. They are seldom used for interior illumination, except in mills, foundries, or similar large buildings. These lamps require a special dynamo, which will generate a constant current at all times, no matter how many lamps are burning, and also a special circuit from the dynamo through the string of lamps and back again. Lamps for connection in series, where all the lamps in the series must receive the same current, are called **constant-current lamps.** A constant current may be either direct or alternating, but its value, in amperes, must remain constant.

70. Some **enclosed-arc lamps** are also made for operation in series, with a constant current through all, the greater number of such lamps being made for alternating current and used for street lighting. Enclosed-arc lamps are also made for parallel operation on the same constant-potential mains as are used for incandescent lighting, and **constant-potential enclosed-arc lamps** are much used for lighting shops, stores, halls, and all buildings where

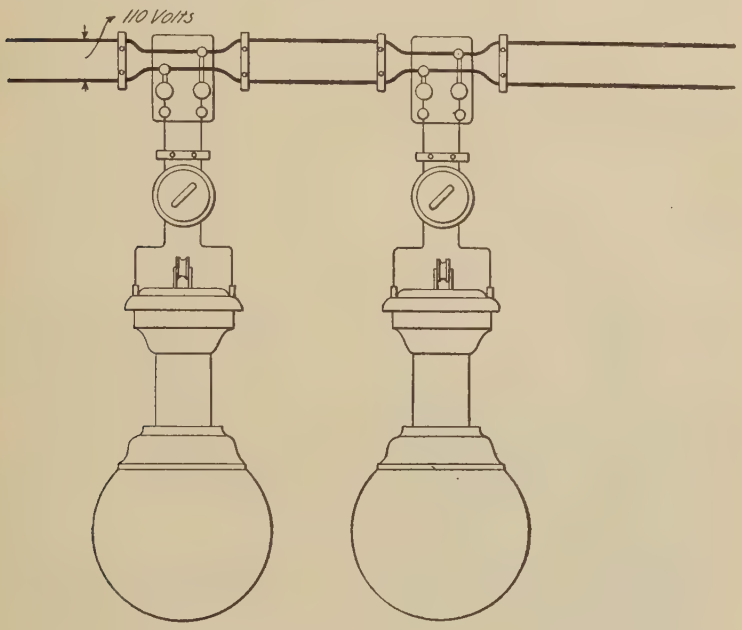

FIG. 31

large single sources of light can be used to good advantage. Fig. 31 shows two such lamps connected in parallel to 110-volt mains.

ELECTRIC MOTORS

71. An **electric motor** is a device for transforming electric energy into mechanical energy. Scarcely any large modern building is complete without the installation of one or more electric motors for running ventilating fans,

elevators, pumps, etc. If only one or two motors are used in a building, the wiring to them is simple, and little extra provision over that necessary for the electric-light wires is needed; but in buildings designed for manufacturing purposes, special provision must be made for *power* wires to the motors. Electric motors are usually rated in horsepower. The appearance of a motor is the same as that of a dynamo built for the same kind of service; in fact, a machine built for a dynamo may be used as a motor or a machine built for a motor may be used as a dynamo. They are not usually exactly the same in design and construction and some slight changes in connections may be necessary when using the one for the other.

STORAGE BATTERIES

72. A **storage battery,** or **accumulator,** is a device consisting of plates of metal, usually lead, immersed in an electrolyte, the materials and the arrangement being such that when an electric current is sent through them, chemical action occurs and the apparatus acquires the properties of a voltaic battery. If then made a part of a closed circuit, the battery will give off current. The process of sending current through a storage battery is called *charging* it; and when it gives off current, it is said to be *discharging*. Fig. 32 shows a storage cell consisting of positive plates a, a, etc., and negative plates b, b, etc., arranged alternately. The positive plates are those at which current enters the cell while charging and from which current

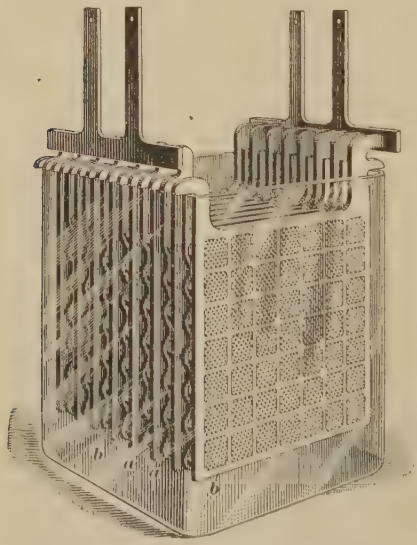

FIG. 32

leaves the cell while discharging. All the positive plates are connected to one connecting strap and all the negative plates to another.

In order to distinguish the two kinds of batteries more readily, voltaic batteries are usually referred to as **primary batteries,** and storage batteries as **secondary batteries.**

CONDENSERS

73. A **condenser** is an apparatus for condensing, or accumulating, a quantity of static electricity on a surface. It consists of two conductors separated by a layer of some non-conducting material called the *dielectric*. The two conductors may be subdivided into many parts and interleaved together so as to present a very large surface to each other. A condenser is usually made by piling together a large number of sheets of tin-foil, as shown in Fig. 33, where a, b represents sheets of tin-foil. Each alternate sheet is connected to a conductor t or t'; and the sheets are separated by sheets of waxed paper, mica, or other insulating material i, so that there is no electrical connection between the two sets of plates.

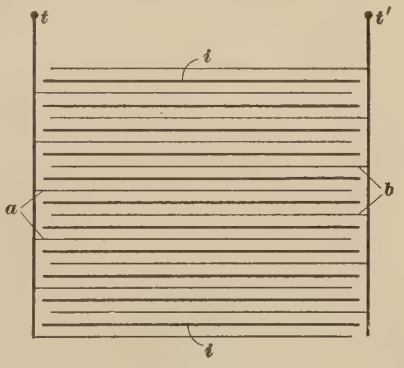

FIG. 33

When a battery or other source of electromotive force is connected to the terminals t, t', a current flows momentarily into the condenser, which becomes charged. The process is somewhat similar to pumping water into a flexible-rubber bag or to compressing a steel spring; the quantity of water forced into the bag or the amount the spring is compressed will depend respectively on the water pressure or the mechanical pressure applied. Likewise, the quantity of charge taken by the condenser will depend on the electric pressure,

or electromotive force, applied to the terminals. The current flowing into the condenser is of very short duration, as the charge becomes complete almost immediately; if the terminals are disconnected from the source of electromotive force and are connected in a closed circuit, the condenser will discharge as suddenly as it was charged, somewhat similar to the action of the compressed spring when released.

TRANSFORMERS

74. It has been shown that the drop of potential in an electric circuit is proportional to the product of the current flowing and the resistance of the circuit, or $E = IR$, and that the energy loss in a circuit is proportional to the product of the square of the current and the resistance, or $P = I^2R$. Then, if a quantity of energy can be transmitted over a line at high pressure and low current, the drop of potential and the energy loss in the line will be much less than if the same quantity is transmitted over the same line at a low pressure and a high current. For example, if 2 amperes at 1,100 volts flow over a line of 10 ohms resistance, there will be transmitted $P = IE = 2 \times 1{,}100 = 2{,}200$ watts; the drop of potential will be $2 \times 10 = 20$ volts; and the line loss will be $2 \times 2 \times 10 = 40$ watts. The same energy could be transmitted by 4 amperes at 550 volts, for $4 \times 550 = 2{,}200$ watts. The drop would then be $4 \times 10 = 40$ volts, and the loss would be $4 \times 4 \times 10 = 160$ watts; that is, with the low pressure and the high current the line loss would be four times as much as with the high pressure and low current, and the drop of potential twice as much.

75. An **electric transformer,** sometimes called a static transformer, is a device for changing an alternating current of one pressure to another of a higher or a lower pressure. It consists essentially of an iron core on which are placed two coils of wire—a primary coil, and a secondary coil. The core is made up of thin iron plates, or laminations of proper shape, punched from large sheets and built up into proper thickness by using a number of laminations

and bolting or clamping them together. Fig. 34 shows a common type of transformer; C is the core and P, S, S_1, P_1 are the coils, each wrapped with insulating material.

The *primary coil* is that into which the current to be transformed is led; the *secondary coil* is that from which the transformed current is taken. In Fig. 34, the primary and the secondary coils are each made in two parts; the two halves of the primary P and P_1 are placed outside of the two halves of the secondary S and S_1.

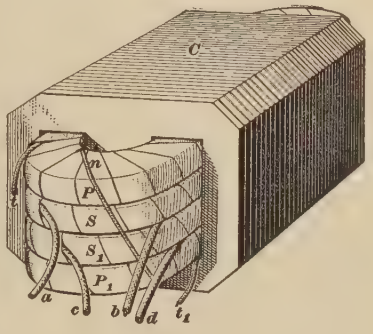

FIG. 34

Each half coil has two terminals, or leads, a–b, c–d, t–n, n–t_1, the two halves of the primary coil being connected in series by the common terminal n. The secondary leads a–b, c–d are brought out separately, so that they may be connected in series or in parallel as desired.

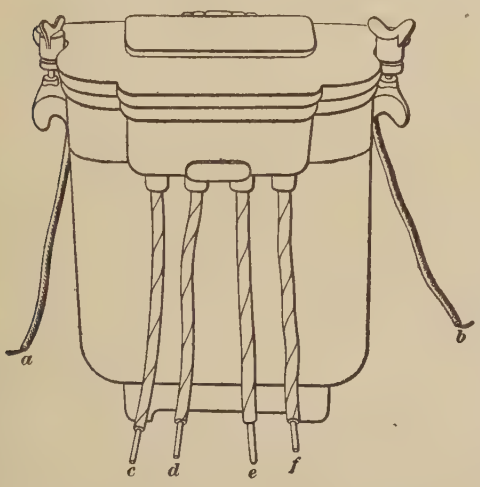

FIG. 35

76. The **transformation** is in proportion to the relative number of complete turns, or convolutions, of wire in the primary coil and in the secondary coil. For example, if the primary coil is made up of 100 turns and the secondary coil of 10 turns, the ratio of transformation will be 100 to 10, or 10 to 1; and if an alternating current were led into the primary coil at a pressure of 1,100 volts, another alternating current very nearly

ten times as large could be taken from the secondary end at 110 volts. Such a device permits the transmission of electric energy at a high pressure and a low current, with correspondingly low losses, from the power station, where the engines, dynamos, etc. are located, to the buildings, possibly miles away, where the energy is to be used. Near the place where the energy is to be used, transformers may be connected so that the pressure will be reduced and the current increased for lamps, motors, etc.

77. Fig. 35 shows a type of transformer for hanging on a pole that carries the overhead wires. The core and coils are enclosed in a weather-proof case, which in some types is filled with an insulating oil. The primary terminals a, b and the secondary terminals c, d, e, f are all brought out through the cover in a downward direction so that it is difficult for water to enter around them.

No similar transforming device exists for changing direct current from one pressure to another, and the greater readiness with which alternating current may be transformed constitutes one of its great advantages.

ELECTRIC WIRING AND BELLWORK

(PART 2)

ELECTRIC-LIGHT AND POWER WIRING

1. The subject of interior wiring includes a study of the various methods for supplying electric current to lamps, motors, etc., also to bells, burglar alarms, and other minor appliances operated by electricity.

In electric wiring, the object is to conduct electricity to the device to be operated; but this should be done in such a manner that danger, unsatisfactory operation, and waste will be avoided.

Four things should be considered in every electric installation; namely, safety, satisfactory operation, convenience and neatness, and economy. The architect and wireman should, therefore, understand the sources of danger in the use of electricity, the precautions necessary to avoid these dangers, as well as how to make the work satisfactory in other respects and profitable.

SAFETY

2. The first consideration in wiring a building is to make the installation safe. The danger of injurious or fatal electric shocks to persons, as well as the danger of fires caused by the electric current, must be made as small as possible. Ordinarily, if the fire-hazard is made small, the danger to life will also be decreased.

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ACCIDENTS CAUSED BY ELECTRIC WIRING

3. The so-called *electrical fires*, or fires that are caused by the presence of electric wires or apparatus within a building, can be divided into three classes, as follows:

1. Fires caused by poor work or improper materials.
2. Fires caused by overloading the apparatus or wire with a higher voltage or with more current than it was designed to carry.
3. Fires caused by lightning striking the outside lines or by the crossing of circuits that should never come in contact with one another.

A good job of interior wiring overcomes all danger due to the first of these sources of hazard and most of the danger due to the second and third. Some kinds of apparatus, however, can be overloaded through no fault of the wireman, and accidents sometimes occur outside of the buildings, against the results of which the present accepted devices for the protection of inside circuits are not sufficient.

The failure of a lighting company to use proper lightning arresters and transformers or to insulate the outside wires thoroughly may cause trouble within a building in which the wiring is properly done. When a lightning discharge strikes line conductors, it selects the path to ground offering least opposition to its passage. If suitable lightning arresters have not been installed at the station, the discharge may enter a building and jump through woodwork to gas or heating pipes and follow them to ground, the woodwork taking fire during the passage. If the transformers or outside wires are not properly insulated or supported, contact may be made between high- and low-pressure circuits; and the wiring in a building may thus be subjected to an electromotive force much higher than that for which it was intended, making it hazardous to both life and property. Many cases are on record where death has resulted from touching a lamp socket or fixture, a fan motor, or a telephone, though ordinarily these appliances can be handled with perfect safety.

THE NATIONAL ELECTRICAL CODE

4. Soon after electric lights first came into general use, the insurance companies discovered that there were many fires of electrical origin, because of poor wiring. On account of this, the various associations of underwriters formulated rules, governing wiring, and refused to insure buildings that failed to fulfil their requirements. In the course of time, these various rules of local associations were reduced to a uniform code, and finally, in 1898, they became known as the **National Electrical Code** and received the indorsement of practically all the inspection bureaus throughout the United States, besides that of the following organizations: the American Institute of Architects, the American Institute of Electrical Engineers, the American Society of Mechanical Engineers, the American Street Railway Association, the Factory Mutual Fire Insurance Companies, the National Association of Fire Engineers, the National Board of Fire Underwriters, the National Electric Light Association, and the Underwriters' National Electric Association. These rules are variously referred to briefly as the Underwriters' rules, the National Code rules, or simply the Code rules. These names will be used interchangeably in the following pages.

A few cities have rules of their own that differ slightly from this Code, but the differences are not vital. Any person, before designing or doing a job of wiring in any city where there is municipal legislation governing such work, should investigate the laws of that particular place. Every architect and electrical contractor should be supplied with a copy of the latest edition of the National Electrical Code, and all wiring and fittings should comply with those rules, whether additional laws exist or not. Copies of the Code and of all other information published by the Underwriters' Association for the sake of reducing the fire-hazard may be obtained from the National Board of Fire Underwriters at Chicago or from the nearest Underwriters' Inspection Bureau. The rules are revised by conventions as often as changes in the electrical art make such revision necessary.

In the following pages, wherever positive statements are made, the meaning is intended that the rules given must be followed in order to make the buildings insurable.

5. Fittings That May Be Used.—In addition to this Code of rules, the National Board of Underwriters publishes twice each year a **list of approved fittings** for use in connection with the Code. This list contains the names of articles that have been found entirely satisfactory, together with the names of the manufacturers. It does not mention all fittings that will pass inspection, and many good articles are not listed in its pages. Specifications for wiring should always call for fittings to be *National Code standard* or those equally good.

6. The rules of the National Electrical Code will be frequently referred to in the following pages. The general suggestions contained in these rules are deemed of sufficient importance to quote in full. They are as follows:

General Suggestions.—In all electric work, conductors, however well insulated, should always be treated as if bare, to the end that under no conditions, existing or likely to exist, can a ground or short circuit occur, and so that all leakage from conductor to conductor, or between conductor and ground, may be reduced to the minimum.

In all wiring, special attention must be paid to the mechanical execution of the work. Careful and neat running, connecting, soldering, taping of conductors, and securing and attaching of fittings, are especially conducive to security and efficiency, and will be strongly insisted on.

In laying out an installation, except for constant-current systems, every reasonable effort should be made to secure distribution centers located in easily accessible places, at which points the cut-outs and switches controlling the several branch circuits can be grouped for convenience and safety of operation. The load should be divided as evenly as possible among the branches, and all complicated and unnecessary wiring avoided.

The use of wireways for rendering concealed wiring permanently accessible is most heartily indorsed and recommended; and this method of accessible concealed construction is advised for general use.

Architects are urged, when drawing plans and specifications, to make provision for the channeling and pocketing of buildings for electric-light or power wires.

EXAMPLES OF ELECTRICAL FIRES

7. Quarterly reports of the National Board of Fire Underwriters contain many examples of fires of electrical origin. A few will be quoted:

1. After a motor circuit had been installed and inspected, drop lights were installed on the same circuit, using paraffin-covered wire for pendants. A short circuit ignited the paraffin and the building was burned.

2. Electric-light wires were run unprotected through damp woodwork in a brewery. The moisture caused a short circuit or ground, and the woodwork was set on fire.

3. A circuit of No. 14 B. & S. wire was supported in a damp place on porous crockery knobs. A partial short circuit caused the wires to

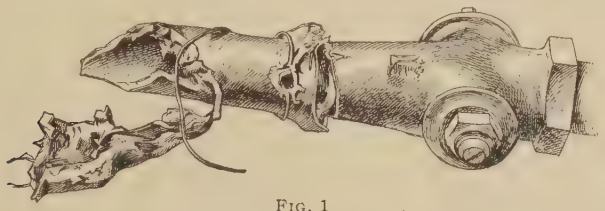


FIG. 1

become overheated. The insulation on the wires was ignited, and this set fire to some adjacent combustible material, resulting in a loss of $15,000.

4. A lighting circuit became grounded on a gas pipe in an attic. An arc formed and burned a $\frac{1}{4}$-inch hole in the pipe and set fire to the escaping gas.

5. A circuit carrying a heavy current came in contact with a signal circuit that was connected to a gas pipe by a poor and unsoldered

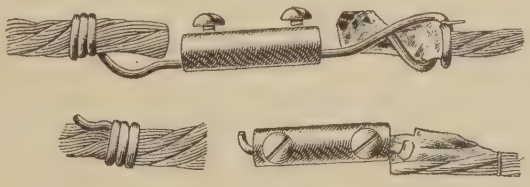


FIG. 2

joint. The current in leaking to ground through the signal wire to the gas pipe formed an arc in passing from the wire to the pipe and melted the latter, as shown in Fig. 1.

6. Poor and unsoldered joints gave rise to heating, and the wires were burned off, as shown in Fig. 2.

7. A short circuit caused by a fuse inside of a fixture canopy melted a hole in the canopy and set fire to the ceiling above. A canopy ruined in this way is shown in Fig. 3.

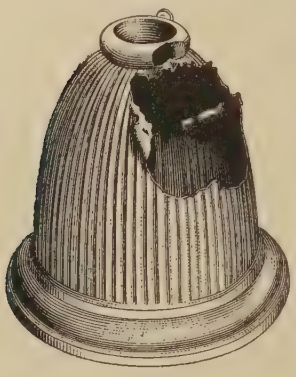


FIG. 3

8. An uncovered fuse in the loft of a livery stable blew and ignited surrounding straw.

NOTE.—Some of the terms used in the foregoing examples may not be entirely clear at present, but they are all explained in the following pages.

8. Many other examples of electrical fires could be given, but these will serve to emphasize the fact that extreme care is necessary in installing and inspecting all wiring in a building, no matter for what purpose the wires are installed. Even though a wire may not be intended to carry a dangerous current, it may accidentally become crossed with one that does carry such a current and may thereby become a source of danger.

OTHER CONSIDERATIONS

9. Satisfactory operation, convenience and neatness, and economy are all secondary considerations to that of safety, but all should receive attention. To install a system of wiring that will comply with all the Underwriters' rules is possible, and, although it may be considered safe, it may not be satisfactory; for, unless the size of the wires in each circuit is so calculated that lamps receive the proper voltage, the lights may be either too dim or too bright, or they may not all glow with the same brilliancy. It is evident that lamps should be located where they will be convenient, and that the job should be neat and workmanlike to be satisfactory to the customer. Finally, all unnecessary expense should be avoided in order to make the job profitable.

MATERIALS FOR LIGHT AND POWER WIRING

CONDUCTORS

10. Dimensions.—Wires for electric conductors are made with certain standard diameters, each diameter usually being specified by a *gauge number*. There are several systems of gauge numbers in use, that most used in the United States being the American, or Brown & Sharpe (usually written B. & S.), gauge.

The diameter of a wire is usually expressed in *mils*, and its area in *circular mils*. One **mil** equals .001 inch; for example, a No. 18 B. & S. gauge wire is 40.3 mils, or .0403 inch in diameter. A **circular mil** is the unit of cross-sectional area of wires, and the area of a wire, in circular mils, is equal to the square of its diameter in mils. For example, a wire 1 mil in diameter has a cross-sectional area of 1 circular mil; a No. 14 B. & S. wire has a diameter of 64.08 mils and an area of $64.08 \times 64.08 = 4{,}106.2$ circular mils. The second and third columns of Table I give diameters and areas of the B. & S. sizes most generally used. The Code rules do not permit the use of wire smaller than No. 14 B. & S. for electric-lighting purposes, except in fixtures and in flexible cords. The odd-sized wires, 7, 9, 11, etc., are seldom used for wiring buildings.

11. An inspection of Table I will show that the change in size of wire, as the gauge numbers increase or decrease, follows a very simple rule, which, if understood, will be very useful in making calculations. Starting from any gauge number as a basis, an increase of three gauge numbers gives a wire approximately one-half as large. For example, a No. 3 wire has approximately one-half the area of a No. 0, and twice the area of a No. 6; a No. 8 wire is approximately one-half as large as a No. 5, and twice as large as a No. 11; etc. Each decrease of one gauge number in the table gives a wire about 26 per cent. larger. For example, No. 13 is about 26 per cent. larger than No. 14; that is, the area of No. 14 multiplied by 1.26 gives very nearly the area of No. 13; etc.

TABLE I

WIRE TABLE

B. & S. Gauge Number	Dimensions		Carrying Capacity		Resistance at 68° F.		Weight per 1,000 Feet
	Diameter Mils	Area Circular Mils	Rubber-Covered Wires Amperes	Weather-Proof Wires Amperes	Ohms per 1,000 Feet	Feet per Ohm	
0000	460.00	211,600.0	210	312	.049	20,440	640.5
000	409.64	167,804.6	177	262	.062	16,210	508.0
00	364.80	133,079.4	150	220	.078	12,850	402.8
0	324.86	105,534.4	127	185	.098	10,190	319.5
1	289.30	83,694.2	107	156	.124	8,083	253.3
2	257.63	66,373.4	90	131	.156	6,410	200.9
3	229.42	52,633.5	76	110	.197	5,084	159.3
4	204.31	41,742.5	65	92	.248	4,031	126.4
5	181.94	33,101.8	54	77	.313	3,197	100.2
6	162.02	26,250.4	46	65	.394	2,535	79.5
7	144.28	20,816.7			.497	2,011	63.0
8	128.49	16,509.7	33	46	.627	1,595	50.0
9	114.43	13,093.8			.791	1,265	39.6
10	101.89	10,381.6	24	32	.997	1,003	31.4
11	90.74	8,233.7			1.257	795.3	24.9
12	80.81	6,530.3	17	23	1.586	630.7	19.8
13	71.96	5,178.2			2.000	500.1	15.7
14	64.08	4,106.2	12	16	2.521	396.6	12.4
15	57.07	3,257.0			3.179	314.5	9.9
16	50.82	2,582.7	6	8	4.009	249.4	7.8
17	45.26	2,048.5			5.055	197.8	6.2
18	40.30	1,624.1	3	5	6.374	156.9	4.9
19	35.89	1,288.1			8.038	124.4	3.9
20	31.96	1,021.4			10.140	98.7	3.1

12. Wire Gauges.—One who is accustomed to handling wires of various sizes soon learns to tell, with considerable certainty, the gauge number of a wire by its appearance; but a wire gauge should frequently be used, so as to avoid mistakes. Fig. 4 shows one style of **wire gauge,** which is made by cutting suitable notches in the rim of a steel disk; a wire should just enter the slot corresponding to its gauge number. Such gauges can be obtained from any hardware dealer.

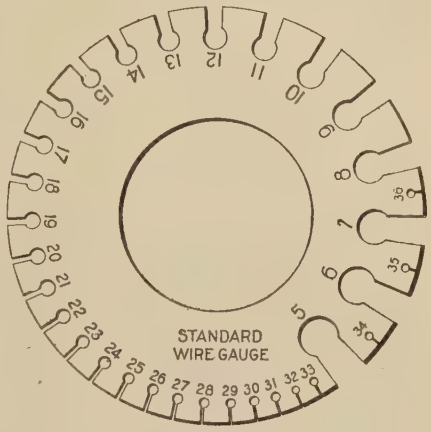


FIG. 4

13. Insulation.—For interior wiring, the Underwriters' rules usually specify **rubber-covered wire,** though **slow-burning weather-proof insulation** is allowed in one or two cases. In applying the rubber covering, the copper is first tinned, then covered with rubber, and on the outside is placed a braiding of cotton soaked in a preservative compound. Slow-burning weather-proof wire, permitted for open work in dry places, is covered with two coatings—a weather-proof braid next to the wire and a fireproof braid on the outside. The outside layer is treated with a compound and smoothed down to a hard, dense finish. Wire obtained from any reputable manufacturer will conform to the rules. Weather-proof wire is less expensive than rubber-covered wire.

14. Carrying Capacity.—As all electric conductors carrying a current are heated to some extent, and as the temperature rise of a conductor depends on the amount of current it is carrying, there is a limit to its safe-carrying capacity. The National Code rules specify the limits given

in the fourth and fifth columns of Table I. Under no circumstances should a wire be installed to carry more current than is permitted by the rules. In most cases, it will not be possible to carry the full amount permitted without excessive drop in potential, so that there will usually be little temptation to violate the rules in this respect.

15. Resistance.—The resistance of a conductor depends on several conditions, among them being the material of which the conductor is made, the temperature of the conductor, and its length and cross-sectional area. Practically, all conductors for interior wiring are made of very nearly pure copper, and all formulas and wire calculations in this Section are based on the use of copper wire. The resistance of copper wire is slightly different at different temperatures, but the change is not great enough to affect ordinary wiring calculations. The only things that need here be considered as affecting the resistance of a conductor are its length and its sectional area: the greater the length, the greater is the resistance; and the greater the sectional area, the less is the resistance. This may be expressed as follows:

Rule.—*The resistance of a conductor varies directly as its length and inversely as its sectional area.*

A copper wire 1 foot long and 1 circular mil sectional area is called 1 **mil-foot** of copper. In wiring calculations, the resistance of 1 mil-foot of copper may be taken as 10.8 ohms. Then, from the rule just stated, a wire 2 feet long and 1 circular mil sectional area has a resistance of $2 \times 10.8 = 21.6$ ohms; and a wire 2 feet long and 2 circular mils cross-section has a resistance of

$$\frac{2 \times 10.8}{2} = 10.8 \text{ ohms}$$

Let R = resistance to be determined;
L = length of wire;
$C.\,M.$ = circular mils sectional area.

Then,
$$R = \frac{10.8\,L}{C.\,M.}$$

EXAMPLE.—Calculate the resistance of a wire 1,000 feet long with 10,381 circular mils cross-section.

SOLUTION.—By the formula

$$R = \frac{10.8 \times 1{,}000}{10{,}381} = 1.04 \text{ ohms. Ans.}$$

NOTE.—Resistances calculated by using the constant 10.8 are slightly higher than those given in Table I. The calculations for the table were made for a temperature of 68° F., at which temperature the constant is about 10.35.

16. Joints.—It is best, wherever possible, to avoid making joints. When they must be made, the rules require that they first be made *mechanically and electrically secure* and then soldered, to keep the contact good. This rule applies to all interior wiring. In making a joint, the insulation is first stripped from the portions to be joined, the wire is scraped until it is clean and bright, and the bright portions are twisted together. Figs. 5 and 6 show joints in common use. After twisting the wires together, they are soldered, and the joint is then covered with insulating tape.

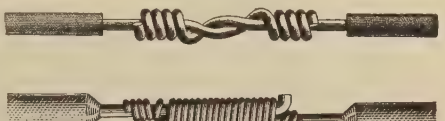


FIG. 5

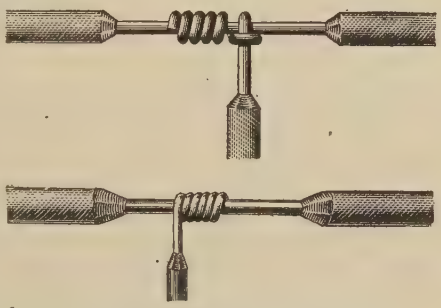


FIG. 6

FIG. 7

The strands of stranded conductors, Fig. 7, must be soldered together before being put under clamps or binding screws; and all conductors, stranded or solid, when their conductivity is greater than that of No. 8 B. & S. wire, must be soldered into lugs, as shown in Fig. 8, for terminals.

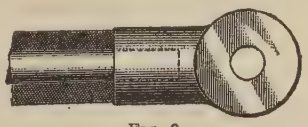


FIG. 8

17. Flexible Lamp Cord.—The National Code rules give very minute instructions for the manufacture and use of flexible lamp cord. Unless properly insulated and installed, flexible cord becomes a source of much trouble. Some of the most important requirements are: flexible cord must not be used where the pressure is over 300 volts; it must not be used as a support for clusters; it may be used only for pendants, wiring of fixtures, and portable lamps or motors, and portable heating apparatus; it must not be used in show windows; it must be protected by insulating bushings where it enters a lamp socket; it must be so knotted inside the lamp socket or the rosette from which it is suspended that the weight supported will not be borne by joints or binding screws. It is recommended that cords for pendant lamps be made no longer than is needed, so that it will not be necessary to loop them up with cord adjusters. Different grades of insulation are made, and a grade of insulated cord should be selected appropriate for the service it is to perform.

SWITCHES AND CUT-OUTS

18. Switches.—An **electric switch** is a device for opening or closing a circuit; it affords a means for interrupting or establishing the flow of current in a circuit. A *single-pole switch* controls one wire only, a *double-pole switch* controls two wires, a *triple-pole switch*, three wires, etc. A switch must not be single-pole except when the circuit that it controls supplies devices that require not over 660 watts, and when the electromotive force is not over 300 volts. Switches for interior wiring may be divided into two general classes, *knife switches* and *snap switches*.

19. Knife switches may be *single-throw* or *double-throw*, according to whether they can be closed one way or two ways. Fig. 9 shows a double-pole single-throw (abbreviated D. P. S. T.) knife switch with enclosed fuses *f*, *f* that melt and open the circuit if the current becomes too high, and Fig. 10 shows a double-pole double-throw (abbreviated D. P. D. T.)

fused knife switch. The circuit wires are soldered into the lugs shown attached to the switch terminals. The blades close between clips so that a good contact is made.

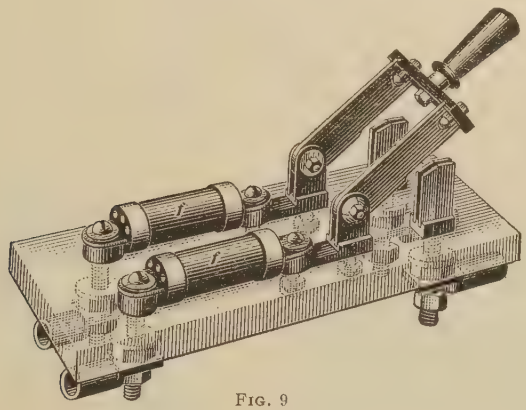


FIG. 9

The rules require that single-throw knife switches be so mounted that gravity will tend to open rather than close them. Double-throw switches may be mounted so that the throw will be vertical or horizontal as preferred. When possible, knife switches should be so connected that the

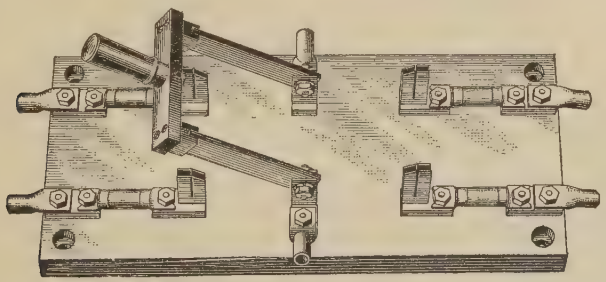


FIG. 10

blades are *dead*, that is, disconnected from the circuit when the switch is open. It is recommended that all knife switches be mounted in cabinets. The rules positively require dust-proof cabinets for knife switches in places where combustible particles may be ignited by a spark on opening the switch.

20. Snap Switches.—There are many styles of good **snap switches** on the market; one or two only will be described. Approved indicating snap switches are recommended by the Underwriters for lighting circuits where the voltage is not over 250, or the current over 30, amperes. By the word *indicating* is meant those that indicate plainly by a rotating dial or by the position of the handle whether the current is on or off. Indicating switches are preferred in all cases.

Fig. 11 (*a*) shows an indicating single-pole snap switch in the on-position, which is plainly indicated, and (*b*) shows the switch with the cover removed. The wires come through

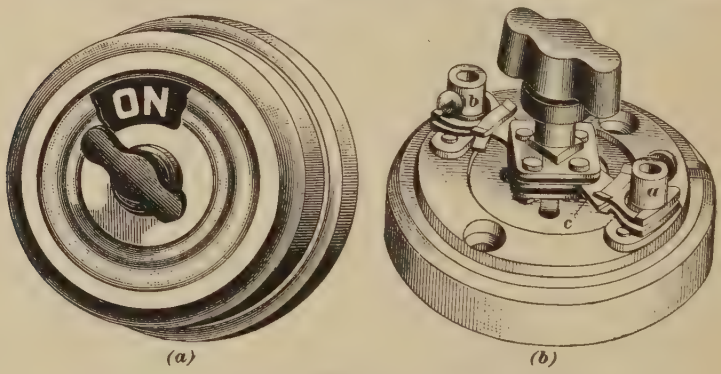


FIG. 11

the porcelain base and are held in the binding posts *a*, *b*, which also carry the switch contacts. When the switch is closed, as shown in (*b*), the blades of the cross-piece *c* rest against the switch contacts, and thus close the circuit. When the handle is turned, the blades stick until the tension on a coiled spring (not shown) causes them to leave the contacts and the cross-piece turns through an angle of 90° with a snap. The style of switch shown in Fig. 11 is also made double-pole.

Fig. 12 (*a*) shows the general appearance of a single-pole flush wall switch of the push-button type, and (*b*) illustrates the mechanism. When the light button is pushed in, cross-piece *c*, *c* swings around and makes contact with the clips *a*, *b*,

and at the same time the dark push button flies out. Pushing in the dark button causes the cross-piece to snap to the off-position and the light button comes out. In each case, a

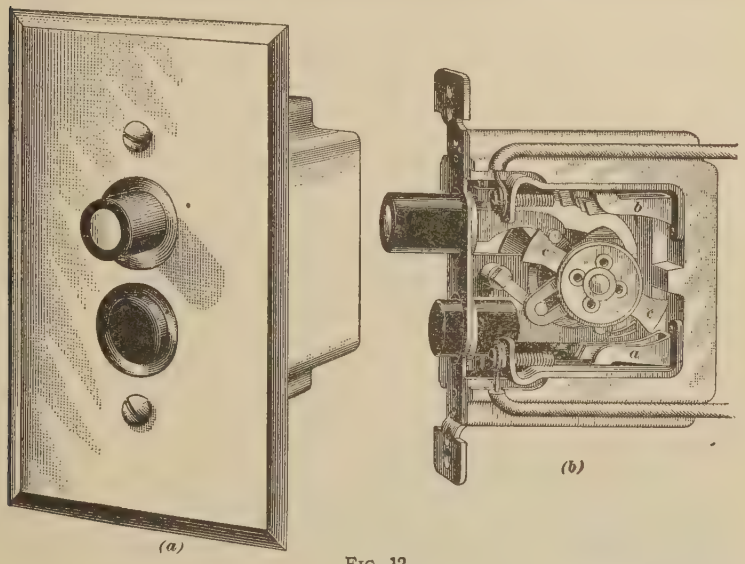


FIG. 12

coiled spring (not shown) makes the action quick and positive, so that there is little sparking.

21. Switch Boxes.—When switches are mounted flush with the wall, an iron or steel box must be installed to enclose the switch mechanism and its porcelain base. Fig. 13 shows a **stamped-steel switch box.** The cover, which carries the switch, is attached to the box by means of screws passing through slotted holes. The slots permit the cover to be set true, even though the box is slightly twisted in setting. Holes are provided in the sides of the box for the entrance of wires.

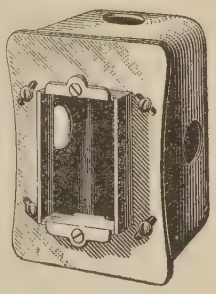


FIG. 13

22. Location of Switches.—The rules require that on all service wires, in a readily accessible place, as near as

possible to the point where they enter the building, shall be placed a switch and cut-out arranged to cut off all the current. All switches throughout the building should be placed in dry accessible places, and should be grouped as far as possible. If located in damp places, they must be mounted on porcelain knobs. Switches must not be placed in the vicinity of easily ignitible material, or where gas or flying particles may be fired by a spark.

23. Automatic cut-outs, or devices for automatically opening overloaded circuits, are of two kinds—*fuses*, or *fusible cut-outs*, and *circuit-breakers*. Branch circuits have a fuse in each of the two wires. A double-pole cut-out is usually installed. The load of incandescent lamps dependent on the branch cut-out must not exceed 660 watts. If possible, all cut-outs should be mounted in dry places; if this cannot be done, they should be mounted on porcelain knobs.

24. Fuses.—The ordinary fusible cut-out consists of a piece of metal wire that melts, or *blows*, when the current through it is larger than any part of the circuit to be protected can carry safely. Fig. 14 shows an **ordinary link fuse,** a type formerly much used and still permitted if mounted on slate or marble bases, but not permitted on porcelain bases. The fuse wire *c* has attached to

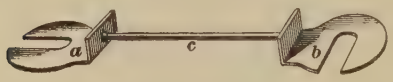


FIG. 14

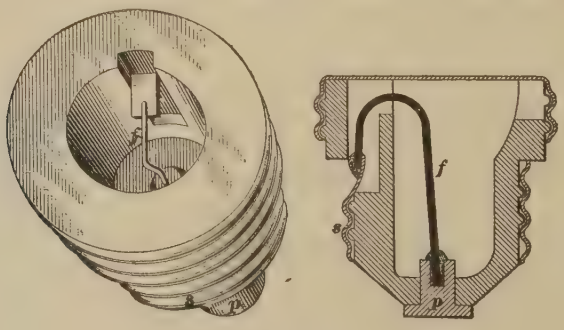


FIG. 15

each end, terminals *a*, *b*, which fit under screw heads to which the circuit wires are attached. Open-link fuses are being superseded by enclosed fuses.

Fig. 15 (*a*) and (*b*) shows an **Edison fuse plug** much used in interior wiring for circuits where the voltage is not over 125 or the amperes over 30. The fuse *f* is mounted inside of a porcelain holder, or plug, and is attached to the screw terminal *s* and the contact *p*. The plug is usually provided with a cover, which sometimes consists of mica, so that the fuse can be seen, and sometimes of metal. The plugs screw into receptacles in the fuse blocks, and a new plug, of the type shown, must be used whenever a new fuse is necessary. Other types of fuse plugs are made so that a new fuse wire can be inserted in an old plug.

25. Most enclosed fuses are of the so-called **cartridge type,** shown at *f*, *f*, Fig. 9. The fuse wire is enclosed in an insulating tube and is connected to metal terminals. The terminals may be provided with blades that will slide under screws head, as in Fig. 9, or they may be in the form of metal rings that fit into clips, as in Fig. 16, where rings *b*, *b* on fuse *a* fit into clips *c*, *c*. An objection to some enclosed fuses is the difficulty of telling whether a fuse has blown or not. In the fuses shown in Fig. 16, this objection is overcome by shunting the main fuse with a small wire that runs under a label on the cartridge. When the main fuse blows, the small wire at once melts and makes a mark on the label.

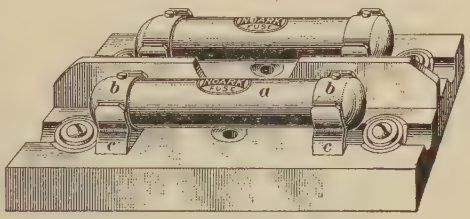


FIG. 16

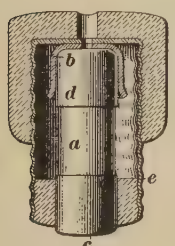


FIG. 17

Fig. 17 shows an adaptation of the cartridge-type fuse to an Edison plug. Such plugs can be used with pressures as high as 250 volts. The cartridge is pushed through the hole in the bottom of

the plug, and is held in place by the clip *b*; current passes through the fuse by way of contacts *c*, *d*, *e*. When a fuse blows, a new cartridge is all that is necessary to be restored.

26. Fuse Blocks.—Fig. 18 (*a*) shows a three-wire **main block,** (*b*) shows a three-wire **branch block,** and

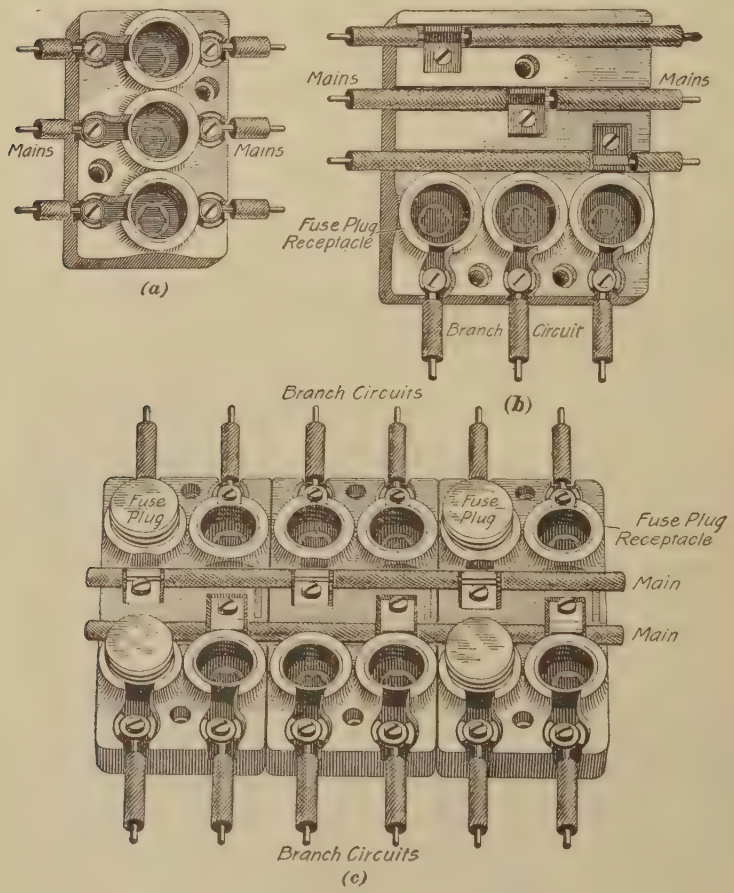


FIG. 18

(*c*) shows three two-wire **double-branch blocks,** grouped together to form a distribution center. The connections of circuit wires and the fuse-plug receptacles are plainly indicated.

Connections between circuit wires and receptacles are in some cases made through channels in the back of the block. This is the case with branch blocks where the receptacle may lie some distance from a wire to which it connects. Fig. 16 shows a two-wire main block with enclosed fuses.

27. A **circuit-breaker** is an automatic switch that is controlled by an electromagnet, so that the switch will open when the current becomes too large. Except in connection with electric motors, circuit-breakers are not much used in interior wiring. The Code rules do not permit their use, unless fuses are also provided, except on switchboards or other places where expert supervision can be maintained.

28. Location of Cut-Outs.—In addition to the requirements already mentioned, cut-outs must be placed at every point where a change is made in the size of wire, unless the nearest cut-out in the larger wire circuit will open before the smaller wire can be injured. Cut-outs should be in plain sight or enclosed in an approved cabinet. They must not be placed in the canopies or shells of fixtures, nor in the vicinity of combustible materials, gas, or dust that might be ignited when a fuse blows.

29. Rating of Cut-Outs.—Every fuse must be plainly marked with its rating in amperes and volts. The rated capacity of the fuses on a circuit must not exceed the safe-carrying capacity (as given in Table I) of the wires to be protected. Circuit-breakers must not be set for more than 30 per cent. above the safe-carrying capacity of the wire, unless a fusible cut-out is also installed in the circuit.

SYSTEMS OF DISTRIBUTION

CONSTANT-CURRENT SYSTEM

30. A **constant-current system** is one on which a number of appliances are arranged to operate in series with one another, the current, which may be either direct or alternating, being maintained nearly constant through all. This system is now used principally for series-arc

lamps. Fig. 19 is a diagram of a circuit of direct-current series-arc lamps. A constant current passes from the positive terminal of the dynamo *A* to the positive terminal *t* of the first lamp in the series, through the lamp and out at negative terminal *t'*, to the positive terminal of the second lamp, and so on through each lamp in the series. If one lamp *B* were connected wrong, as shown, current would enter at the lower carbon, and the lamp would burn upside

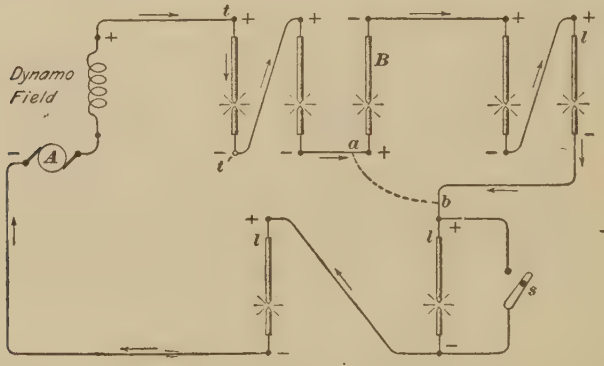


FIG. 19

down until the lower carbon was exhausted and the carbon holder destroyed. The current through all the lamps is exactly the same, unless there is a leakage through ground from one portion of the circuit to another, as indicated by the dotted path *a–b*. A short-circuiting switch is usually installed in the top of each lamp, so that the lamp can be cut out of service at any time; *s* shows the connections of a switch of this kind.

CONSTANT-POTENTIAL SYSTEMS

31. A **constant-potential system** is one on which the electromotive force is maintained nearly constant. The current may be either direct or alternating. The constancy with which the electromotive force is maintained, or the *regulation*, depends on the use to which the current is to be put. The regulation on systems that feed incandescent lamps must be very close.

32. The National Code rules divide constant-potential systems into three classes: (1) **low-potential systems,** 550 volts or less; (2) **high-potential systems,** 550 to 3,500 volts; and (3) **extra high-potential systems,** over 3,500 volts. Almost all wiring with which architects have to deal is covered by the first class, or low-potential systems. Wires of the second class sometimes enter buildings, but those of the third class never, except in power stations and substations.

33. Two-Wire Systems.—Fig. 20 is a diagram of connections of a dynamo supplying a two-wire system through *feeders A, B* and *centers of distribution C, D.* Incandescent lamps are represented by small circles connected to the *mains E, F* leading from the centers of distribution. The dynamo, or generator, supplies current at 125 volts, and the

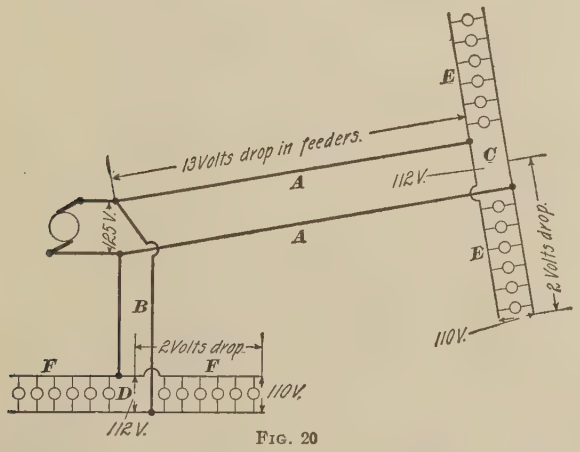


FIG. 20

feeders are calculated for a drop of 13 volts, so that the current reaches the centers of distribution at 112 volts. Such an arrangement is known as the **feeder-and-main system.** Feeders carry the current from the dynamo, or possibly from a transformer near the building to be lighted, to a central point to which all the lamp circuits connect.

34. Centers of Distribution.—In lighting large buildings, feeders are brought into the building to a *feeder panel*

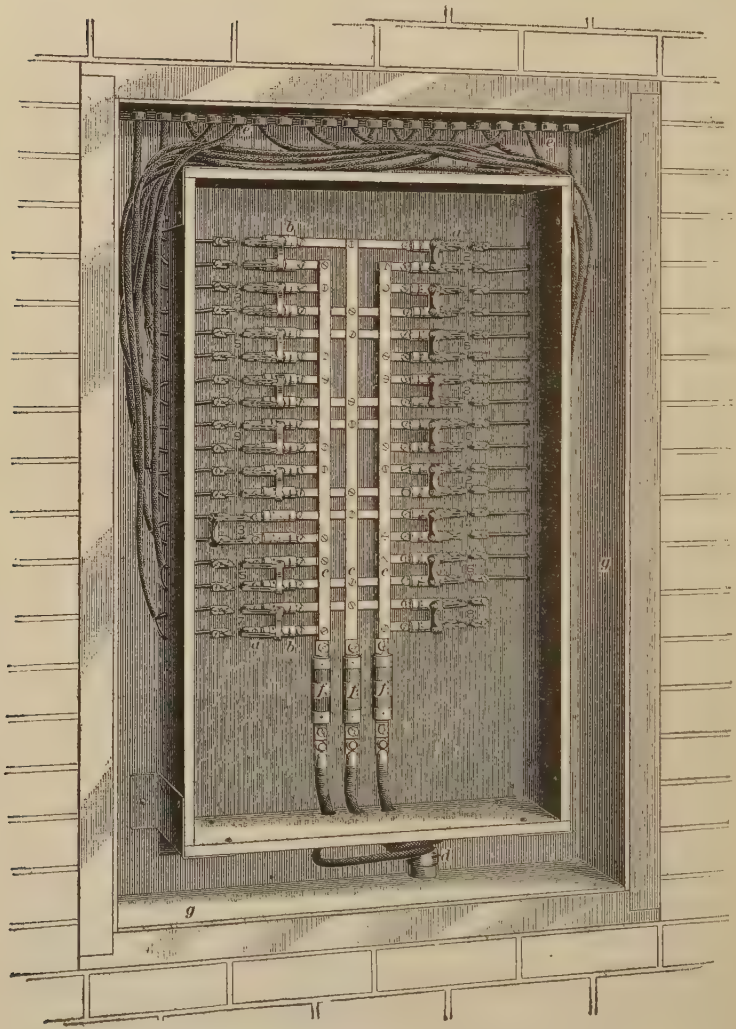


FIG. 21

located in a dry room, often a dry basement, from which mains are run to **centers of distribution** located at convenient points throughout the building. Sometimes, there is one center of distribution on each floor, and sometimes several are required. They are often located in hallways; they should be easily accessible without being too conspicuous. **Distribution circuits** lead from such centers to the lamps.

35. Panel Boards.—Centers of distribution are usually equipped with **cut-out cabinets,** or **panel boards.** Fig. 21 shows a cabinet containing a panel board for a three-wire system of conduit wiring, that is, with the wires run inside of iron tubes. The board is equipped with eighteen double-pole knife switches *a* and thirty-six enclosed branch-circuit fuses *b*. The mains leave the conduit *d*, enter the bottom of the cabinet, and are connected by fuses *f*, *f*, *f* to the bars *c*, *c*, *c*. The two-wire distribution, or branch, circuits pass out through conduits *e* and lead to the lamps. The horizontal copper bars to which the distribution circuits are connected are in contact with the vertical copper bars *c*, *c*, *c* only where the screw heads are shown, and are completely separated and insulated from them at other points.

The door and casings of the cabinet are removed in order to show the wires in the gutter *g*. The panel board constitutes the back of the cabinet, and the sides and ends are of $\frac{1}{2}$-inch slate. The lining of the gutter is of $\frac{1}{16}$-inch enameled iron or $\frac{1}{4}$-inch slate or marble. For some purposes, wooden cabinets completely lined with $\frac{1}{8}$-inch asbestos board firmly attached by shellac and tacks are permissible, but slate or marble is strongly recommended in all cases.

36. If a building is to contain constant-potential electric devices other than lamps, motors for example, they can all be connected to convenient centers of distribution, provided the feeders to the centers are properly proportioned to carry the current necessary for both the lamps and the other devices. For large motors whose demand for current is variable, or for a large number of motors operating under variable

conditions, it is often necessary to install power feeders and mains separate from those carrying current for the lamps. Otherwise a change of load on the motors will so affect the current, and consequently the drop in the feeders, that the brilliancy of the lamps will vary. This effect can be seen in the lamps of a trolley car operating some distance from the power station; when the car is starting or going up a grade, the motors are taking a large current, and the drop in the trolley wire or feeders to the trolley is so great that the lamps become dim. If current for the lamps could be brought to the car over lines separate from those supplying the motors, the change in the lights could be made much less; if the quantity of current required by the car motors were always the same, current for both lamps and motors could be brought over the same conductors without causing a variation in the brilliancy of the lamps. The same idea applies to the operation of motors or other devices and lamps in buildings. If the demand for current is practically constant, or at least if there are no sudden variations in demand, the same feeders and mains can be installed to carry all the current through the building.

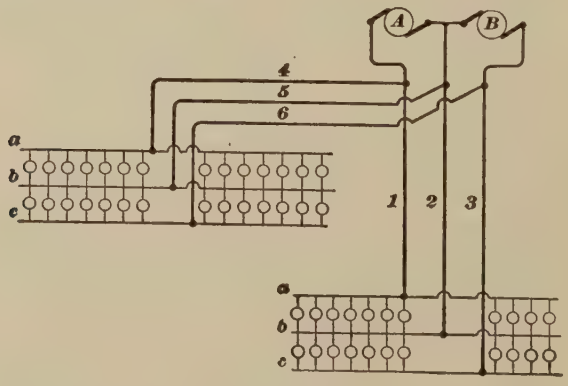


FIG. 22

37. Three-Wire System.—Fig. 22 is a diagram of connections for two dynamos *A* and *B* supplying an **Edison three-wire system** through wires *1*, *2*, *3*, *4*, *5*, *6*, and *a*, *b*, *c*. Conductors *1*, *3*, *4*, *6*, *a*, and *c* are called **outside wires;**

conductors *2*, *5*, and *b* are **neutral wires.** The simple two-wire system requires but one dynamo and two sets of conductors, one positive and the other negative. The simple three-wire system may be operated with one or two dynamos. Two dynamos were formerly thought necessary, and the system can be more clearly explained with two, but by using a balancing device, of which there are several available, one dynamo can be used. If two dynamos are used, they are connected in series, so that the voltage between outside wires is double what it would be with but one such machine; that is, the voltage between each outside wire and the neutral wire is the same as that of a two-wire system for the same lamps. If 110-volt lamps are used on a three-wire system, the voltage between each outside wire and the neutral wire must be 110 volts, and that between the two outside wires, 220 volts. This is called a **220-volt three-wire system,** and is probably the one most used. Either two 110-volt dynamos in series or one 220-volt dynamo with a balancer may be used. In some recent plants, 220-volt lamps are used, requiring 440 volts between outside wires.

38. The three-wire system is much used in cities where a large quantity of current is required within a comparatively small space. Its principal advantage over the two-wire system lies in the saving of copper needed for conductors. To illustrate this, suppose fifty 110-volt lamps, each requiring $\frac{1}{2}$ ampere, are connected to a two-wire circuit. Each wire must then carry $50 \times \frac{1}{2} = 25$ amperes. If, instead of this, the lamps were connected to a 220-volt three-wire circuit, twenty-five lamps on each side of the neutral, each outside wire would carry $25 \times \frac{1}{2} = 12.5$ amperes; that is, each outside wire need have but one-half as much carrying capacity, or be one-half as large, as one of the wires of the two-wire circuit. If all three wires of the three-wire circuit were made the same size, they would together be $3 \times \frac{1}{2} = 1\frac{1}{2}$ times as large as one wire of the two-wire circuit or three-fourths as large as both wires. But this is not all; electric-light wires are so calculated that there shall be a certain *per cent.*

drop of potential between the dynamo and the lamps. With a given per cent. drop, the volts lost in the outside wires of the three-wire 220-volt feeders will be double that in the two-wire 110-volt feeders. For example, 5 per cent. of 220 volts is 11 volts, and 5 per cent. of 110 volts is 5.5 volts. As the 220-volt feeders carry only one-half as much current as the 110-volt feeders, and, at the same time, have twice as much drop, they must each be one-fourth as large as one of the 110-volt feeders. But there are three of the 220-volt feeders and only two of the 110-volt feeders; hence, the three 220-volt wires must be three-fourths as large as one 110-volt wire or three-eighths as large as both.

39. This conclusion is based on the assumption that the lamps are equally divided between the two sides of the three-wire circuit, in which case the neutral wire carries no current. Such a system is called a **balanced three-wire system,** and the current passes from one outside wire to the other through pairs of lamps in series, the same current lighting both lamps in each pair. The middle, or neutral, wire might then be removed, and the system would then be called *parallel-series* or *multiple-series.* Such systems have been used, but are now forbidden by the Underwriters' rules.

If no neutral wire were used, each lamp would be dependent on another for its current; if one lamp of a pair should burn out or be switched off, both would go out. With a neutral wire, any number of lamps on either side of the system may be switched off without putting out any of those on the other side. In a perfectly balanced three-wire system, no current flows in the neutral wire, and hence it may be made smaller than the outside wires, thus effecting even greater saving over the two-wire system than that calculated in Art. **38.** Feeders to large buildings are sometimes installed with a neutral wire smaller than either of the outside wires; but for smaller circuits inside of buildings, where the load is more likely to become unbalanced, the three wires are made the same size.

40. Only two wires, a positive and a negative, need run to any lamp or cluster of lamps; hence, the three-wire system

terminates at the nearest center of distribution. Two-wire-distribution circuits, leading from the centers to the lamps or clusters, should be so connected that the system is as nearly balanced as possible. When motors are used on a three-wire system, they are usually connected to the outside wires, so that no part of the motor current passes over the neutral wire. Fig. 23 is a diagram of the connections at a three-wire-distribution center. The positive (+), negative (−), and neutral (∓) wires from the main switch and cut-out terminate in the distribution center. The small circles here represent terminals, or screws, under which wires and fuses are connected; the wavy lines between the circles represent fuses. Three two-wire distribution circuits are shown leading from each side to 110-volt lamps, and a circuit is also shown leading to a 220-volt motor.

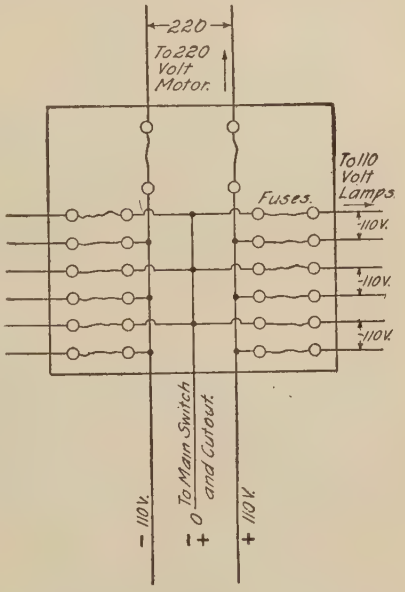


Fig. 23

41. In order to be able to control a part of a group of lamps on a two-wire system without running two extra

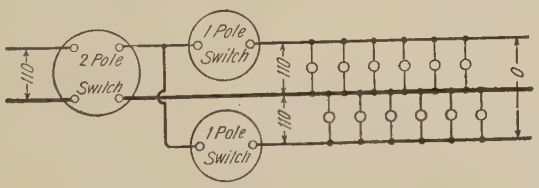


Fig. 24

wires, connections are sometimes made as shown in Fig. 24. A double-pole switch controls all the lamps in the group; a single-pole switch controls each of the two sections into which

the whole group is divided. The middle wire carries current for all the lamps in the group, and so must be made as large as the two outside wires together. Each outside wire carries only the current for the lamps on its side. This system is sometimes mistaken for the three-wire system. The rules will not permit its use where more than 660 watts is required by either of the two sections.

42. A combination of the two- and the three-wire system and a modification of the system described in Art. **41** is shown in Fig. 25. The lamps are supplied from a three-wire system when the triple-pole double-throw switch *S* is in the upper contacts, and from a two-wire system when the switch is in the lower contacts. Lower contacts *a*, *b* are connected by a conductor, which is represented by the dotted line. The switches are shown in position to supply current from the two-wire system. Current from the positive terminal of dynamo *A* flows through one side of the single-pole single-throw switch *S'* to *c*, where it divides, part going directly to the lamps on one side of the system and part through switch *S*–*a*–*b* to the lamps on the other side. Thus, current flows out to the lamps on the outside wires, through the lamps to the middle wire, and then the whole current comes back to *d*, whence it flows through switch *S'* to the negative terminal of the dynamo. The cross-sectional area of the middle wire must therefore be as great as

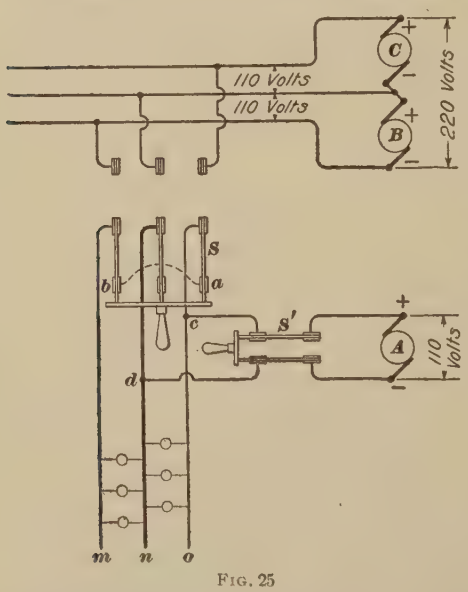


FIG. 25

the combined areas of the two outside wires. When the current is to be taken from the three-wire system, switch S' is opened and switch S is closed in the upper contacts. This method of connecting is useful where a building is to be lighted part of the time from three-wire street mains and part of the time from its own dynamo. When lighted from the two-wire system, the dynamo A must generate a slightly higher electromotive force than is supplied by the street mains of the three-wire system; since, with the two-wire system, the return current through the middle wire causes therein a drop of potential, which does not occur when the three-wire system is used. The current in the outside wires is the same in value in either case, though not necessarily the same in direction; hence, the drop of potential in the outside wires is the same with either system. In the three-wire system, only the drop in the outside wires need be considered, since whatever current flows in the middle wire is deducted from that in one of the outside wires. It is often advisable to have a building wired in this way, so that a change from one system to the other can be made at a future time without requiring expensive changes in the wiring.

ALTERNATING-CURRENT SYSTEMS

43. The connections already illustrated and described for constant-potential systems may be used for either direct or alternating current. On account of the ease with which alternating current may be transformed from one pressure to another, it is usually transmitted at a high pressure to a point near the buildings in which it is to be used, and is then transformed down to a working pressure. Alternating-current systems are classified as *single-phase* and *polyphase*, the latter class being subdivided into two- and three-phase systems. A higher number of phases are sometimes used in large power stations, but the alternating-current systems used for ordinary light and power purposes will come under the classes named.

44. Single-Phase System.—Ordinary interior wiring for the **single-phase system** does not differ materially

from that for the constant-potential two-wire system already described. Single-phase current may, however, be used to feed a three-wire system, as shown in Fig. 26. The current comes from the alternator at a high pressure on the feeders *A*, *B*. It then passes through primary fuses *f*, *f*, *f*, *f* and through the primaries of transformers *T*, *T*. The transformer secondaries are connected to the three-wire mains *C*, which carry the current to the centers of distribution throughout the building. The pressure between either outside wire

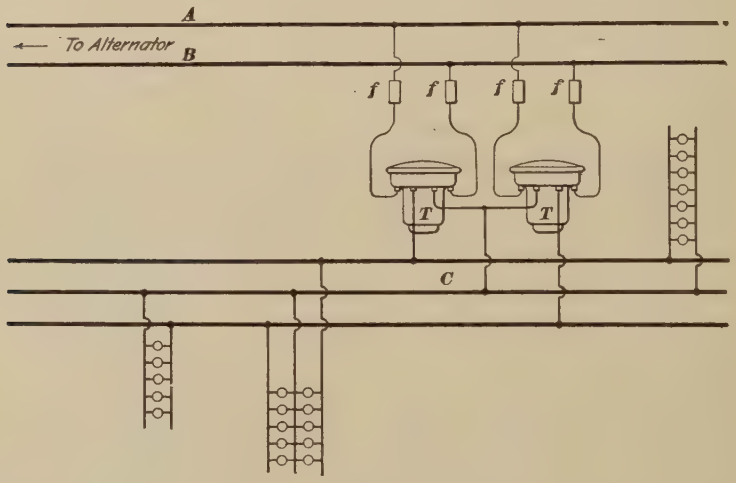


FIG. 26

and the middle wire is that delivered by the secondary of one transformer, and that between the two outside wires is the combined electromotive force of the two secondaries. A very common practice is to use from 2,000 to 2,200 volts primary pressure and from 100 to 110 volts secondary pressure. This makes the pressure between an outside and a middle wire from 100 to 110 volts, and that between the outside wires from 200 to 220 volts.

45. Polyphase Systems.—The **two-phase system** may be looked on as two separate single-phase systems. Four wires are generally used—two for each phase. If desired, transformer connections may be so made to only

one phase that the house wiring to the distribution centers can be three-wire, as shown in Fig. 26.

A **three-phase system** usually requires three wires, though sometimes, when motors and lamps are to be connected to the same circuit, a fourth wire is used for the lamp connections. This is to prevent the varying current required by the motors from affecting the pressure at the lamps.

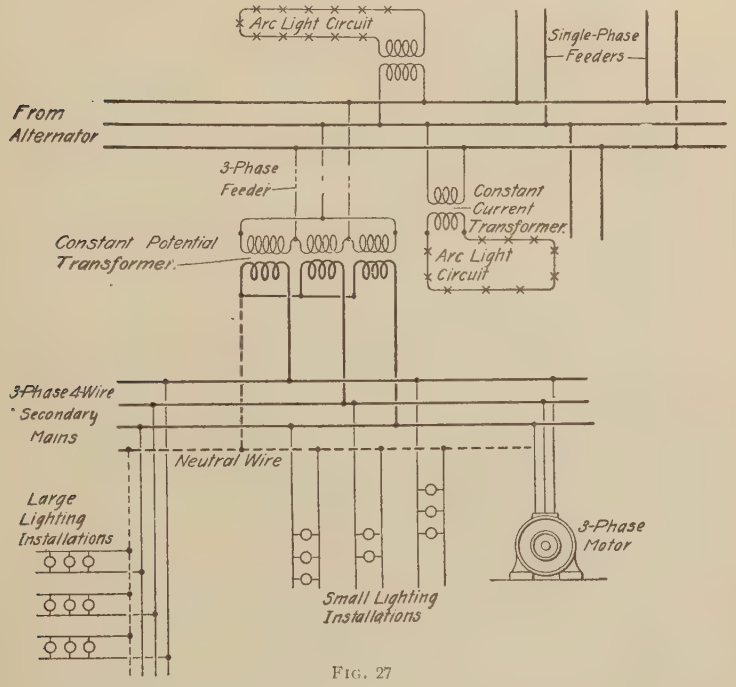


FIG. 27

Fig. 27 is a diagram of connections of a three-phase system using a fourth, or neutral, wire for the lamp connections. The lamps are all connected between the neutral wire and one of the other three; the motors are connected only to the three-phase wires. Separate leads are carried to the larger lighting installations. In this case, the four wires are fed by the constant-potential transformer, but in some cases all four wires run to the alternator, which then has four collector rings. Connections are also shown for constant-current

transformers, for supplying current to arc lamps. Single-phase feeders may be connected to any one of the three phases. All the devices should be so distributed that the system will be balanced as nearly as possible.

DETERMINING THE SIZE OF WIRES

ESTIMATING THE CURRENT

46. Current for Lamps.—Before calculating the size of wire, it is necessary to know the quantity of current to be carried. This may be determined very closely by estimating the number of lamps, motors, or other devices to be supplied and the current required by each. The quantity of current, either direct or alternating, required by incandescent lamps depends on the candlepower and the efficiency of the lamps. Instead of stating the efficiency of lamps, it is customary to state their consumption, in watts per candlepower. The total power, in watts, required by all the lamps is the product of three factors—the number of lamps, the candlepower, and the consumption. The current equals the total watts divided by the volts of the system.

Let I = total current, in amperes;
N = number of lamps;
C = candlepower;
W = consumption, in watts per candlepower;
E = volts.

Then, $$I = \frac{NCW}{E}$$

On a balanced three-wire system, the voltage E must be that between the outside wires. The candlepower times the watts per candlepower, or CW, equals the total watts per lamp, and this value, if known, may be used instead of the factors CW. An ordinary 16-candlepower 110-volt lamp of the type heretofore much in use consumes from 3.1 to 3.5 watts per candlepower, making a total consumption of from 50 to 56 watts per lamp. Assuming an average of 55 watts per lamp, the current consumption on 110 volts is $55 \div 110 = .5$

ampere per lamp. This is a fair average value for 16-candlepower 110-volt carbon-filament lamps, but is not correct for the newer lamps now coming into use. The formula just given is general, and may be used for all types of lamps.

EXAMPLE.—Current is carried by three-wire feeders to a center of distribution, from which distribution circuits run to the following groups of 110-volt incandescent lamps:

NUMBER OF LAMPS	CANDLEPOWER	CONSUMPTION WATTS PER CANDLEPOWER
12	10	3.6
18	16	3.3
10	32	3.2

If the voltage between the outside wires is 220, determine the current carried by each of the three wires when the system is balanced and all the lamps are burning.

SOLUTION.—Substitute values for each size lamp in the formula as follows:

1. For the 10-c.-p. lamps, $I = \frac{12 \times 10 \times 3.6}{220} = 1.96$ amperes
2. For the 16-c.-p. lamps, $I = \frac{18 \times 16 \times 3.3}{220} = 4.32$ amperes
3. For the 32-c.-p. lamps, $I = \frac{10 \times 32 \times 3.2}{220} = 4.65$ amperes

Total current for all lamps, 10.93 amperes

When the system is balanced, each outside wire will carry 10.93 amperes and the middle wire will carry no current. Ans.

47. For ordinary calculations, with carbon-filament incandescent lamps, the data in Table II may be used.

TABLE II

POWER CONSUMPTION OF INCANDESCENT LAMPS

Candlepower	Voltage	Current Amperes	Watts
8	110	.27	30
10	110	.32	35
16	110	.50	55
16	52	1.00	52
16	220	.30	66
32	110	1.00	110

This data will not be correct for the higher-efficiency lamps now coming into use.

The current taken by enclosed-arc lamps varies with the make and size of the lamp. About 5 amperes is a fair average for constant-potential enclosed arcs, though in some cases, lamps are designed for 6 amperes, while in others where a long arc is used, the current is as low as 4 amperes.

48. Current for Motors.—Motor ratings in *horsepower*, volts, and amperes are nearly always marked on the name plates attached in some conspicuous place to the frames of the motors. One horsepower is equivalent to 746 watts; but in order to give an output of 1 horsepower, a motor must be supplied with more than 746 watts, because some of the power consumed by the motor is converted into heat instead of into mechanical energy. The energy converted into heat is called the *motor losses*. For estimating purposes, an allowance of 1 kilowatt, or 1,000 watts, input for each horsepower output will answer for small direct-current motors, but for large motors, from 825 to 900 watts is sufficient. For example, a 5-horsepower direct-current motor will require about 4,500 watts input, and the current at 220 volts will be $4{,}500 \div 220 = 20.4$ amperes. This is slightly more than will be required by a good commercial motor. A 20-horsepower 220-volt motor will require about

$$\frac{20 \times 825}{220} = 75 \text{ amperes}$$

49. The method of calculating the current required by alternating-current motors is beyond the scope of this Section, but Table III will give sufficient information for practical purposes. The values in the table are approximate, and will vary according to the efficiencies of the motors. Alternating-current motors require slightly more input, in watts, for the same output, that is, they are slightly less efficient than direct-current motors. The figures in the columns are the ampere current carried by each wire. For example, a 5-horsepower 220-volt direct-current motor requires 20 amperes, that is, each wire carries 20 amperes when the

TABLE III

CURRENT, IN AMPERES, REQUIRED BY MOTORS

Horsepower	Direct-Current Motors			Alternating-Current Motors								
				Single Phase			Two Phase (Four Wire)			Three Phase (Three Wire)		
	110 Volts	220 Volts	500 Volts	110 Volts	220 Volts	500 Volts	110 Volts	220 Volts	500 Volts	110 Volts	220 Volts	500 Volts
1	9	4.5	2.0	14	7	3.1	6.4	3.2	1.4	7.4	3.7	1.6
2	17	8.5	3.7	24	12	5.3	11	5.7	2.5	13	6.6	2.9
3	26	13	5.6	34	17	7.5	16	8.1	3.5	19	9.3	4.1
5	40	20	8.8	52	26	11	26	13	5.5	30	15	6.4
$7\frac{1}{2}$	60	30	13	74	37	16	38	19	8.1	44	22	9.3
10	76	38	17	94	47	21	44	22	10	50	25	12
15	112	56	25				66	33	15	76	38	17
20	150	75	33				88	44	19	102	51	22
30	226	113	50				134	67	29	154	77	33
40	302	151	66				178	89	39	204	107	45
50	368	184	81				204	102	45	236	118	52
75	552	276	122				308	154	68	356	178	77
100	736	368	162				408	204	90	472	236	104
150	1,110	555	244				616	308	135	710	355	156
200	1,474	737	324				818	409	180	940	470	208

motor is working at full load; a 5-horsepower single-phase motor at full load requires 26 amperes; a 5-horsepower two-phase (four-wire) motor at full load requires 13 amperes per phase, that is, each wire carries 13 amperes; and a 5-horsepower three-phase (three-wire) motor requires 15 amperes per phase. The voltage in each case is assumed to be 220. Wires for carrying current to motors should have a safe carrying capacity from 25 to 50 per cent. greater than the current required by the motor at full load, in order to carry safely the current taken by the motor at occasional overloads, as at starting.

THE SIZE OF WIRES

50. The flow of electric current through a conductor is always accompanied by some drop of potential. This drop in some cases may be considerable, and unless something were done to make up, or compensate, for it, electric lamps would burn dim and motors would run too slow. The dynamo is usually adjusted to generate a pressure enough higher than is needed for the lamps, so as to allow for the drop. If a number of circuits of different lengths, or carrying different quantities of current, are fed from the same dynamo, it becomes necessary to calculate the size of wire to be used on each circuit, so that each lamp or other device shall receive the pressure for which it was intended. If an incandescent lamp receives a pressure higher than that for which it was intended, it will soon burn out; hence, it is important that the size of electric-light wires be carefully determined. The size of wires leading to motors need not be so carefully determined, for a little more or a little less drop in them does not very appreciably affect the operation of the motor.

51. In ordinary wiring, the drop can be made small by making the conductors large, but large conductors are expensive. In order to make the building in which wires are installed safe against fire, the wires must at least be large enough to have the carrying capacity called for by Table I; but they should ordinarily be larger than indicated by the

table, or the drop will be excessive. The amount of drop advisable in the conductors depends on the price of electric energy. If the energy is cheap, a larger drop may be allowed; hence, smaller conductors may be used than if energy is expensive. A drop of 3 per cent. in the feeders from the street mains to the distribution centers and 2 per cent. in the distribution circuits to the lamps is a fair allowance. This means that on a 110-volt system, the drop in the feeders is $\frac{3}{100} \times 110 = 3.3$ volts, and in the distribution circuits, $\frac{2}{100} \times 110 = 2.2$ volts, or a total of 5.5 volts. The current should then enter the building at 115.5 volts.

The distinction between per cent. drop and volts drop is shown in Table IV. The left-hand column shows the per cent. drop, the figures at the heads of the columns are different line voltages, and the figures in the columns indicate the volts drop.

TABLE IV

PER CENT. DROP AND VOLTS DROP

Per Cent. Drop	Line Voltages					
	52	104	110	150	220	250
1	.52	1.04	1.1	1.5	2.2	2.5
2	1.04	2.08	2.2	3.0	4.4	5.0
3	1.56	3.12	3.3	4.5	6.6	7.5
5	2.60	5.20	5.5	7.5	11.0	12.5
7	3.64	7.28	7.7	10.5	15.4	17.5
10	5.20	10.40	11.0	15.0	22.0	25.0
15	7.80	15.60	16.5	22.5	33.0	37.5

52. Fig. 28 is a layout of the wiring in a wagon factory, in which a two-wire system is used. The dynamo and switchboard are located at *D* in the engine room. The various centers of distribution are to be at or near the centers of the various floors, and a separate pair of feeders is to be run to each distribution center. If elevator shafts are convenient, they are used to run *risers*, or feeders, to the upper

floors. In the case illustrated, there are fourteen pair of feeder wires, each pair being represented by one line in the figure.

A 115-volt dynamo and 110-volt lamps are to be used. A loss of 3 volts is to be allowed in the feeders, and a loss of

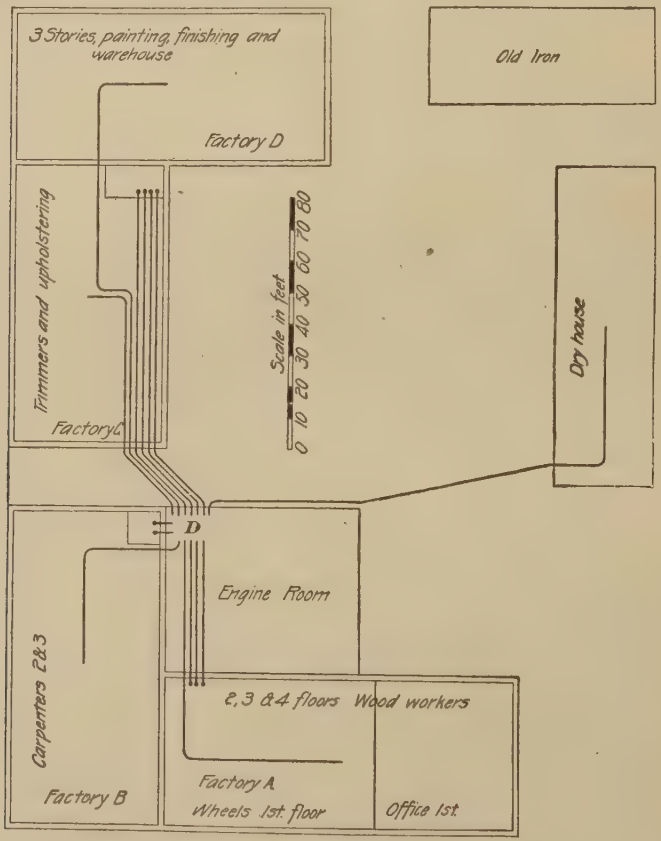


Fig. 28

2 volts in the distribution wires, irrespective of their length. The figure shows the plan of the feeders on one floor only; the small dots indicate risers.

The data for calculating feeders on the first floor are as follows:

Factory	Lamps	Amperes	Distance Feet	Length of Wire Feet
A	50	25	130	260 (.26 thousand)
B	40	20	75	150 (.15 thousand)
C	40	20	85	170 (.17 thousand)
D	40	20	175	350 (.35 thousand)

Since two wires are used, the length, in feet, must be twice the distance; the current flows to the center of distribution over one wire and comes back over the other.

53. Calculations Based on Resistance per 1,000 Feet.—If the drop in a feeder circuit were e volts and the current I amperes, then by Ohm's law the resistance would be $R = \frac{e}{I}$; and if this were the resistance for L thousand feet, for 1,000 feet the resistance would be the total resistance divided by L, or $\frac{e}{IL}$. If the drop in each feeder is to be 3 volts and the current and length of wire are as given in the foregoing table, the resistances per 1,000 feet of the feeders to each factory may be calculated as follows:

$$A, \frac{3}{25 \times .26} = .461, \text{ No. 6 has } .394 \text{ ohm}$$

$$B, \frac{3}{20 \times .15} = 1.000, \text{ No. 10 has } .997 \text{ ohm}$$

$$C, \frac{3}{20 \times .17} = .882, \text{ No. 10 has } .997 \text{ ohm}$$

$$D, \frac{3}{20 \times .35} = .429, \text{ No. 6 has } .394 \text{ ohm}$$

The sizes of wire for the foregoing, when the resistances per 1,000 feet are known, may be selected from Table I.

This method of calculating required sizes of wires can be applied to any kind of wiring for any practical purpose; but to avoid the necessity of figuring out each case, wiring tables have been prepared by which the proper size can be determined without calculation.

Calculations based on resistance per 1,000 feet may be put in the shape of a formula, as follows:

$$r_m = \frac{1{,}000\,e}{2\,D\,I}$$

TABLE V

TABLE FOR DETERMINING SIZE OF WIRE

Amperes	Distances, in Feet (One Way), for Loss of 1 Volt															
	20	25	30	35	40	45	50	60	70	80	90	100	120	140	160	180	200
	Size of Soft-Copper Wire, in B. & S. Gauge																
1														14	14	14	14
1.5												14	14	12	12	12	12
2									14	14	14	14	12	12	12	10	10
3							14	14	12	12	12	12	10	10	10	8	8
4						14	14	12	12	12	10	10	10	8	8	8	8
5				14	14	12	12	12	10	10	10	10	8	8	8	6	6
6			14	12	12	12	12	10	10	10	8	8	8	6	6	6	6
7		14	12	12	12	12	10	10	10	8	8	8	6	6	6	6	5
8		14	12	12	10	10	10	10	8	8	8	8	6	6	6	5	5
9	14	12	12	12	10	10	10	8	8	8	8	6	6	6	5	4	4
10	14	12	12	10	10	10	10	8	8	8	6	6	6	5	5	4	4
12	12	12	10	10	10	8	8	8	6	6	6	6	5	4	4	3	3
14	12	10	10	10	8	8	8	6	6	6	6	5	4	4	3	3	2
16	12	10	10	8	8	8	8	6	6	5	5	5	4	3	3	2	2
18	10	10	8	8	8	8	6	6	6	5	5	4	3	3	2		
20	10	10	8	8	8	6	6	6	5	5	4	4	3	2	2		
25	10	8	8	6	6	6	6	5	4	4	3	3	2				
30	8	8	6	6	6	5	5	4	3	3	2	2					
35	8	6	6	6	5	5	4	3	3	2	2						
40	8	6	6	5	5	4	4	3	2	2							
45	6	6	5	5	4	4	3	2	2								
50	6	6	5	4	4	3	3	2									
55	6	5	4	4	3	3	2	2									
60	6	5	4	3	3	2	2										
65	6	5	4	3	3	2	2										
70	5	4	3	3	2	2											
75	5	4	3	2	2												
80	5	4	3	2	2												
90	4	3	2	2													
100	4	3	2														

in which r_m = resistance of 1,000 feet of wire to be used;
e = drop, in volts;
D = distance, in feet, one way;
I = current, in amperes.

For example, to carry 10 amperes 600 feet ($600 \times 2 = 1{,}200$ feet of wire) with 3 volts drop, the resistance

$$r_m = \frac{1{,}000 \times 3}{2 \times 600 \times 10} = .25 \text{ ohm per } 1{,}000 \text{ feet}$$

or about that of No. 4 wire.

54. Size of Wire for 1 Volt Drop.—In Table V, distances, in feet, are given in the top horizontal line. Beneath these distances are columns containing numbers that designate the proper size of wire to use to obtain a drop of 1 volt when the wire carries the current given in the corresponding line in the left-hand column.

For example, to find the size of wire necessary to carry 20 amperes a distance (not the length of wire) of 140 feet with a drop of 1 volt, look in the column headed 140 and find opposite 20 amperes the figure 2; No. 2 B. & S. wire will be required. If 2 volts drop are required, look in the column headed 70 ($140 \div 2$) and find opposite 20 the figure 5, or look in the column headed 140 and find opposite 10 ($20 \div 2$) the figure 5. Both ways give the same result, namely, No. 5 wire. The table is sufficiently accurate for all practical purposes, but if very great exactness is desired, it is better to calculate the sizes. Only the nearest even sizes of wires larger than No. 6 are given, because the odd sizes are not ordinarily used.

55. Calculation of Area in Circular Mils.—In the Underwriters' table of safe carrying capacities (see Table I), the wires are listed both by number (B. & S. gauge) and by **circular mils.** Cables larger than the wires given in the table and having no B. & S. gauge number are listed by circular mils only. Large cables of any desired cross-section in circular mils are made by all the leading manufacturers of insulated wires.

It is often more convenient to calculate the size of wires or cables in terms of circular mils than in terms of resistance

TABLE VI

DISTANCE, IN FEET (ONE WAY), PRODUCING A DROP OF 1 VOLT FOR GIVEN CURRENTS AND GIVEN SIZES OF WIRE

Size of Wire, B. & S. Gauge	Amperes 1	2	3	4	5	6	7	8	9	10	12	15
18	75.2	37.6	25.1	18.8	15.0							
16	120.0	60.0	40.0	30.0	24.0	20.0						
14	190.0	95.0	63.3	47.5	38.0	31.7	27.1	23.8	21.1	19.0	15.8	12.7
12	302.0	151.0	101.0	75.5	60.4	50.3	43.1	37.7	33.6	30.2	25.2	20.2
10	480.0	240.0	160.0	120.0	96.0	80.0	68.6	60.0	53.3	48.0	40.0	32.0
8	764.0	382.0	255.0	191.0	153.0	127.0	109.0	95.5	84.9	76.4	63.7	51.0
6	1,215.0	607.0	405.0	304.0	243.0	202.0	174.0	152.0	135.0	121.0	101.0	81.0
5	1,533.0	766.0	511.0	383.0	307.0	255.0	219.0	192.0	170.0	153.0	128.0	102.0
4	1,933.0	966.0	644.0	483.0	387.0	322.0	276.0	242.0	215.0	193.0	161.0	129.0
3	2,437.0	1,219.0	812.0	609.0	487.0	406.0	348.0	305.0	271.0	244.0	203.0	162.0
2			1,024.0	768.0	615.0	512.0	439.0	384.0	341.0	307.0	257.0	205.0
1			1,291.0	969.0	775.0	646.0	554.0	484.0	431.0	387.0	323.0	258.0
0				1,222.0	978.0	815.0	698.0	611.0	543.0	489.0	407.0	326.0
00					1,232.0	1,027.0	880.0	770.0	685.0	616.0	513.0	410.0
000						1,295.0	1,110.0	971.0	863.0	777.0	648.0	518.0
0000							1,400.0	1,225.0	1,089.0	980.0	817.0	653.0

Size of Wire, B. & S. Gauge	Amperes 18	20	25	30	35	40	45	55	65	75	85	95	100
18													
16													
14													
12	16.8	15.1											
10	26.7	24.0	19.2	16.0	13.7								
8	42.5	38.3	30.6	25.5	21.8	19.1	17.0						
6	67.5	60.8	48.6	40.5	34.7	30.4	27.0	22.1					
5	85.0	76.6	61.3	51.1	43.8	38.4	34.1	27.8	23.6	20.4			
4	107.0	96.7	77.3	64.4	55.1	48.3	43.0	35.1	29.7	25.8	22.7		
3	135.0	122.0	97.4	81.1	69.6	60.9	54.0	44.3	37.5	32.5	28.7	25.7	24.4
2	171.0	154.0	123.0	102.0	87.9	76.9	68.3	55.9	47.3	41.0	36.2	32.4	30.7
1	215.0	194.0	155.0	129.0	111.0	96.9	86.0	70.3	59.6	51.7	45.6	40.8	38.7
0	272.0	245.0	195.0	163.0	140.0	122.0	109.0	88.7	75.2	65.1	57.5	51.4	48.9
00	342.0	308.0	246.0	205.0	176.0	154.0	137.0	112.0	94.8	82.0	72.5	64.8	61.6
000	432.0	388.0	311.0	259.0	222.0	194.0	173.0	141.0	119.0	104.0	91.4	81.7	77.7
0000	544.0	490.0	392.0	326.0	280.0	245.0	218.0	178.0	151.0	131.0	115.0	103.0	98.0

per 1,000 feet; and calculations in terms of circular mils are applicable to wires or cables of any size or shape.

The size of wire, in circular mils, may be calculated from the formula,

$$\text{circular mils} = \frac{21.6\,DI}{e}$$

in which D = distance, in feet, one way;
I = current, in amperes;
e = drop of potential, in volts.

This formula will generally be found as useful as any that can be given for interior-wiring calculations. It will be well to commit it to memory, because wiring tables are not always available, and besides, calculations have often to be made that are beyond the range of the tables. The formula can be applied to any two-wire system or to the three-wire system, as illustrated by the following examples:

EXAMPLE 1.—Calculate the size of wire necessary to supply eighty 55-watt 110-volt 16-candlepower lamps situated at a distance of 200 feet from the center of distribution. The allowable drop is 3 volts.

SOLUTION.— $D = 200$ and $e = 3$. Each 16-c.-p. lamp will take $\frac{55}{110} = \frac{1}{2}$ ampere; hence, $I = 40$, and by the formula

$$\text{cir. mils} = \frac{21.6 \times 200 \times 40}{3} = 57{,}600$$

or, as shown by Table I, between No. 2 and No. 3 B. & S. wire will be required. No. 2 wire would likely be used. Ans.

EXAMPLE 2.—Calculate the size of wire necessary to supply one hundred carbon-filament 16-candlepower lamps on a 110–220-volt three-wire system. The distance from the center of distribution to the lamps is 250 feet, and the drop on each side of the system is not to exceed 3 volts. The system is to be balanced, fifty lamps on each side.

SOLUTION.—This problem should be solved by the foregoing formula as if it concerned a two-wire system. Assuming that each pair of lamps takes $\frac{1}{2}$ ampere, the current in the outside wires, when all the lamps are burning, will be $100 \div 4 = 25$ amperes instead of $100 \div 2 = 50$ amperes, as it would have been if a two-wire system had been used. The allowable drop on each side of the circuit is 3 volts; hence, the total drop in the outside wires will be 6 volts. Then, by the formula,

$$\text{cir. mils} = \frac{21.6 \times 250 \times 25}{6} = 22{,}500$$

No. 6 wire will be large enough and the same size would likely be installed for the neutral. Ans.

The same method may be used for 220–440-volt three-wire system, except that in estimating the current, allow about .3 ampere for each pair of lamps instead of .5 ampere, as in the previous case.

56. There are several methods of arranging wiring tables for selecting the wire other than that shown in Table V. Table VI gives the exact distance one way over which a current may be transmitted with a drop of 1 volt for each size wire ordinarily used. To use this table, divide the number of amperes transmitted by the number of volts drop desired. Find the nearest number to this result in the line of amperes at the head of the table; in the column below this number, find the distance, in feet, most nearly corresponding to the given distance; in the same line in the first column of the table is given the number of the required wire. For example, to find the size of wire to transmit 15 amperes 140 feet with 3 volts loss, divide 15 by 3 and find the quotient 5 in the line of amperes. In the column below, we find the nearest distance 153, and to the left of this, in the first column, the size of wire required, No. 8.

57. Figs. 29 and 30 are diagrams of a graphic method of determining the size of wire. The vertical lines represent amperes, and are numbered along the bottom of the diagrams; the horizontal lines represent feet, and are numbered along the left-hand margin. To use these diagrams, find the point of intersection of the line representing the number of amperes per volt drop, that is, the $\frac{\text{number of amperes}}{\text{volts drop}}$, with the line representing the distance, in feet, and use the wire indicated by the curve lying nearest this point. Unless the wire curve is very close, use the curve representing the larger wire of the two.

For example, to find the wire required for 7 volts drop in a distance of 125 feet, with 21 amperes, divide 21 by 7, which gives 3; the line of 3 amperes (Fig. 30) intersects the line of 125 feet about midway between the curves representing

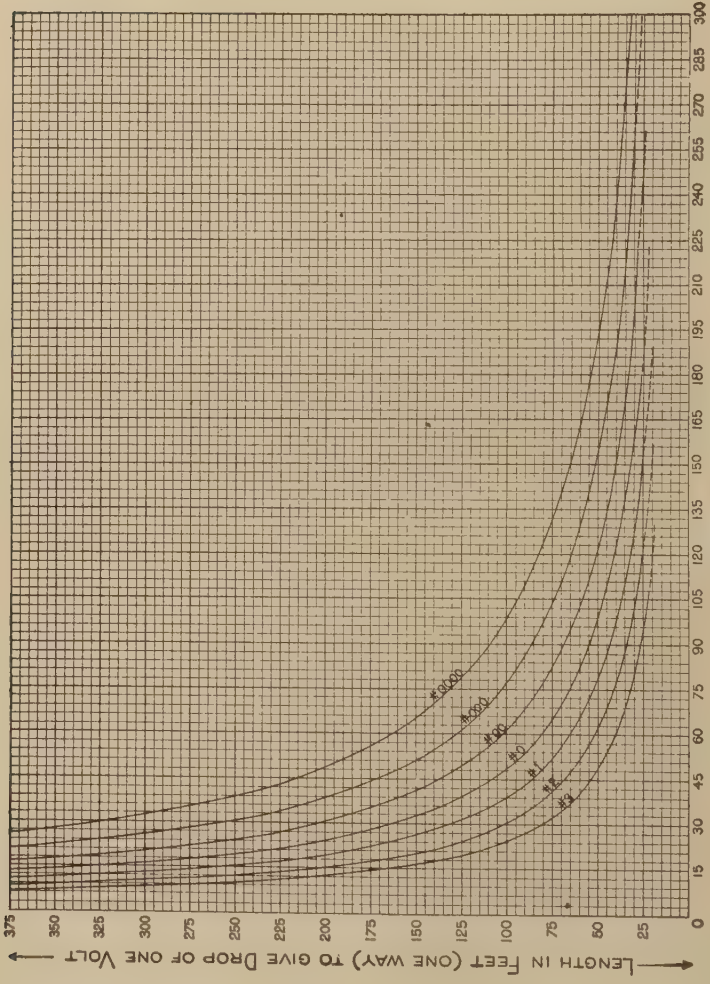

LENGTH IN FEET (ONE WAY) TO GIVE DROP OF ONE VOLT
CURRENT—AMPERES
#0000
#000
#00
#0
#1
#2
#3
0 15 30 45 60 75 90 105 120 135 150 165 180 195 210 225 240 255 270 285 300
375 350 325 300 275 250 225 200 175 150 125 100 75 50 25

FIG. 29

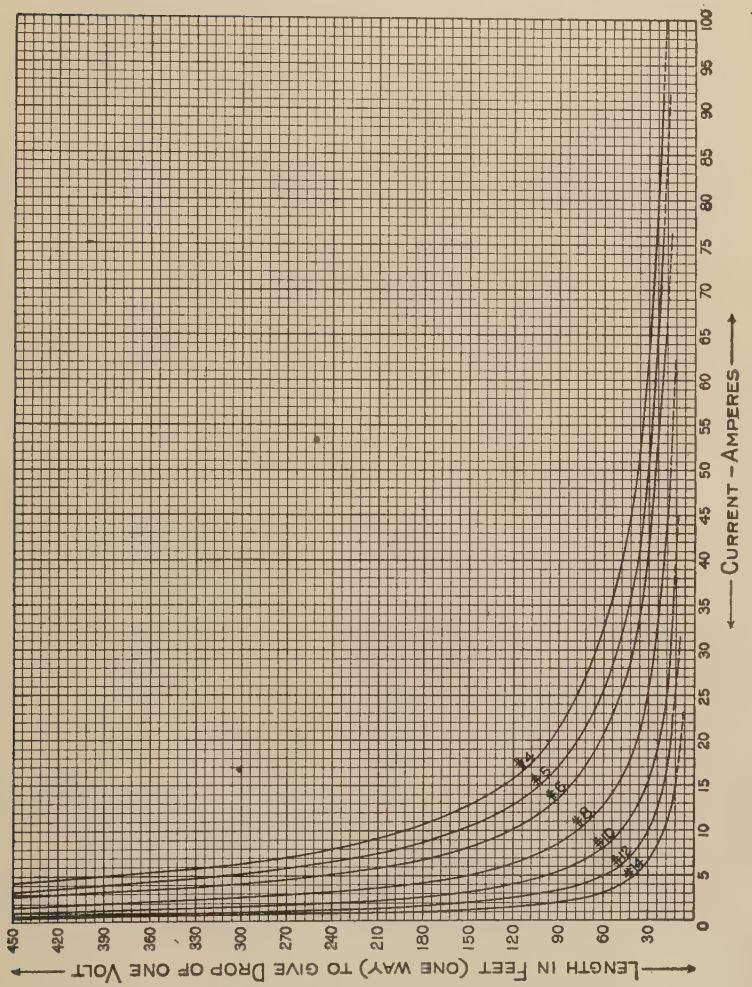

LENGTH IN FEET (ONE WAY) TO GIVE DROP OF ONE VOLT
450 420 390 360 330 300 270 240 210 180 150 120 90 60 30
CURRENT - AMPERES
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
#4
#5
#6
#8
#10
#12
#14

FIG. 30

No. 10 and No. 12 wire; hence, the larger size of wire, No. 10, would be used.

58. Distance Used in Calculations.—In making calculations relating to wiring, the distance to be taken is the *average distance* through which the current supplied can be considered as flowing. For example, Fig. 31 is a diagram of a system where a circuit is run from a distributing point *A* 100 feet before it reaches the first of a group of lamps *B*, the lamps being distributed along the circuit 50 feet farther. In making calculations for such a circuit, the distance should be

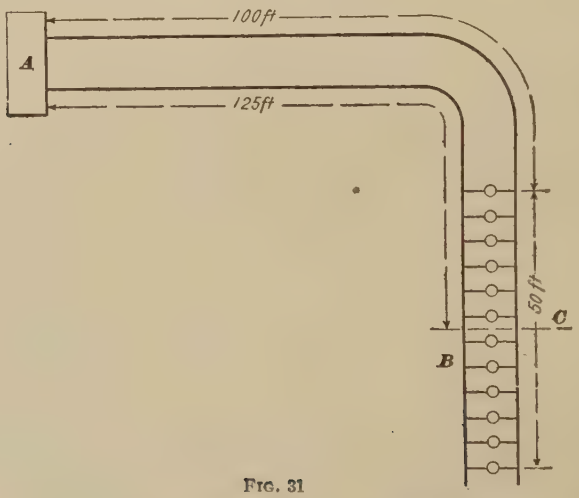


FIG. 31

taken neither to the nearest nor to the farthest lamp, but to a point *C* such that the number of lamps on each side is the same. With lamps distributed as shown, the point *C* lies in the middle of the 50-foot space, and the distance used for calculations should be 125 feet. If the lamps were all bunched at the farthest end, the distance would be 150 feet. If they were all distributed along the entire distance from *A* to the end of the line, the distance would be $150 \div 2 = 75$ feet.

59. It is best to be liberal in determining the size of wire. The tendency is nearly always to add more lamps or other appliances to a system, and if wires are no larger than

TABLE VII

EQUIVALENT CROSS-SECTION OF WIRES

Number of Wire B. & S. Gauge	Equivalent Cross-Section, in Terms of Smaller Wires								
0000	00 + 1	2 — 0	4 — 3	8 — 6	16 — 9	32 — 12	64 — 15	128 — 18	1 + 2 + 2
000	0 + 2	2 — 1	4 — 4	8 — 7	16 — 10	32 — 13	64 — 16	128 — 19	2 + 3 + 3
00	1 + 3	2 — 2	4 — 5	8 — 8	16 — 11	32 — 14	64 — 17	128 — 20	3 + 4 + 4
0	2 + 4	2 — 3	4 — 6	8 — 9	16 — 12	32 — 15	64 — 18		4 + 5 + 5
1	3 + 5	2 — 4	4 — 7	8 — 10	16 — 13	32 — 16	64 — 19		5 + 6 + 6
2	4 + 6	2 — 5	4 — 8	8 — 11	16 — 14	32 — 17	64 — 20		6 + 7 + 7
3	5 + 7	2 — 6	4 — 9	8 — 12	16 — 15	32 — 18			7 + 8 + 8
4	6 + 8	2 — 7	4 — 10	8 — 13	16 — 16	32 — 19			8 + 9 + 9
5	7 + 9	2 — 8	4 — 11	8 — 14	16 — 17	32 — 20			9 + 10 + 10
6	8 + 10	2 — 9	4 — 12	8 — 15	16 — 18				10 + 11 + 11
8	10 + 12	2 — 11	4 — 14	8 — 17	16 — 20				
10	12 + 14	2 — 13	4 — 16	8 — 19					
12	14 + 16	2 — 15	4 — 18	8 — 20					
14	16 + 18	2 — 17	4 — 20						
16	18 + 20	2 — 19							

needed for the first installations, they may soon become overloaded. If the required cross-section comes between two standard sizes, it is usually best to use the larger of the two. If the area required is about that of an odd size smaller than No. 6 B. & S., use the next larger even-numbered wire.

60. Calculations for Alternating Current.—Wires for carrying alternating current should be so insulated that all the wires of one circuit, two or three as the case may be, may be placed not over 1 inch apart; and if the wiring is done in iron pipes, or conduits, *all the wires of a circuit must be drawn in the same conduit.* If these instructions are followed, the size of wire for alternating current may be calculated in exactly the same way as for direct current. Care should be taken that the various appliances or distribution circuits are so connected as to *balance* a two- or a three-phase system. When a conductor of large cross-section is required, it is better to use two or more strands of a smaller size wire with a total area equivalent to that of the larger wire. Some engineers avoid using a single strand of No. 6 or larger; but in place of No. 6 use two strands of No. 9 or four strands of No. 12, and in place of No. 5 use two strands of No. 8, etc. Table VII shows the number of strands of smaller wires equivalent in cross-section to each size of larger wire.

To use the table, find in the first column the B. & S. gauge number required; then, in the columns to the right, find various combinations of wires that have equivalent cross-sectional areas. For example, in place of one strand of No. 1, could be used one strand of No. 3 and one strand of No. 5 together, or two strands of No. 4, or four strands of No. 7, etc., reading in the same lines in successive columns to the right; the last column shows a combination of one strand of No. 5 and two strands of No. 6 that could be used.

USING SEVERAL WIRES IN PARALLEL

61. Small wires have more surface, hence more radiating capacity, in proportion to their cross-section than do larger ones. Therefore, if the carrying capacity will not permit

the use of one large conductor, it may be possible to use several smaller ones in parallel. For example, the carrying capacity of one No. 5 rubber-covered wire is 54 amperes (see Table I); but two No. 8 wires, equal in cross-section to one No. 5, will carry 66 amperes.

62. Calculations.—The combined cross-section of several wires in parallel must very nearly equal or exceed the required cross-section as determined by the formula of Art. **55**, or from wiring tables. For example, if a current of 40 amperes is to be transmitted over a distance of 150 feet with a loss of 3 volts and only No. 12 wire is readily available, the number of strands to be used in parallel would be determined as follows: The number of circular mils are first determined by the formula of Art. **55**, in which $D = 150$, $I = 40$, and $e = 3$; hence,

$$\text{circular mils} = \frac{21.6 \times 150 \times 40}{3} = 43{,}200$$

The cross-section of No. 12 wire is 6,530 circular mils, and the number of strands required are $43{,}200 \div 6{,}530 = 6.6$; hence, seven strands, the nearest whole number, would be used.

Sometimes, it is desired to reinforce the wire in a circuit so as to decrease the drop or to increase the carrying capacity. Suppose that a circuit of No. 8 wire is to be reinforced until it will carry 20 amperes 150 feet with a drop of 2 volts; from the formula of Art. **55**,

$$\text{circular mils} = \frac{21.6 \times 150 \times 20}{2} = 32{,}400$$

No. 8 wire has a cross-section of 16,509 circular mils (see Table I); hence, there must be added to the original circuit $32{,}400 - 16{,}509 = 15{,}891$, which is approximately the cross-section of No. 8. Another No. 8 wire added to the original circuit will produce the desired result.

63. Fuse Protection.—When several wires insulated from one another are used in parallel in place of one larger wire, a fuse must be placed in series with each wire, and not one large fuse for all the wires considered as one conductor.

Fig. 32 (*a*) illustrates the correct method of installing such fuses, and (*b*) illustrates the incorrect method. This rule

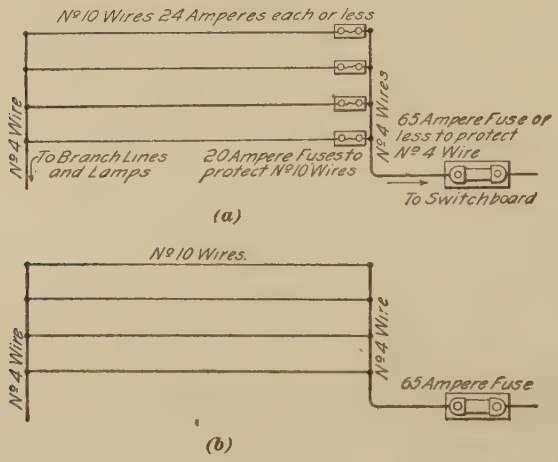

No 10 Wires 24 Amperes each or less
No 4 Wire
No 4 Wires
65 Ampere Fuse or less to protect No 4 Wire
To Branch Lines and Lamps
20 Ampere Fuses to protect No 10 Wires
To Switchboard
(a)
No 10 Wires.
No 4 Wire
No 4 Wire
65 Ampere Fuse
(b)

Fig. 32

does not apply to stranded conductors in which the separate strands of bare copper are in continuous contact with one another.

EXAMPLES FOR PRACTICE

1. Determine, by the formula of Art. 55 and by Table I, the size of wire required to carry 30 amperes a distance of 150 feet (one way) with a drop of 3 volts. Ans. No. 5 B. & S.

2. If a circuit 200 feet long (single distance) carries 25 amperes and is of No. 6 B. & S. wire, what will be the drop in volts? Ans. 4.1 volts

3. If a circuit of No. 10 B. & S. wire carries 20 amperes a distance of 200 feet (single distance), what size of wire must be connected in parallel with the existing wire to limit the drop to 2 volts? Ans. No. 5 B. & S.

4. A current of 40 amperes is to be carried 300 feet (single distance) with a drop of 3 volts. Assuming that No. 10 B. & S. is the only size of wire available, how many wires must be connected in parallel to carry the current with the specified drop? Ans. 8 wires

5. By the aid of Table V, determine the size of wire that will be required for a line (two wires) extending a distance of 120 feet and

carrying 30 amperes with a drop not exceeding 3 volts. Verify result by Table VI. Ans. No. 6 B. & S.

6. What size B. & S. copper wire should be used, allowing a drop of 2 volts, to supply a group of eighty 110-volt 16-candlepower incandescent lamps at a distance (one way) of 200 feet? Assume that each lamp requires $\frac{1}{2}$ ampere. Ans. No. 1 B. & S.

7. Calculate the size of wire necessary to supply fifty 16-candlepower 110-volt $\frac{1}{2}$-ampere lamps located in a group at a distance of 150 feet (one way) from the center of distribution, allowing a drop not to exceed 2 volts. Ans. No. 4 B. & S.

INSTALLATION OF WIRES

OPEN WORK

64. Wiring in Dry Places.—The several systems of wiring may be generally classified as *open work* and *concealed work*. **Open work** may be considered under two heads: *wiring in dry places* and *wiring in damp places*. Open, or exposed, wiring is generally used in factories, mills, and warehouses, and often in stores and in old buildings where the expense of concealing the wires is considered too great. Open wiring is the cheapest kind and is often the safest.

The wire must be either rubber covered or have slow-burning weather-proof insulation. The latter insulation is the cheaper, and is good enough for dry places if the wire is rigidly supported on non-combustible non-absorptive insulators. The wires must be held at least $\frac{1}{2}$ inch from the surface over which they pass and $2\frac{1}{2}$ inches apart for voltages up to 300, and 1 inch from the surface and 4 inches apart for voltages from 300 up to 550.

65. Supporting and Insulating Fittings.—Exposed wires must be put up as though they were bare; that is, the covering on the wire must not be depended on for insulation. The wires must not be allowed to come in contact with anything except insulating supports.

Figs. 33 to 42, inclusive, show a few of many varieties of porcelain fittings suitable for installing exposed wiring.

These and many other styles are made in various sizes to suit different conditions. Fig. 33 shows a section of a porcelain **knob,** which may be attached to the wall or ceiling

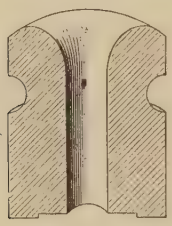


FIG. 33

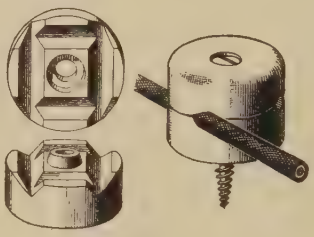


FIG. 34

by a screw or nail through the center hole. The wire is fastened in the groove on one side by a piece of wire, called a *tie-wire*, the insulation of which must be as good as that of the wire being tied. Fig. 34 shows a **knob cleat** that gives a neater appearance than the knob with tie-wires. Fig. 35 shows another form of single-wire cleat

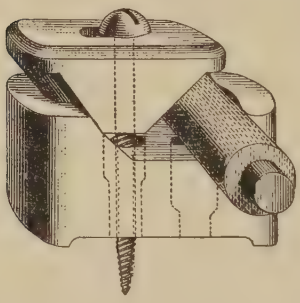


FIG. 35

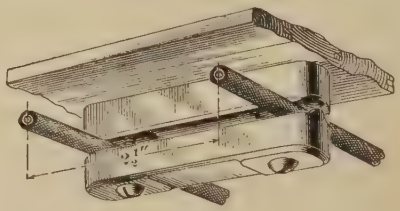


FIG. 36

more used for larger size wires. Fig. 36 shows a two-wire cleat designed to separate the wires $2\frac{1}{2}$ inches, according to Code rules. Similar cleats are made for three wires. It is always best to use screws in putting up knobs and cleats, but nails with leather washers under the heads, are permissible.

FIG. 37

Fig. 37 shows a porcelain **tube,** or **bushing,** for insulating holes through joists. Fig. 38 shows a longer tube, to be used through floors or partitions; the shoulder *a* leaves enough of the end of the tube projecting from the wall or

floor so that the wire will be well protected. Fig. 39 shows a form of tube used in bringing a wire through an outside wall. The downward curve of the outside end of the tube and the *drip loop a* in the wire prevent the entrance of water into the tube. Fig. 40 shows a double-headed tube used where wires cross. As near as possible to such crossings, each wire should be securely fastened to knobs or by cleats. The tube shown in Fig. 37 may be used for crossings in place of a double-headed tube, provided that it is securely fastened by taping it to the wire on which it is placed.

Fig. 38

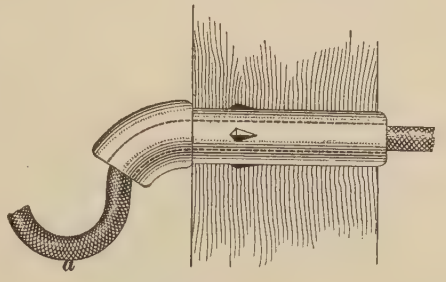


Fig. 39

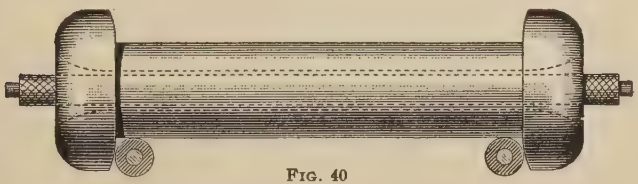


Fig. 40

Fig. 41 shows a fused **ceiling rosette** for a drop light. The rosette is made in two parts: a base *a*, which is fastened to the ceiling and which carries terminals *c*, *c'* to which the circuit wires fasten, and a cap *b* from which the lamp is suspended. The lamp cord is connected to terminals *d*, *d'*,

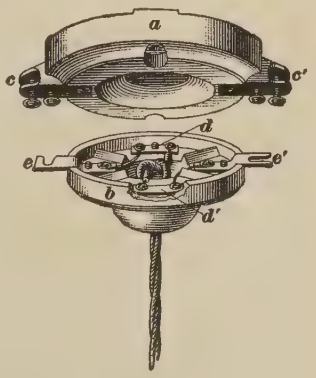


Fig. 41

which are connected by fuses to the contact pieces e, e'. These contact pieces, when the cap is in position, are held by screws to the terminals c, c'. A knot is tied in the cord above the cap so that the weight of the lamp will not hang on the terminal screws. Rosettes with link fuses, such as shown in Fig. 41, must not be used where the pressure is over 125 volts or if the current through each rosette is over 2 amperes; they must not be located near inflammable goods or where dust or gas may be fired, and the next fuses back of them toward the dynamo must not be over 25 amperes capacity. Fuseless rosettes are preferable if proper protection can be obtained by line cut-outs.

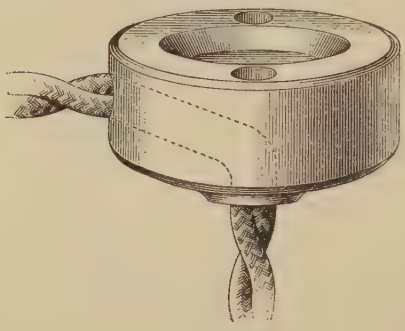


FIG. 42

Fig. 42 shows a **ceiling button** for suspending a lamp a few feet (not over 2 or 3) from the ceiling rosette.

66. Wiring in Damp Places.—Under the heading, Wiring in Damp Places, are usually considered not only places where wires may be affected by moisture, as damp cellars, cold-storage rooms, breweries, etc., but also places where acid or other fumes are likely to injure the insulation on the wires. For protection against water, rubber insulation must be used; for protection against corrosive vapors the insulation may be either rubber or weather-proof. The insulating supports are the same as those used in dry places, except that the wire must be separated at least 1 inch from the surface wired over. Lamp sockets for use in damp places must be waterproof; and if inflammable gases are present, both the socket and the lamp must be enclosed

FIG. 43

in a vapor-tight globe. Fig. 43 shows a waterproof socket *a* with rubber-covered terminal wires *b*, *b* for attaching direct to the circuit. Fig. 44 shows a gas- and waterproof glass globe for enclosing an incandescent lamp.

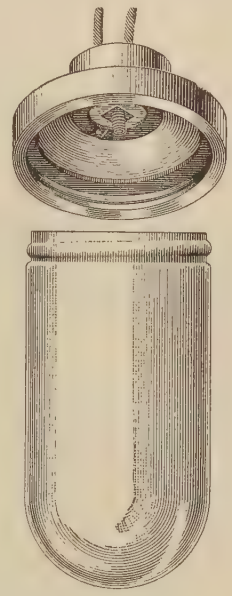


Fig. 44

In wiring damp rooms, only a few lamps should be made dependent on any one fuse, so that the blowing of a fuse will not put out many lights. If possible, all cut-outs should be placed outside in a dry place; otherwise they should be placed in waterproof boxes.

67. In stringing exposed wires, either in dry or in damp places, insulating supports, that is, knobs or cleats, must be placed at least every $4\frac{1}{2}$ feet. If the wires are likely to be disturbed, this distance must be shortened. In buildings of mill construction, overhead wires larger than No. 8, if not likely to be disturbed, may be separated from each other about 6 inches and run from timber to timber, the supports being attached to each timber only, as shown at *a*, Fig. 45. If the wires are smaller than No. 8, or if they may be disturbed by piling boxes, etc. up

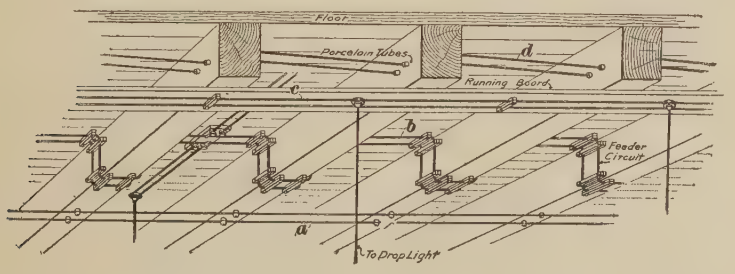


Fig. 45

against the ceiling, they should be carried around the timbers as shown at *b*. One feeder circuit carried across the

ceiling in this way may, by means of branch circuits, be made to supply all the lamps in a room. Small wires crossing ceiling joists are sometimes placed on running boards, as shown at *c*, and sometimes through tubes, as shown at *d*; but either of the last two methods is expensive and is to be avoided if possible.

CONCEALED WORK

68. Concealed wiring is usually installed according to one or more of the following methods: *concealed knob-and-tube*, *conduit*, and *molding*. Only approved rubber-covered wire is permissible for any kind of concealed wiring. Concealed knob-and-tube work has been used in the past more than either of the other methods; it is the cheapest of the three, and is quite safe if properly installed. This method is much used in dwelling houses where it is desirable to avoid unnecessary expense. In some large cities, local rules prohibit this class of work, but the Underwriters' rules permit it.

Conduit wiring represents the best and the most expensive method, and is the only one to be considered for fireproof office buildings, hotels, or similar structures. Sometimes, the knob-and-tube system and the conduit system can be combined to good advantage, knob-and-tube work being used where no mechanical disturbance is likely to occur, and conduits where mechanical protection is necessary.

Molding is useful in old buildings where it is difficult to conceal the wires otherwise. The molding may be made to match the woodwork of the rooms, so that its presence will scarcely be noticed.

69. Concealed Knob-and-Tube Work.—Fig. 46 is a plan view showing three joists and the method of supporting and insulating conductors by means of porcelain knobs and tubes. In running lengthwise of the joists, each wire is run along a separate joist if possible. The knobs (on any wire) should not be over $4\frac{1}{2}$ feet apart, and should be less if there

is any danger of disturbance to the wires. The wires must be held at least 1 inch from the surface wired over and 5 inches apart. Porcelain or glass tubes are always used in passing through joists or partitions. The tubes should fit tightly in the holes, which must be 5 inches apart, and far enough from either edge of the joists so that no nails are likely to penetrate to the tubes. If the holes are not bored horizontal, the tubes must be put in with their heads on the high side. The tube should be long enough to reach through the joist and project $\frac{1}{2}$ inch on each side. In large timbers, it is permissible to bush the hole with one continuous length of iron pipe with porcelain tubes in each end so that the wire cannot touch the iron.

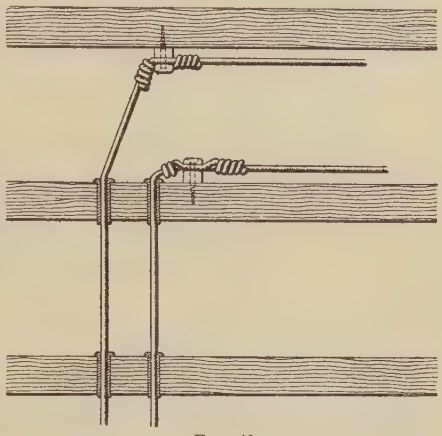


FIG. 46

70. Outlets.—Particular care should be exercised to keep the insulation good wherever wires are brought out through plaster, especially where the same outlet is used for both gas and electricity. Fig. 47 shows a method of arranging a ceiling outlet for both gas and electricity. Flexible insulating tubes, or conduits, cover and protect the wires from a point near the porcelain knobs nearest the outlet to a point flush with the end of the gas pipe or just beyond it. Fig. 48 (*a*) and (*b*) shows two forms of combination gas-and-electric side-wall

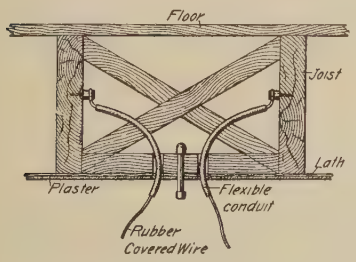
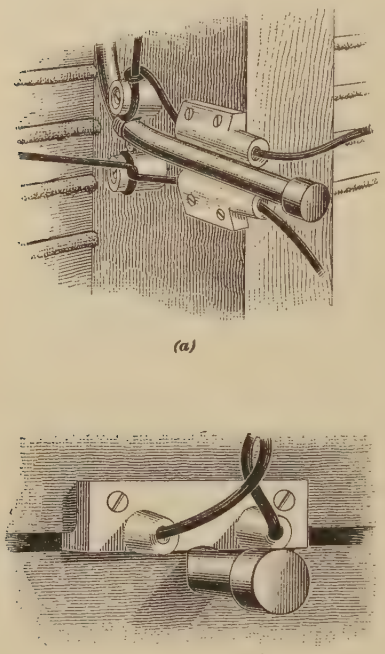


FIG. 47

outlets. In form (*b*) wooden cleats, or cross-pieces, are nailed between the studding back of the lath to support the outlet and the fixture.



(*a*)

(*b*)

FIG. 48

71. Sometimes, in wiring completed buildings with knob-and-tube work, there are places where it is difficult to place necessary insulating supports. If the pressure does not exceed 300 volts (and it very seldom does in interior wiring for lighting work) and if the places are dry, it is permissible to use flexible tubing and pull the wires through by a process to be explained later, and known as *fishing* them through. In this case each wire must be separately incased in tubing. Flexible **circular-loom tube** is very useful for this purpose. It is a woven tube without metal covering, and is treated outside and inside with insulating material that makes it hold its shape. This tube is neither weather-proof nor nail-proof, but it may be used in dry places, as explained, or for outlets, as shown in Fig. 47.

72. Fig. 49 shows a section of insulated iron pipe, as used for some kinds of electric wiring, and is called **iron-armored conduit.** The iron armor *a* is about $\frac{1}{8}$ inch thick, the same as ordinary gas pipe, and not less than $\frac{5}{8}$ inch inside diameter; the insulating lining *b* is not less than $\frac{1}{32}$ inch thick, and it must adhere firmly to the armor. Unlined conduit is permissible, and is now more used than lined conduit. Unlined conduit

FIG. 49

is practically the same as gas pipe, except that the interior is made smooth and less liable to rust by galvanizing or enameling it. The wire used in unlined conduit must have an extra-heavy braiding; *twin wire* is generally used, each wire being separately insulated and the two then bound together with insulating braid.

73. Fig. 50 shows a section of flexible *steel-armored cable*, which consists of two rubber-covered wires having a common covering of heavy insulation with an interlocked steel strip wound outside, so as to form a continuous protection. This

FIG. 50

cable may be used as an auxiliary to the knob-and-tube system or it may be used for the complete wiring of a building. When used in so-called fireproof buildings or in damp places, there must be a lead sheathing at least $\frac{1}{32}$ inch thick underneath the armor.

74. Specifications for Concealed Electric-Light Wiring.—The following typical specifications cover all the main points necessary for wiring an ordinary dwelling house by the concealed knob-and-tube system. Other details, such as the location of additional switches, the use of particular kinds of cut-outs, etc., may be added to these specifications if desired. The specifications cover only the concealed work.

SPECIFICATIONS FOR 110- OR 220-VOLT TWO-WIRE SYSTEMS

Distribution Cabinet

A distribution cabinet is to be located on some inside wall, in a readily accessible place, in the second story or the attic, as near the center of the building as possible.

The cabinet must be lined on the ends and sides with slate $\frac{1}{4}$ inch thick, and must be fitted with a door covered on the inside with slate $\frac{1}{4}$ inch thick.

Circuits

From this cabinet, separate circuits must be run to the outlets in such a manner that not more than 660 watts shall be required to supply all the lamps in any circuit. Wherever the number of lamps is not marked on the plans or otherwise specified, pendants shall be considered as intended to carry four lamps each and brackets one lamp each.

Fuses

All fuses must be located on a panel board placed in the distribution cabinet. The panel boards must be of slate at least $\frac{3}{4}$ inch thick and be provided with terminals designed for enclosed fuses. Both sides of all lines must be fused, and the fuses must be of a type suitable for use on 220 volts and capable of interrupting the arc due to a 220-volt short circuit.

Wires

All circuits running from the distribution center must be of No. 14 B. & S., or larger, rubber-covered copper wire of a make accepted by the National Board of Fire Underwriters.

Mains

From the distribution cabinet to the attic, and also to the basement, a pair of mains must be run, the size of which will depend on the total current required by all the lamps in the house, as follows:

10 amperes or less	No. 14 or larger
10 to 15 amperes	No. 12 or larger
15 to 20 amperes	No. 10 or larger
20 to 30 amperes	No. 8 or larger
30 to 40 amperes	No. 6 or larger

If more than 35 amperes current is required, it is advisable to have more than one distribution center and pair of mains.

All mains and distribution wires must be so proportioned that the electromotive force at the terminals of each lamp shall not be less than 110 or 220 volts when the current enters the building at 115 or 230 volts.

Extra Wire

A third wire, two sizes smaller than the mains (except that no wire smaller than No. 14 B. & S. shall be used),

SPECIFICATIONS FOR 110- OR 220-VOLT TWO-WIRE SYSTEMS—(Continued)

must also be run from the attic to the basement, through the distribution cabinet, to make possible the use of the three-wire system.

Manner of Fastening Wires

Wires running parallel to joists must be fastened on porcelain knobs, placed on different timbers, and kept as far apart as possible. In passing through joists, floors, and other woodwork, the holes must be bushed with porcelain tubes, which must extend at least $\frac{1}{2}$ inch through the wood and be so arranged that their weight will tend to keep them in place rather than to cause them to slip out.

Space Between Wires

All wires must be kept at least 5 inches away from one another, from gas or water pipes, iron beams, bell or annunciator wires, speaking tubes, furnace pipes, and other conducting materials, except at the distribution cabinet and fixture outlets. Where wires cannot be kept this far apart, they must be run in conduits.

Outlets

Flexible insulating conduits must be used at outlets. Special care must be taken to insulate from the gas pipe at outlets.

Running Along Brick or Stone Walls

Brick or stone walls should be avoided wherever possible; but if such walls cannot be avoided, the wires must be incased in approved conduits.

Main-Line Cut-Out and Fuse

There must be supplied and installed by the contractor a main-line cut-out and a quick-break switch, both double-pole, to be located in the attic at the end of the feeder lines. These devices must be approved by the Underwriters as capable of breaking the current for the total number of lamps wired, at either 115 or 230 volts; and both the cut-out and the switch must be mounted in an asbestos- or slate-lined box provided with a similarly lined door. Knife switches, if used, must be so connected that they open downwards, and the blades must be "dead" when the switch is open.

Inspection, Certificate, and Payment

The contractor must notify the Underwriters' Association of the progress of his work in time to have a thorough inspection made at least 2 days before work is concealed. He must secure a certificate from that Association stating that the work is suitable for use on 110- or 220-volt service, two- or three-wire systems, before any payment shall be made to him.

75. Conduit Wiring.—Interior wiring has passed through a gradual evolution. At first, wires with very inferior insulation were fastened to walls with iron staples; then came wooden cleats; then, improvements in insulation; and later, porcelain knobs, cleats, and tubes. In concealed work, wires were pulled through holes in joists, sometimes two wires in one hole, with no protection other than their insulating covering, which was none of the best. This was disastrous, and improvements both in insulation of the wire and in methods of stringing wire were demanded, and the knob-and-tube system is one result. The provision of a continuous insulated raceway, or **conduit,** soon came. The first of these consisted of stiff paper tubes, which were good when first installed and which would last some time in dry places if not disturbed mechanically. To overcome their brittleness, they were next covered with a thin brass shell; but this protection was not sufficient, as such tubes would still absorb moisture. Now, the only forms of conduits approved by the Underwriters are iron pipes, as shown in Fig. 49, or flexible-steel armored cables.

76. Iron or steel conduit, whether lined or unlined, requires a complete system of piping throughout the building, something similar to a good job of gas-fitting, except that greater care must be taken to see that no burrs or rough places are left anywhere on the inside of the pipe and that all curves are easy (not less than $3\frac{1}{2}$-inch radius). Whenever possible, the pipe should be bent in preference to using an elbow and threaded joints; this is easily done for curves of large radii.

77. Outlet and Junction Boxes.—One of the primary objects in using the conduit system is to be able to withdraw the wires at any time. Joints in the wire would hinder their withdrawal; hence, wherever a joint is necessary, a **junction box** of some kind is provided. In order to avoid the necessity of joints, conduit wiring is done on the so-called **loop system;** that is, no branches are taken off between outlet boxes but the conduit is looped to each outlet.

Fig. 51 (*a*) illustrates, diagrammatically, the method of leading a branch by the knob-and-tube system to lamps *L*, *L*

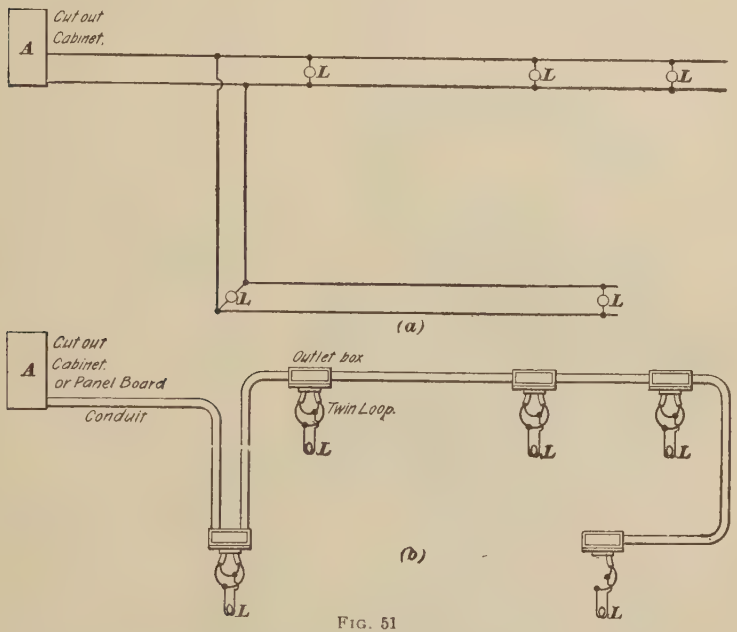


Fig. 51

off the line of the main circuit, and (*b*), the way the same lamps would be reached by the loop system in conduit wiring.

Outlet and junction boxes are made in great variety. Fig. 52 (*a*) shows a cast-iron junction box provided with

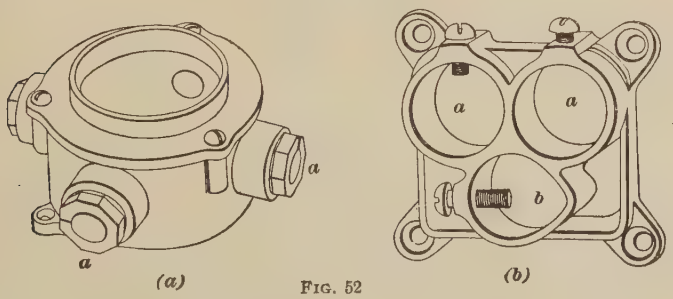


Fig. 52

split nuts *a* for holding the conduit in place; (*b*) shows an **outlet plate** suitable for clamping two conduits in

openings *a*, *a* and a gas pipe in opening *b*. Outlet plates must not be used where it is practicable to install outlet boxes. All outlet boxes must be enameled, galvanized, or otherwise treated both inside and outside to prevent rust, and those used with lined conduit must also be provided with an insulating lining. There must be no openings in the boxes other than those in use.

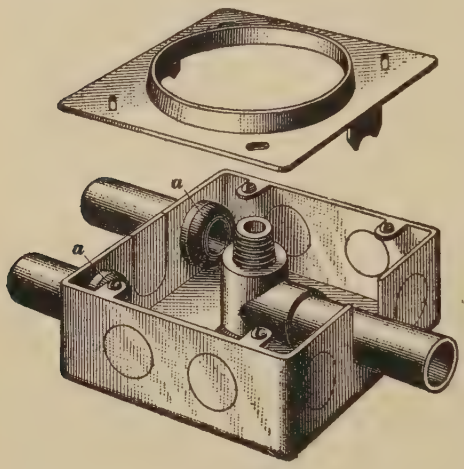


FIG. 53

Stamped-steel boxes, with holes made and plugged so that a blow with a hammer will knock out a plug and leave an opening for the conduit, are now much in use. Fig. 53 shows a combination steel box for gas and electric fixtures. Similar boxes can be obtained for electric fittings only, and with such boxes it is unnecessary to carry a large variety in stock, since holes can easily and

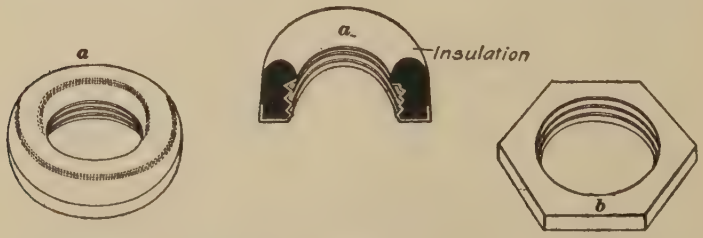


FIG. 54

quickly be made on any side desired by simply striking the stamped-steel disks a blow with a hammer.

The conduit projects inside the box enough to be held in place by an insulating nut *a*. This nut is shown both in perspective and in section at *a*, *a*, Fig. 54. A check-nut *b*

screws up against the outside of the box to hold the box firmly to the ends of the conduit. If the size of wire is changed in passing through a junction box, a cut-out must be installed to protect the smaller wire. Special cut-outs are made for use in junction boxes. So far as possible, however, all fuses are grouped in cut-out cabinets. Junction boxes must always be installed where they are accessible.

78. Arrangement of Outlets.—There are many methods of arranging outlets. Fig. 55 shows one method of arranging a ceiling outlet for a combination fixture in a fireproof building where iron-armored conduit is used. The floors are made of hollow tile placed between I beams. On top of the I beams are laid sleepers, between which are placed the gas pipes and electric conduit, the remaining space being filled nearly flush with the tops of the sleepers with concrete.

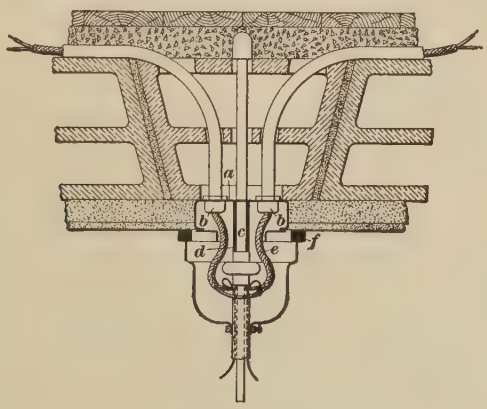


FIG. 55

The flooring is then laid and fastened to the sleepers. The conduit and gas pipes are brought down through the tile to the steel outlet box *a*, which is firmly secured to the ends of the conduit. Insulating nipples *b*, *b* and an insulating sleeve *d* on the gas pipe *c* serve as additional protection against grounds. The twin wire *e* is brought down on the loop system and connected to the fixture wire. The canopy is separated from the ceiling by an insulating ring, or canopy

insulator, *f*. The fixture is supported by attaching it to the gas pipe by means of an **insulating joint,** shown complete and in section in Fig. 56. Piece *a* screws on the gas pipe, and the fixture screws into piece *b*. The two pieces are separated by insulating material *e*, and the outside of the joint is covered with molded insulation *d*.

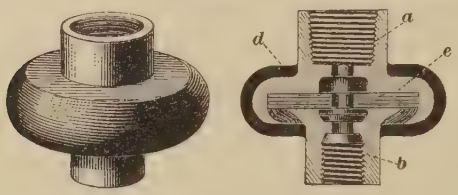


FIG. 56

79. Fig. 57 shows an outlet for an electric fixture mounted against a brick wall. A threaded stud is fastened to the back of the outlet box. The box is fastened by screwing it to wooden plugs driven into holes drilled in the wall or by expansion bolts, and the fixture is screwed on to the threaded stud. In some cases, it may be necessary to sink the box into the brick so as to bring its outer edge flush with the plaster, but generally the wooden strips, or furring, nailed to the wall to take the lath make sufficient depth to take the box between the surface of the plaster and the brick wall. The loop system makes two conduits necessary, one to bring the twin wire down to the fixture and the other for the return. A double-pole wall switch would require the same number of conduits and wires; in this case the wires would be cut and the ends fastened into the switch terminals.

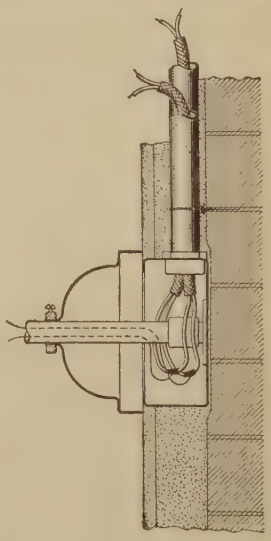


FIG. 57

80. A complete system of interior conduits, including outlet and junction boxes, must be installed before any wires are drawn in. The system must also be in complete electrical connection; that is, there must be no break in the electrical

continuity of the conduit. If at any place, as at cut-out cabinets or junction boxes, the ends of the conduit may not be in good electrical contact, they should be bonded by running a piece of wire around the cabinet and securely soldering an end to each conduit end. The whole conduit system must have a permanent and efficient ground connection. This may be made by connecting the ground wire from the conduit system to a metal plate buried in the earth below permanent moisture level, or to water or other piping systems *on the street side* of main cocks, meters, etc. Grounding the conduit is a safety precaution, as no injury other than the blowing of fuses is likely to result should the electric-light wire become grounded to the conduit.

81. Flexible Metallic Conduit.—Fig. 58 shows a section of **Greenfield flexible metallic conduit**, and at the same time shows a method of making bends. This conduit is more easily installed than rigid conduit, and is in very extensive use, though it cannot be used where moisture is present. It is made of steel strip wound spirally and interlocked, the same as the armor of the cable shown in Fig. 50. Both the conduit and the cable are made by the same company, and both are permitted by the Underwriters, provided installation is made according to the rules for rigid conduit. Outlet boxes and junction boxes must be provided where necessary; the armor on flexible steel conduit must be all connected electrically and must be grounded.

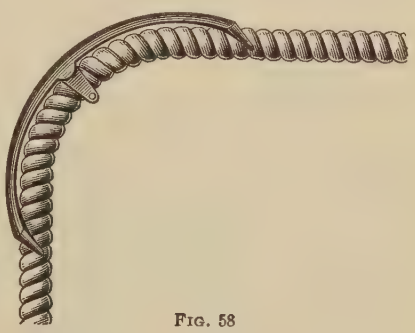


FIG. 58

82. Drawing Wires Into Conduits.—A conduit system is put in before the lathing is done, but the wire is not drawn in until all but the finishing coat of plaster has been applied. For alternating-current systems, it is a very essential

requirement of the Code rules that both wires of a two-wire circuit or all three wires of a three-wire circuit be drawn into the same conduit; and it is advised that this be done for direct-

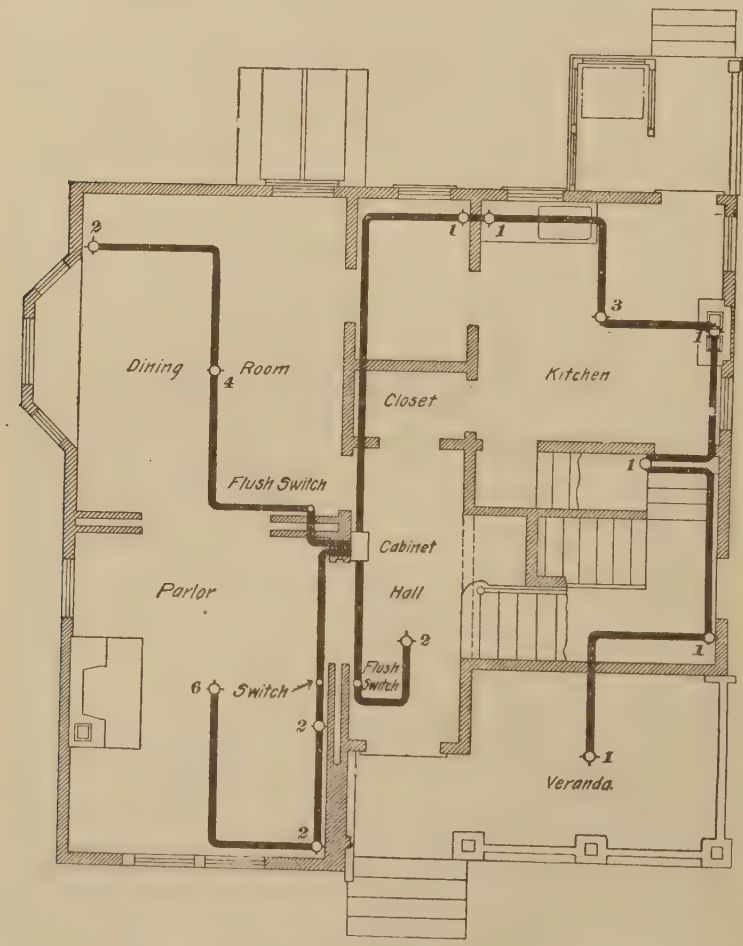


FIG. 59

current systems also. Since it is much cheaper to install but one conduit for all the wires of a circuit, there is little occasion for breaking this rule. Two- or three-conductor cable is generally used. Before drawing in the wires,

soapstone is blown through the conduit in order to make the wires slide more easily.

The wire is drawn into the conduit by a process called **fishing.** A piece of flexible steel ribbon about $\frac{1}{8}$ inch wide and $\frac{1}{16}$ inch thick, called a *snake*, having a steel ball about $\frac{1}{4}$ inch in diameter fastened to one end, is first pushed through the conduit. The steel ball prevents the end of the snake from catching in the insulation, and the snake will follow the bends so that it is possible to push it through several hundred feet of conduit if necessary. The cable or wire is then fastened to the end of the snake and drawn into the conduit.

83. Fig. 59 shows one floor of a dwelling house wired with conduit. The numbers on the various outlets indicate the number of lamps supplied. The wiring is carried out on the loop system; no branches are taken off between outlets. Four circuits are used in order that there may not be more than ten lamps on any one circuit.

84. Molding Work.—Fig. 60 shows a section of two-wire wooden molding in which wires may sometimes be encased for mechanical protection, or where a neater appearance than that given by exposed wires is desired. The molding must have a backing *a* at least $\frac{3}{8}$ inch thick under

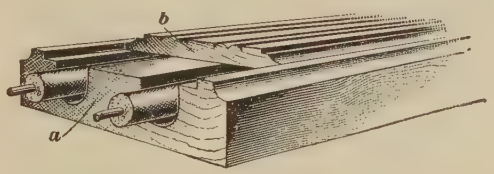


FIG. 60

the wire grooves. The tongue between the groove must be at least $\frac{1}{2}$ inch thick, and those on the sides at least $\frac{1}{4}$ inch. There must also be a cap *b*, and both pieces must be covered outside and inside with at least two coats of waterproof material, or be impregnated with a moisture repeller. Hardwood moldings are recommended.

Wooden molding must not be used in concealed or damp places, and it should not be placed directly against a brick

wall, as sweating may introduce moisture. Only approved rubber-covered wire is permitted in moldings; weather-proof insulation is quite likely to fail and allow the wires to become short-circuited, thus perhaps starting fires.

TESTS

85. It is of the utmost importance that frequent tests be made during the process of installing a job of wiring. Each individual circuit should be tested as soon as it is completed, and each fixture before it is connected to the circuit. If these tests are not made while the work is in progress, serious faults may be found to exist when the job is completed, resulting in much trouble and loss of time.

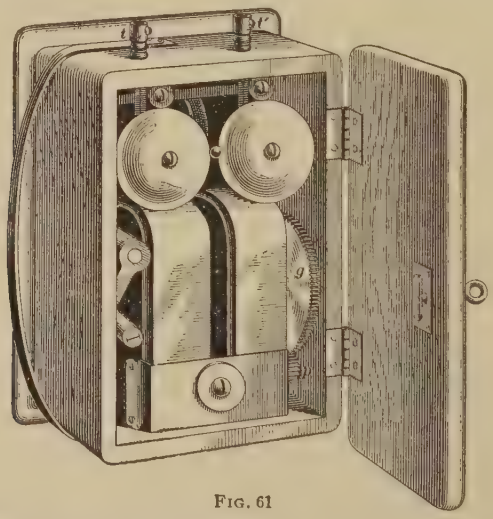


FIG. 61

86. The tests required are for short circuits, **continuity,** and grounds. All may be made with a **magneto-bell,** a small hand-power electric generator connected with a bell similar to the call bell on a telephone. Fig. 61 shows an ordinary testing magneto, on which $t\,t'$ are the terminals to which the circuit to be tested is connected. If the circuit is complete between these terminals, the bells will ring

when the crank-handle (not shown) on the side of the box is turned; if the circuit is not complete, no ring can be obtained. These instruments are made to ring through resistances of from 5,000 to 10,000 ohms, or more. Usually, a few feet of flexible terminal wire is connected to each magneto-terminal, so that contact may easily and quickly be made with any circuit to be tested.

To test a circuit for short circuits or for continuity, connect the magneto-terminals to the two ends of the circuit and turn the crank-handle, first with all lamps or other appliances switched off from the circuit. The bells should not ring; if they do, the lines must be short-circuited. If no short circuit appears, test with the two wires of the circuit connected at their farthermost end by temporarily short-circuiting a lamp socket or switching on a lamp. The bells should ring; if they do not, the circuit is not continuous, that is, it is open. To test for grounds, connect one magneto-terminal to the wire to be tested and the other to any convenient system of piping. The bells should not ring; if they do, the wire is grounded. If any of the defects mentioned—short circuits, open circuits, or grounds—are found to exist, they should be located and removed.

87. Underwriters' Tests.—Insurance inspectors seldom require more accurate tests than can be made with a magneto. In addition to the tests just described, an inspector usually ascertains that no wire will have to carry more than its allowed capacity when all lamps, etc. are switched on. He also inspects the cut-outs, switches, and other devices and materials, as well as the workmanship, in order to see that all rules have been complied with. Concealed work must be inspected before the lath and plaster are applied.

WIRING FOR MINOR ELECTRIC APPLIANCES

BELL WIRING

APPLIANCES

88. Electric bells, formerly a luxury, have become a convenience and a necessity. Few dwellings, office buildings, hotels, factories, etc. are without them. Interior bell-wiring systems are of all stages of complexity, from the simplest door bell to the complex signaling systems of large hotels. Putting in bell-wiring systems is a comparatively simple matter; and yet, if it is not properly done, the system may become a source of trouble, annoyance, and even danger, instead of a convenience.

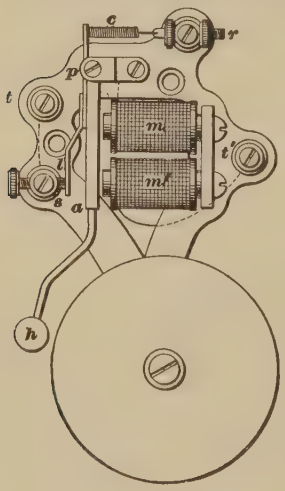


FIG. 62

89. The action of an electric bell may be explained by referring to Fig. 62. The terminals to which wires are connected are at t, t'. The electric current enters, say, at t and passes through the mechanism by way of the adjustment screw s–the spring l–the pivot p–the electromagnets m, m'–out at terminal t'. As soon as current passes through the magnets, the armature a is drawn toward the magnets and the hammer h strikes the bell. The movement of the armature causes spring l to break contact with screw s, and the flow of current ceases. This causes the magnets to lose their attractive power, so that spring c, which may be

adjusted by screw *r*, draws the armature away from the magnets, and contact is again made between the spring *l* and the screw *s*. Current again flows through the magnets and the hammer *h* again strikes the bell, etc., these actions being repeated in quick succession until the circuit is opened at some other point. Every time contact is broken between *s* and *l*, a spark occurs, and most metals would soon become so burned that they would not make good electrical contact; to prevent this, the contact points are tipped with platinum, which is not easily injured by heat.

90. The bell just described is an illustration of *vibrating bells;* in another class, called *single-stroke bells*, the hammer strikes the bell only one blow each time the circuit is closed at the push button, or switch. No automatic contact-breaker is necessary in a single-stroke bell. The bell shown in Fig. 62 could be made single-stroke by changing one wire connection from terminal *t* direct to magnet *m*, or by connecting a piece of wire from *t* across to *p*. Single-stroke bells are usually so constructed that the hammer moves through a considerable distance and strikes the bell a hard blow.

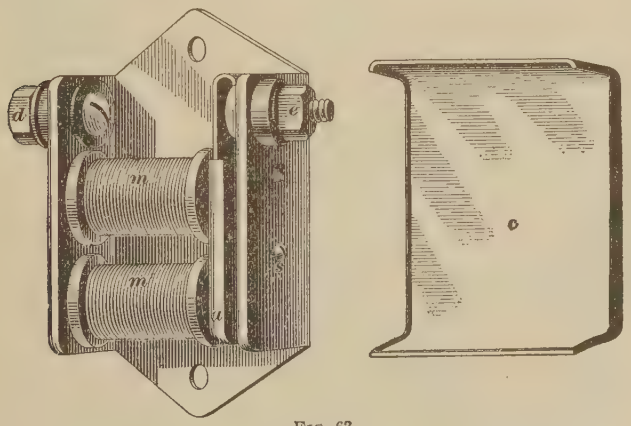
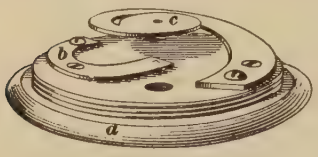


FIG. 63

91. Buzzers.—In places where the noise of an electric bell would be objectionable, a **buzzer** may be used. Fig. 63 shows a buzzer with the cover *c* removed. The wires are

connected to terminals d, e; m, m' are the magnets, and a is the armature, which is adjusted by means of screw s. When current is flowing, the armature vibrates rapidly and thus causes a buzzing sound.



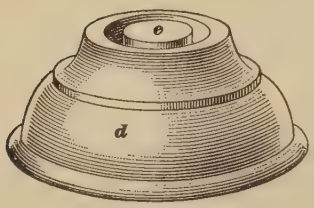


FIG. 64

92. Push Buttons.—Bell-ringing circuits are usually closed, and the bells are rung by means of a **push button,** a very simple form of which is shown in Fig. 64. The wires are brought through a hole in the wooden base a and connected to contact springs b, c. When the cap d is screwed into place over the base, a slight pressure applied to the button e forces contact spring c against contact b, thus closing the circuit.

Push buttons are made in a great variety of styles and finish. The one shown in Fig. 64 is the cheapest kind of wooden button. Buttons may be had of the same general form but with ornamental metal caps; or, if desired, buttons mounted on beautifully engraved bronze plates with spaces arranged for names may be obtained. Smaller buttons, or springs, are made for mounting in the casings of doors or windows so that the opening of a door or the raising of a window will close a contact and ring a bell. Fig. 65 shows a floor push, for mounting in the floor underneath a table so that a slight foot pressure on the cap a will force the plunger b into contact with springs c, d, thus closing a circuit and causing a

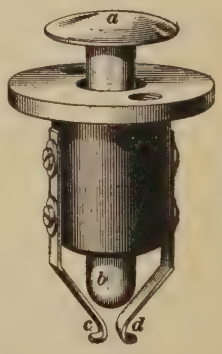


FIG. 65

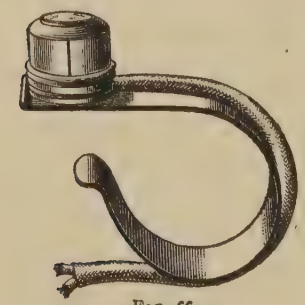


FIG. 66

bell to ring. Fig. 66 shows a button with a clamp for fastening to the edge of a table or a desk.

93. Sources of Current.—It requires but very little current at a low pressure to ring an electric bell, and voltaic cells are nearly always used as the source of current. The cells most used are those of the *open-circuit type*, which will furnish current for short intervals only and will run down if used continuously. Fig. 67 shows a Leclanché cell, a type much used for bell ringing. *B B'* are the terminals, *C* is the carbon electrode contained in a porous cup *P*, and *Z* is the zinc electrode. The space surrounding the carbon electrode, or cathode, inside of the porous cup is filled with a chemical substance mixed with crushed carbon. The chemical absorbs or unites with the free hydrogen gas that would otherwise gather on the surface of the cathode, and thus acts as a depolarizer.

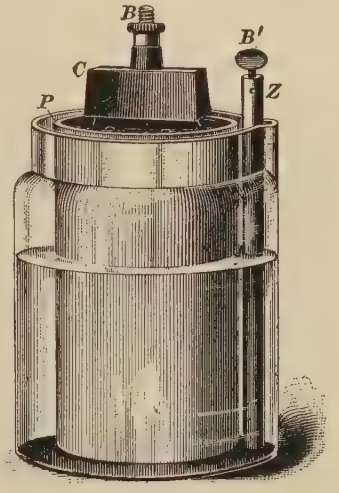


FIG. 67

94. Bell ringing is sometimes accomplished by using current from the lighting system. Fig. 68 (*a*) and (*b*) shows

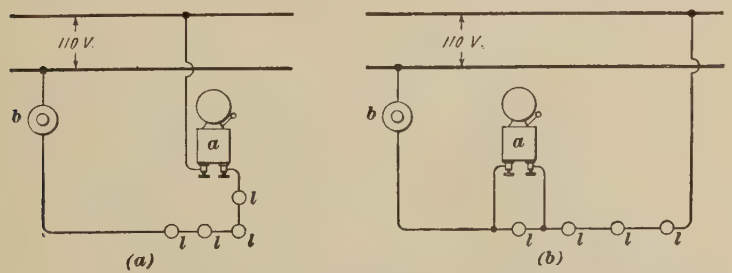


FIG. 68

two methods of making such connections; *a* is the bell, *b*, the button, and *l*, *l*, *l*, *l* are incandescent lamps. In (*a*), the

automatic contact-breaker on the bell must break the full voltage of the lighting circuit and may thereby soon be destroyed; in (*b*), only the voltage across one lamp has to be broken. The Underwriters' rules will not permit the connection of bell circuits to lighting circuits unless all the bell wiring is installed according to the rules for electric-light wires.

95. Fig. 69 shows connections for ringing bells by means of storage batteries so that the bell circuits are never connected to the lighting circuits, and hence the bell wiring need not conform to the rules for electric-light wiring. If

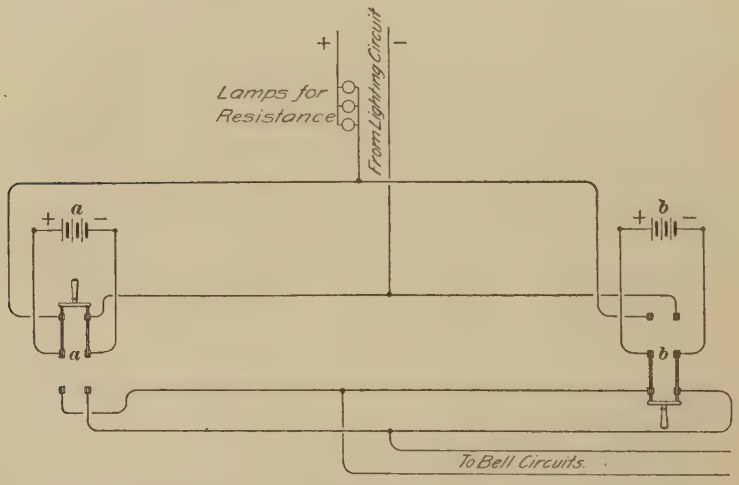


FIG. 69

the bells are not in use during certain hours of the day, only one battery will be needed, as it can be charged while the bells are idle; but if they must be in readiness for use at any moment, two batteries, as shown, will enable one to be charged while the other is in service. The connections show the double-pole double-throw switch *a* closed in the upper, or battery, contacts, while a similar switch *b* is closed in the bell-circuit contacts; so that battery *a* is receiving a charging current from the lighting circuit, and battery *b* is in readiness to supply ringing current to the bell circuit The charging current is limited by the resistance of

the lamps. Storage cells cost more than primary, or voltaic, cells, but each storage cell has about the same voltage as two voltaic cells.

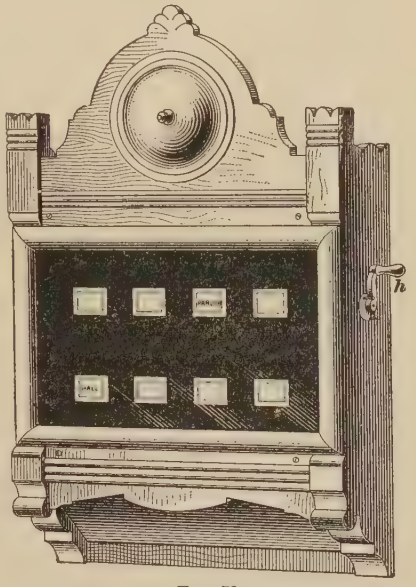


FIG. 70

96. In large bell-ringing systems, a small dynamo or motor generator is sometimes installed. A motor generator is two machines, a dynamo and a motor, direct-connected. For ringing bells, the motor would be run on the light or power circuits of the building, and would drive the dynamo to supply ringing current. The two machines are sometimes combined into one, called a *dynamotor*. Ordinary bell wiring must not be connected to any source of pressure exceeding 10 volts, otherwise the wiring would come under the low-potential classification of the Code rules and would have to be put up the same as electric-light wires.

97. Annunciators.—An **annunciator** is a device that indicates the source of a signal when it is possible to ring the same bell from several points. An ordinary house-style annunciator is shown in Fig. 70. Each time the bell rings, the location of the button is indicated on the front of the annunciator by a *drop*, which remains down until replaced by means of the handle *h*. Fig. 71 shows a single drop. A hinged arm *a*, bearing a card with the name or number of the location of a button,

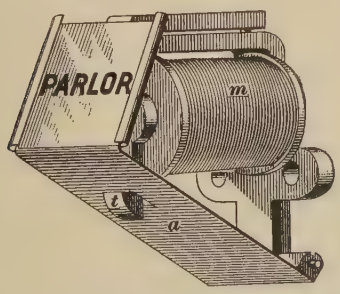


FIG. 71

is held up by a counterbalanced trip t in front of an electromagnet m. When the button is pushed, the ringing current passes through the electromagnet, the trip is released, and the drop falls as soon as the bell begins to ring. Fig. 72 shows a **needle annunciator,** much used in hotels and for elevators. Pivoted armatures on the back carry needles outside, and when a button is pressed, a needle indicates the source of the signal, as at P. The needles are reset to a horizontal position by pushing a knob k located below the case.

FIG. 72

98. Self-restoring annunciators are in use, in which each time a button is pressed not only does the corresponding drop fall, but also any drop that may have been down from a preceding ring, is reset. These prevent any confusion that might arise from forgetting to reset an annunciator.

BELL-WIRING CIRCUITS

99. Fig. 73 shows a system of bell wiring with no annunciator, which is suitable for a small dwelling. Fig. 74 shows connections for a simple annunciator system. The buttons

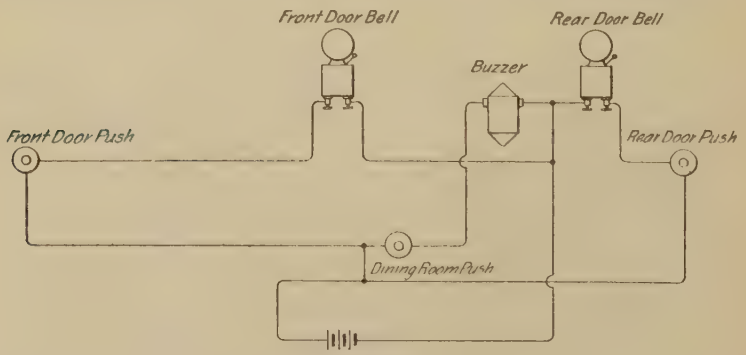


FIG. 73

1, *2*, *3*, etc. are located at convenient points in the various rooms or at outside doors. One terminal of each button is connected to the common *battery wire b*, which goes to one battery terminal. The other terminal of each button is connected by a *leading wire l* to the proper electro-magnet in the annunciator, whence a wire goes to one bell terminal. The other bell terminal is connected to the vacant battery terminal. The battery *B* may have any convenient and accessible location where it will not freeze or be exposed to excessive moisture.

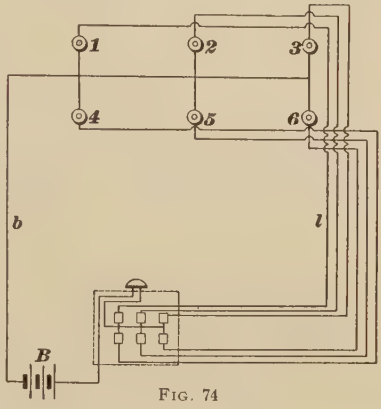


FIG. 74

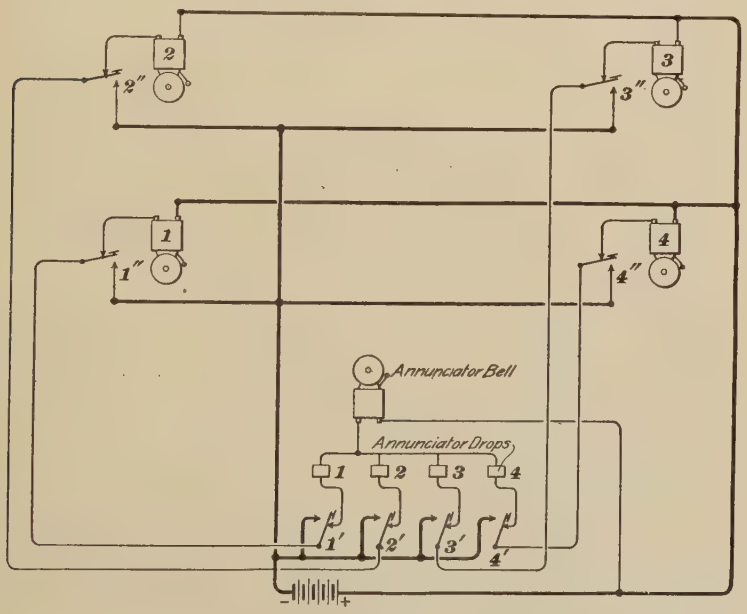


FIG. 75

100. Return-Call Annunciator System.—Fig. 75 is a diagram of connections for a return-call annunciator system.

Each station is equipped with a bell and a double-contact push button, such as shown in Fig. 76; metal spring *t* normally remains in contact with a point *c*, but breaks contact with *c* and touches *c'* when the button *k* is pressed. The buttons below the annunciator, Fig. 75, are also double contact. If the button *1''* in room *1* is pressed, current flows from the positive side of the battery through the annunciator bell and drop *1*–the permanent contact of button *1'*–the depressed button–back to the negative battery terminal; this rings the annunciator bell and operates drop *1* but does not ring the bell in room *1*. When button *1''* is released, it returns to its permanent contact, and a return signal may then be sent by depressing button *1'*; current will then flow from the positive battery terminal through bell *1*–the permanent contact of button *1''*–the depressed button *1'*, back to the negative battery terminal; this rings bell *1* but does not ring the annunciator bell.

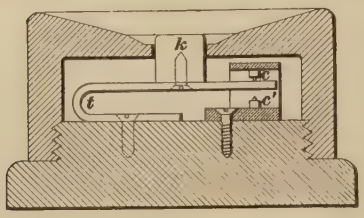


FIG. 76

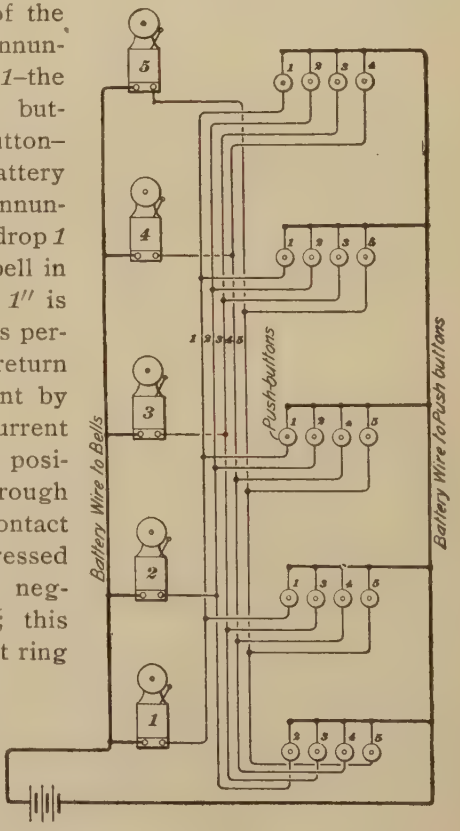


FIG. 77

101. Wiring for Speaking-Tube System.—Fig. 77 shows a plan of wiring for electric bells used in conjunction with speaking tubes. There

are five stations, each provided with a bell and four push buttons, one button for each of the other stations; thus, from any station can be rung any bell except its own.

102. Wiring for Fire-Alarm Gongs.—Fig. 78 shows connections by which widely distributed gongs may all be rung at the same time by depressing a single button; at the same time an annunciator drop near each bell will reveal

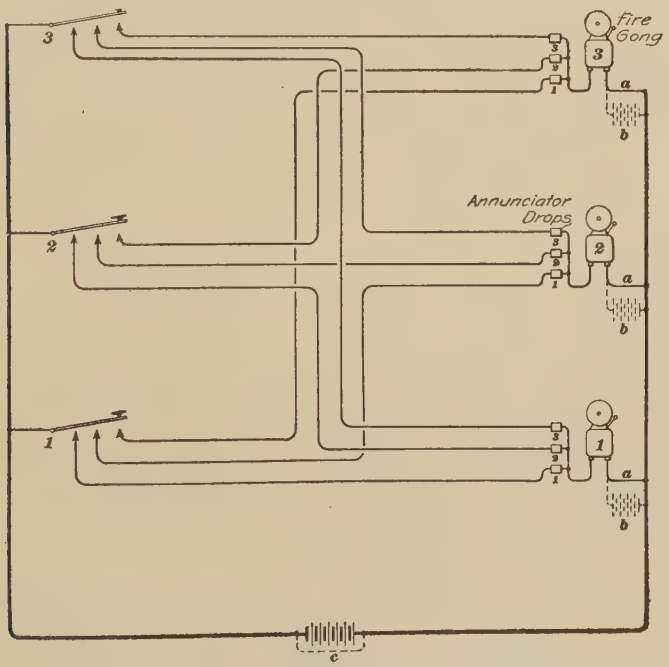


FIG. 78

the source of the alarm. The full lines indicate the connections if a single battery is used, but better results would be obtained by dividing the battery and placing a part at each station, as shown by the dotted lines. Connections a, a, a would then be replaced by batteries b, b, b, and the main battery by a connection c.

103. Wiring for Flats.—Fig. 79 shows a plan of wiring for four flats, each provided with a bell, a buzzer, and

two push buttons. At the rear door on each flat is a button for sounding the buzzer, and at the front door a button for ringing the bell. On the lower landing near the front door of the building are four push buttons—one for the bell in each flat.

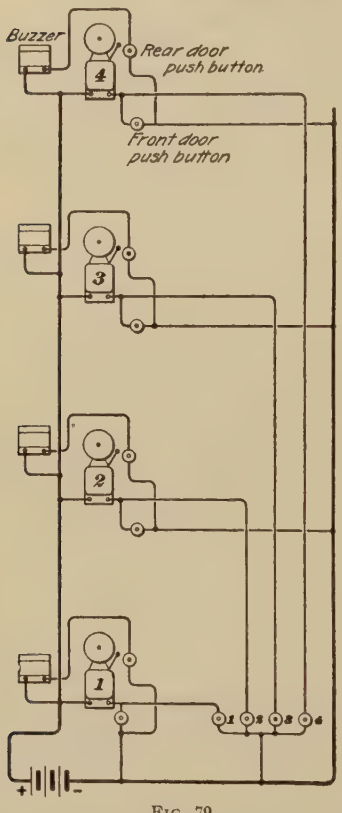


FIG. 79

INSTALLING BELL WIRES

104. No. 16 or No. 18 B. & S. wire covered with two layers of cotton treated with paraffin is generally used for bell-wire circuits in dry places. For good work, or where dampness is present, weather-proof office wire should be used; if the wires are to be exposed to much moisture they should be rubber covered. If considerable current is to be carried on a wire, as on the main battery wire in a large system, it will pay to use No. 14 B. & S., so as to obtain less resistance.

105. There are not many Code regulations for putting up electric-bell wires. The wires must be neatly arranged and securely fastened in place in some convenient workmanlike manner. They must not come nearer than 6 inches to any electric-light or power wire in the building unless they are encased in approved tubing. When bunched together in any building, bell wires must have a fire-resisting covering or else be incased in a non-combustible tube or shaft. They should not be in the same tube with electric-light or power wires, and if in the same shaft, must be kept at least 2 inches from such wires. Wire ducts or shafts must be thoroughly

stopped at each floor or wall to prevent the spread of fire. Both push-button and electric-light switches must not be mounted on the same wall plate.

106. Bell wires are often stapled to woodwork. The staples should not be driven hard, and both wires should not be fastened under one bare staple; but special insulated staples, as shown in Fig. 80, may be so used. In new buildings, the bell wires can be grouped and run through holes in the woodwork, thus avoiding the use of staples. In the best class of work, bell wires are run in conduits. All bell circuits, when complete, should be tested for breaks, grounds, or crosses.

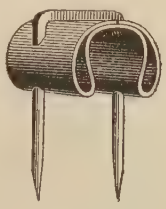


FIG. 80

DOOR OPENERS

107. In apartment houses, banks, and other places, it is often convenient to have the latch on a door arranged so that the door may be unlocked from some distant point. For this purpose **door openers** are used. These are made in a number of styles, the mechanism differing with the different makes. In all of them, however, the unlocking is effected by means of an electromagnet, which is connected to the push and battery in the same way as an ordinary bell.

BURGLAR ALARMS

108. Wiring for **burglar alarms** does not differ materially from ordinary bell wiring. A simple connection can be made to each window so that any attempt to raise the window will close a contact and ring a bell. The wiring can all be connected to an annunciator that will reveal the point at which the entrance has been attempted. Fig. 81 shows one of many kinds of window springs made for such purposes. The plate, with insulated spring *s* on the inside, is let into the window frame just above the lower sash, so that only the cam *c* projects on the outside. The cam is normally kept from making electrical contact with the spring

by means of insulating wheel w, but when the sash is raised it turns the cam on pivot p until contact is made with the spring, and, as the wires are connected to the plate and spring, respectively, the bell rings.

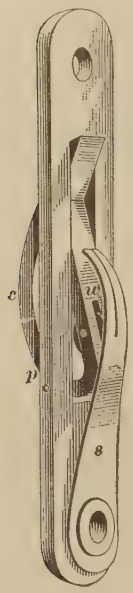


FIG. 81

109. Fig. 82 shows connections for the windows of two rooms to an ordinary annunciator. During the day, when the alarm is not in use, switch s is left on the middle contact, as shown. When setting the alarm, switch s is first placed on the lower contact a, and keys k_1 and k_2 are closed, when, if any window is open, the corresponding drop will fall. For example, if a parlor window is open, thus closing one of the contacts e, current will flow from the battery B through s–a–electromagnet a_2–key k_2–contact e–back to the battery. This causes the drop of a_2 to fall, but the bell v does not ring. When the windows are all closed, the drops all remain in position, and switch s is placed on the upper contact c. Any attempt to open a window will then cause the bell to ring and a drop to fall, since current from the battery will then go through the bell before reaching the drop. The dotted lines show con-

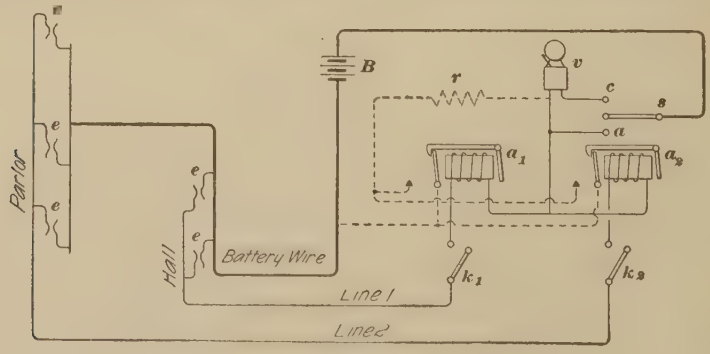


FIG. 82

nections sometimes made through a resistance r, so that the falling of any drop will cause the bell to ring continuously.

ELECTRIC CLOCKS

110. A watchman's clock consists of a number of electromagnets so arranged that when any magnet is excited, the core moves and causes a punch to make a hole in a paper dial. A clock movement causes the dial to rotate so that the exact time a hole is punched may be known by inspecting the dial. In large establishments, the watchman's stations are located in various places that the watchman is expected to visit. The stations may be switches, if direct-current magnets are used in the clock, or they may be magnetos, if alternating current is used. The switchman carries a switch handle, key, or a magneto-handle, as the case may be, with which, at each station, as he visits them in regular order, he closes a circuit leading to one of the clock magnets, or turns the magneto and sends an alternating current through the magnet. The clock itself is usually located in the main office where the watchman cannot get to it. This furnishes a system by which the movements of the watchman can be known with certainty. If he fails to visit a station at the appointed time, the absence of a corresponding hole in the clock dial will expose his failure.

111. Electric time clocks are used in many large establishments where it is desirable that a uniform time shall govern employes in a number of rooms or buildings. A *master time clock* is located in one of the offices, and at regular intervals, usually once each minute, the master clock closes an electric circuit that passes through a series of electromagnets. The core of each magnet actuates a ratchet and pawl, by means of which a clock hand is moved forwards 1 minute every time the magnet is excited. Each magnet, with its accompanying mechanism arranged in a suitable case behind a clock dial, constitutes a *secondary time clock*. The secondary clocks may be placed wherever desired; there must, however, be sufficient electromotive force in the clock circuit to send through all the electromagnets in series a current large enough to operate each magnet.

ELECTRIC GAS LIGHTING

PARALLEL SYSTEM

112. Electric gas lighting is accomplished by causing an electric spark to pass through a jet of gas. In the **parallel system**, each burner is independent of all the others. There are three styles of burners, the *pendant*, the *ratchet*, and the *automatic*.

113. Fig. 83 shows a **pendant burner.** A well-insulated wire is brought from one terminal of a battery to the burner, and is secured to the insulated contact piece *c* by screw *s*. Piece *c* is fastened to the frame *f* by screw *l*, and is insulated from the frame by washers *w*. The other battery terminal is connected to the gas pipe so that when the pendant *r* is pulled and wire *a* forced upwards until it makes contact with *c*, the battery circuit is closed and a current flows from the battery through *s*–*c*–*a*–*f*–the gas pipe–back to the battery. When *a* slides across *c* and leaves it on the upper side, the circuit is broken, thus causing a spark and igniting the escaping gas. Pulling the pendant *r* does not turn on the gas; it only causes ignition after the gas is turned on.

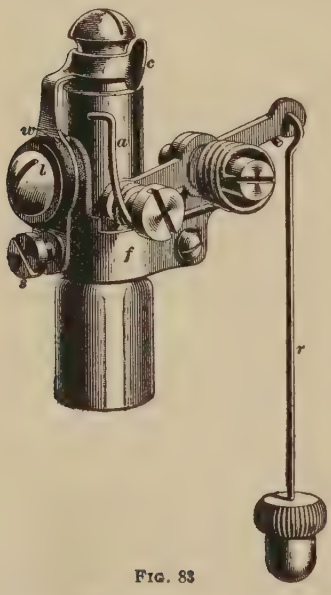


FIG. 83

114. The **ratchet burner** is provided with a ratchet and pawl, so that a downward pull turns on the gas and ignites it, while a second pull turns off the gas.

115. The **automatic burner** is provided with two electromagnets. The movement of the armature of one

magnet turns on the gas and breaks the circuit in the jet, thus causing a spark that ignites the gas; the movement of the other armature turns off the gas. To operate this burner, two push buttons are required, one terminal of each button being connected to an electromagnet terminal and the other to a common-battery terminal. The other battery terminal is connected to the gas pipe. When one button is pressed, the gas is turned on and ignited; and when the other is pressed, the gas is shut off. These burners are not very safe because they are liable to leak gas. They are used principally in hallways, and are wired so that the gas may be ignited or extinguished from one or more rooms in the house.

116. Arrangement of Apparatus.—In order to produce a spark intense enough to ignite a gas jet with the parallel system, a **spark coil** is used. This is a coil of wire wound on an iron core and connected as shown in Fig. 84, where *a* is the gas jet, *b* an insulated wire leading

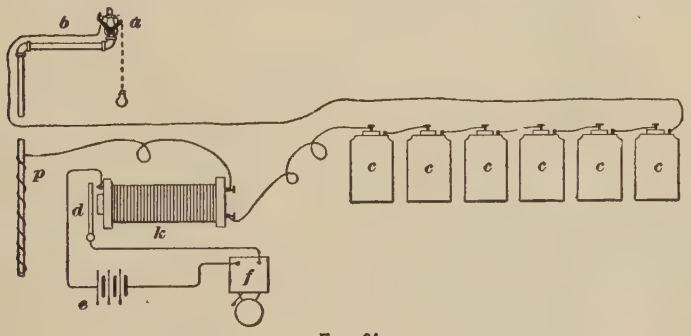


FIG. 84

from one terminal of the battery *e*, *c*, *c* direct to a spark contact at the jet, and *k* is the spark coil, one terminal of which is connected to the battery and the other to the gas pipe *p*.

With these connections, an accidental ground of wire *b* to the gas pipe would close a circuit through the battery, the spark coil, and the gas pipe, and a continuous current would flow until the battery was exhausted. In order to give notice of such current leakage, the spark coil may be provided with an armature *d* that is normally held away from

the core; when current flows through the coil, the armature is attracted, and a circuit is closed through a battery *e* and a bell *f*, and the bell rings.

The wires may be run on the outside of the gas fixtures, the insulation being so colored that it does not show against the fixture, or they may be run between the fixture shell and the gas pipe. Electric gas-lighting wires must not be installed on combination fixtures for both gas and electric lights.

SERIES SYSTEM

117. The **series, or flash, system** of gas lighting is used in large halls, churches, theaters, etc. where many lights are installed in groups. Fig. 85 shows the style of burner used; *a* and *b* are fixed sparking points, each insulated from the other and from the gas pipe. The body of the burner is made of an insulating substance, and a flange of mica *m* is added to give further protection. A fine bare copper wire is stretched from burner to burner and secured in the small holes at the lower ends of *a*, *b*, so that all the burners in a group are connected in series, as shown in Fig. 86. The first jet in the series is connected to a terminal of an induction coil *i* and the last to the gas pipe. An induction coil is wound like a small transformer, there being

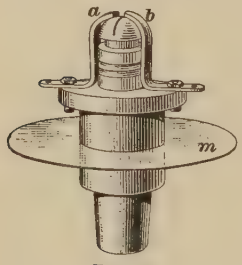


FIG. 85

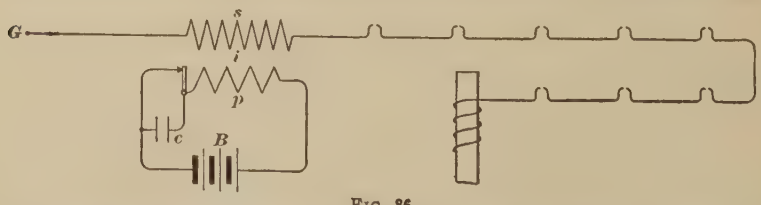


FIG. 86

a primary winding *p* of a few turns of comparatively coarse wire and a secondary winding *s* of many turns of fine wire. One terminal of the secondary is connected through the series of gas jets to the gas pipe, and the other *G* direct to the

STANDARD SYMBOLS FOR WIRING PLANS

AS ADOPTED AND RECOMMENDED BY

THE NATIONAL ELECTRICAL CONTRACTORS ASSOCIATION OF THE UNITED STATES

COPIES MAY BE HAD ON APPLICATION TO THE SECRETARY, UTICA, N. Y.

Ceiling Outlet; Electric only. Numeral in center indicates number of Standard 16 C. P. Incandescent Lamps.

Ceiling Outlet; Combination. $\frac{4}{2}$ indicates 4-16 C. P. Standard Incandescent Lamps and 2 Gas Burners.

Bracket Outlet; Electric only. Numeral in center indicates number of Standard 16 C. P. Incandescent Lamps.

Bracket Outlet; Combination. $\frac{4}{2}$ indicates 4-16 C. P. Standard Incandescent Lamps and 2 Gas Burners.

Wall or Baseboard Receptacle Outlet. Numeral in center indicates number of Standard 16 C. P. Incandescent Lamps.

Floor Outlet. Numeral in center indicates number of Standard 16 C. P. Incandescent Lamps.

Outlet for Outdoor Standard or Pedestal; Electric only. Numeral indicates number of Stand. 16 C. P. Incan. Lamps.

Outlet for Outdoor Standard or Pedestal; Combination. $\frac{6}{6}$ indicates 6-16 C. P. Stand. Incan. Lamps; 6 Gas Burners.

Drop Cord Outlet.

One Light Outlet, for Lamp Receptacle.

Arc Lamp Outlet.

Special Outlet, for Lighting, Heating and Power Current, as described in Specifications.

Ceiling Fan Outlet.

S^1 S. P. Switch Outlet.

S^2 D. P. Switch Outlet.

S^3 3-Way Switch Outlet.

S^4 4-Way Switch Outlet.

S^D Automatic Door Switch Outlet.

S^E Electrolier Switch Outlet.

Show as many Symbols as there are Switches. Or in case of a very large group of Switches, indicate number of Switches by a Roman numeral, thus; S^1 XII; meaning 12 Single Pole Switches.

Describe Type of Switch in Specifications, that is Flush or Surface, Push Button or Snap.

Meter Outlet.

Distribution Panel.

Junction or Pull Box.

Motor Outlet; Numeral in center indicates Horse Power.

Motor Control Outlet.

Transformer.

Main or Feeder run concealed under Floor.

Main or Feeder run concealed under Floor above.

Main or Feeder run exposed.

Branch Circuit run concealed under Floor.

Branch Circuit run concealed under Floor above.

Branch Circuit run exposed.

Pole Line.

Riser.

Telephone Outlet; Private Service.

Telephone Outlet; Public Service.

Bell Outlet.

Buzzer Outlet.

Push Button Outlet; Numeral indicates number of Pushes.

Annunciator; Numeral indicates number of Points.

Speaking Tube.

Watchman Clock Outlet.

Watchman Station Outlet.

Master Time Clock Outlet.

Secondary Time Clock Outlet.

Door Opener.

Special Outlet; for Signal Systems, as described in Specifications.

Battery Outlet.

Circuit for Clock, Telephone, Bell or other Service, run under Floor, concealed.
Kind of Service wanted ascertained by Symbol to which line connects.

Circuit for Clock, Telephone, Bell or other Service, run under Floor above, concealed.
Kind of Service wanted ascertained by Symbol to which line connects.

NOTE—If other than Standard 16 C. P. Incandescent lamps are desired, Specifications should describe capacity of Lamp to be used.

SUGGESTIONS IN CONNECTION WITH STANDARD SYMBOLS FOR WIRING PLANS

Indicate on plan, or describe in specifications, the height of all outlets, located on side walls.

It is important that ample space be allowed for the installation of mains, feeders, branches and distribution panels.

It is desirable that a key to the symbols used accompany all plans.

If mains, feeders, branches and distribution panels are shown on the plans, it is desirable that they be designated by letters or numbers.

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FIG. 87

pipe. One terminal of the primary is connected to a terminal of a battery B, and the other, to an automatic contact-breaker, similar to the contact-breaker on an electric bell. The contact-breaker is connected to the other battery terminal. A condenser c is connected in parallel with the contact-breaker between the primary-coil terminal and the battery terminal.

118. In order to force a spark across the spark gaps of several burners in series, a high electromotive force is necessary; hence, the wiring must be installed with the greatest precaution. If it is necessary for the wire to approach nearer than $1\frac{1}{4}$ inches from the gas pipe, except where purposely grounded to it, the wire should be enclosed in glass tubing.

STANDARD SYMBOLS FOR WIRING PLANS

119. Confusion has often been encountered by different architects using different symbols on plans to designate the same piece of electrical apparatus. To avoid this confusion, the National Electrical Contractors' Association has arranged the standard set of symbols shown in Fig. 87. These symbols have been adopted by various technical societies, and are used by the leading architects throughout the country. The letters C. P. used in the figure stand for candlepower, while S. P. and D. P. stand for single pole and double pole, respectively. By standard 16-candlepower lamps are meant ordinary carbon-filament lamps requiring about .5 ampere each.

DESIGN OF BEAMS

ELEMENTS OF MECHANICS

1. The design of beams is part of the subject of mechanics. To properly understand the principles that underlie this subject, it is necessary to have knowledge of the simple elements of mechanics. **Mechanics** is that science which treats of the action of forces on bodies and the effects that they produce; and the laws that govern the movement and state of rest of bodies.

2. A body is **at rest** when it constantly occupies the same place in space.

3. A body is **in motion** when the body or its parts occupy successively different positions in space.

4. A **force** is any cause that changes or tends to change a body's state of rest or motion.

5. **Equilibrium** is that state of a body when all the external forces acting on it are balanced and will not, therefore, move the body or any part thereof. All bodies that are at rest are in equilibrium and the forces acting on them are balanced.

6. **Statics** is that division of the science of mechanics which treats of the relation between the forces acting on a body at rest.

This subject of statics includes the design of beams and, in fact, the entire theory of architectural engineering. An engineer practicing this profession must design economically the parts of a structure so that it will never move, and he is concerned with nothing else.

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VECTOR QUANTITIES

7. A **vector quantity** is a quantity to give a complete description of which it is necessary to give a *direction*. Thus, when it is said that a ship is moving with a velocity of 10 miles an hour, its motion is not completely described. To do this, it is necessary to add that the ship is moving northeast or southeast or in a circle, as the case may be. Therefore, *velocity* is a vector quantity.

To saw a certain stick of wood with a hand saw requires 2 minutes of time. Now the 2 minutes is not a vector quantity because it has no direction. When it is said that for a certain event to happen it takes 2 minutes, the time is completely described and nothing more need be added.

8. Force.—The quantity of most importance to the architectural engineer is **force**, which is a vector quantity because it acts in a certain specific direction.

9. Effect of a Force on a Body.—The effect of a force on a body may be compared with the effect of another force when the three following conditions are fulfilled in regard to both forces:

1. *The point of application, or point at which the force acts on the body, must be known.*
2. *The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.*
3. *The magnitude of the force, when compared with a given standard, must be known.*

In this and the succeeding Sections, the unit of force will always be taken as 1 pound, and the magnitudes of all forces will be expressed in pounds.

10. Representation of Force.—A force may be represented by a line. Thus, in Fig. 1, let A be the *point of application* of the force, let the length of the line AB represent its *magnitude*, and let the arrowhead indicate the *direction* in which the

FIG. 1

force acts; then the line AB fulfils the three required conditions in regard to point of application, direction, and intensity, and the force is fully represented.

11. Changing the Point of Application.—There is one property of a force that is yet to be mentioned. The point of application of a force may be changed in the direction of its line of action without changing the external effect of that force. Thus, in Fig. 2, let a rod press in the direction of its length on the block a at the point b. The force exerted by this rod presses the block down on the table with a certain definite pressure on each square inch of the table that is under the block. Now, imagine that the rod is moved and presses

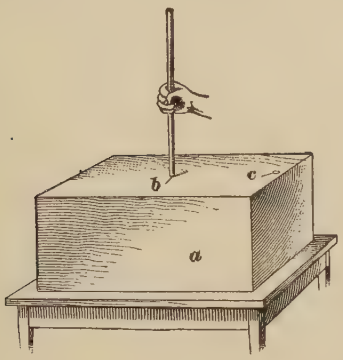


FIG. 2

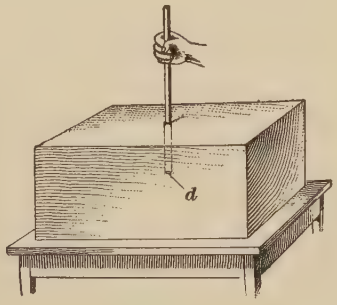


FIG. 3

on the block at c. It can be seen that in this case the pressure will change on each square inch of the table and that one edge of the block will bear much more heavily than the other. Now, suppose that at the point b a hole is bored directly down into the block to a point d, as shown in Fig. 3, and that the rod is inserted in this hole and pressed down. Then, the rod pressing at d will have precisely the same effect on the pressure between the block and the table as the rod pressing at b, Fig. 2, because the point of application is moved in the direction of the line of action of the force, which is not the case with the point c.

12. If several forces act on a body, and if they are replaced by a single force that has the same effect on the

body as the several forces combined, the single force is called the **resultant** of the several forces; and, conversely, the several forces are called the **components** of the single force.

THE COMPOSITION OF FORCES

13. Parallelogram of Forces.—When two forces act on a body at the same time, but at different angles, their final effect may be obtained as follows:

In Fig. 4, let A be the common *point of application* of the two forces, and let AB and AC represent the *magnitude* and *direction* of the forces. The final effect of these two forces will be the same whether they act singly or together. Let, for instance, the line AB represent the distance that the force AB would cause the body to move; similarly, let AC represent the distance that the force AC would cause the body to move, when both forces were acting separately. The force AB, acting alone, would carry the body to B; if the force AC were now to act on the body, it would carry it along the line BD, parallel to AC, to a point D, at a distance from B equal to AC. Join C and D, then CD is parallel to AB and $ABDC$ is a parallelogram. Draw the diagonal AD. The body will stop at D, whether the forces act separately or together, but if they act together, the path of the body will be along AD, the diagonal of the parallelogram. Moreover, the length of the line AD represents the magnitude of a force, which, acting at A in the direction AD, would cause the body to move from A to D; in other words, AD, measured to the same scale as AB and AC, represents the magnitude and direction of the combined effect of the two forces AB and AC.

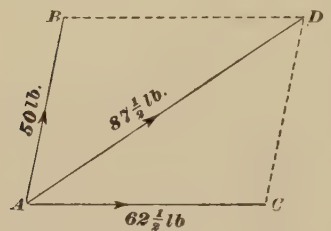


FIG. 4

The force represented by the line AD is the resultant of the forces AB and AC. Suppose that the scale used was 50 pounds to the inch; then, if AB = 50 pounds, and AC

$= 62\frac{1}{2}$ pounds, the length of AB would be $50 \div 50 = 1$ inch, and the length of AC would be $62.5 \div 50 = 1\frac{1}{4}$ inches. If the line AD measures $1\frac{3}{4}$ inches, the magnitude of the resultant, which it represents, would be $1\frac{3}{4} \times 50 = 87\frac{1}{2}$ pounds.

Therefore, a force of $87\frac{1}{2}$ pounds, acting on a body at A, in the direction AD, will produce the same result as the combined effects of a force of 50 pounds acting in the direction AB and a force of $62\frac{1}{2}$ pounds acting in the direction AC.

14. The foregoing method of finding the resulting action of two forces acting on a body at a common point, is correct, whatever may be their direction and magnitudes. Hence, to find the resultant of two forces when their common point of application, their direction, and magnitudes are known:

Rule.—*Through an assumed point draw two lines parallel with the direction of the two forces. With any scale, measure from the point of intersection, in the direction of the forces, distances corresponding to the magnitudes of the respective forces, and from the points thus obtained complete the parallelogram. Draw the diagonal of the parallelogram from the point of intersection of the two forces; this diagonal will represent the resultant, and its direction will be away from the point of intersection of the two forces. To find the magnitude of the resultant, the diagonal must be measured with the same scale that is used to lay off the two forces.*

EXAMPLE.—If two forces act on a body at a common point, and the angle between them is 80°, what is the value of the resultant, the magnitude of the two forces being 60 pounds and 90 pounds, respectively?

SOLUTION.—Draw two indefinite lines having an angle of 80° between them (Fig. 5). With any convenient scale, say 10 lb. to the in., measure off

$$AB = 60 \div 10 = 6 \text{ in.}$$

and

$$AC = 90 \div 10 = 9 \text{ in.}$$

Through B draw BD parallel to AC, and through C draw CD parallel to AB, intersecting at D. Then draw AD, and AD will be the resultant; its direction is toward the point D, as shown by the arrow. By measuring, the length of AD is found to be 11.7 in. Hence, the magnitude of the resultant is

$$11.7 \times 10 = 117 \text{ lb.} \quad \text{Ans.}$$

15. Triangle of Forces.—Let Fig. 5 represent a parallelogram of forces. AB and AC are the two component forces and AD is the resultant. Now, to find this resultant AD, instead of drawing the entire parallelogram, we may first lay off AB and then from the point B lay off BD equal to AC, and parallel to its line of action; then, draw AD, which completes the triangle ABD and makes it unnecessary to draw AC and CD. Or, we might first have drawn the force AC and from C drawn CD, thus completing the triangle ACD and not drawing AB and BD. In either case, it is seen that, if desired, the resultant of two forces can be found by drawing a triangle instead of a parallelogram.

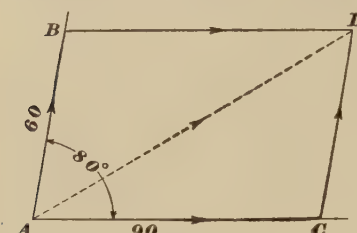


FIG. 5

Rule.—*Through any point draw a line to represent either of the forces in magnitude and direction. At the extremity of this line draw a second line parallel to the line of action of the second force and representing this force in magnitude and direction. Join the extremities of the two lines by a straight line; then this joining line will represent the resultant, and its direction will be opposite to that of the two forces.*

When we speak of the resultant being opposed in direction to the other two forces, we mean that, starting from the point where we began to draw the triangle, and tracing each line in succession, the action of the component forces will have the same general direction around the triangle as if passing around a circle, from left to right, or from right to left, but the closing line or resultant must have an *opposite direction;* that is, *the two arrowheads, the one on the resultant and the other on the last side, must point toward the intersection of the resultant and the last side.*

16. Resultant of Several Forces.—When three or more forces act on a body at a given point, their resultant may be found by the following rule:

Rule.—*Find the resultant of any two forces; treat this resultant as a single force, and combine it with a third force to find a second resultant. Combine this second resultant with a fourth force, to find a third resultant, etc. After all the forces have been thus combined, the last resultant will be the resultant of all the forces, both in magnitude and direction.*

EXAMPLE.—Find the resultant of all the forces acting on the point O, Fig. 6, the length of the lines being proportional to the magnitude of the forces.

SOLUTION.—Draw OE parallel and equal to AO, and EF parallel and equal to BO; then OF is the resultant of these two forces, and its

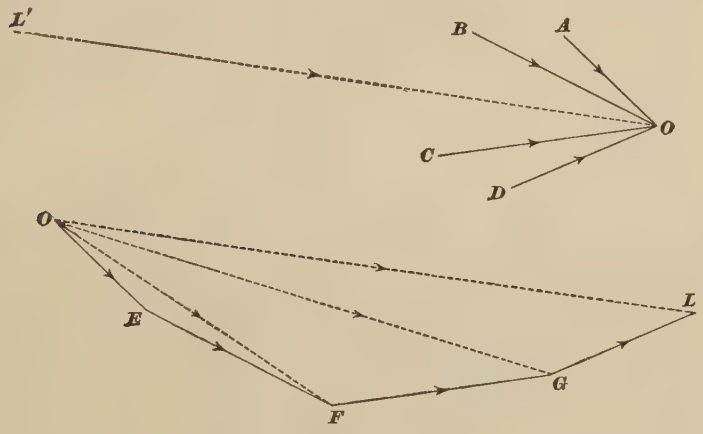


FIG. 6

direction is from O to F, opposed to OE and EF. Consider OF as replacing OE and EF, and draw FG parallel and equal to CO; OG will be the resultant of OF and FG; but OF is the resultant of OE and EF; hence, OG is the resultant of OE, EF, and FG, and likewise of AO, BO, and CO. The line FG, parallel to CO, could not be drawn from the point O to the right of OE, for in that case it would be opposed in direction to OF; but FG must have the same direction as OF, in order that the resultant may be opposed to OF and FG.

For the same reason, draw GL parallel and equal to DO. Join O and L, and OL will be resultant of all the forces AO, BO, CO, and DO (both in magnitude and direction) acting at the point O. If $L'O$ is drawn parallel and equal to OL, and having the same direction, it will represent the effect produced on the body by the combined action of the forces AO, BO, CO, and DO. In this solution, we

have, for brevity, spoken of the forces AO, BO, etc., and the resultants OF, OG, and OL. It must be remembered, however, that these are merely lines that represent the forces in magnitude and direction.

17. Method of the Polygon of Forces.—In Fig. 6, it will be noticed that OE, EF, FG, GL, and LO are sides of a polygon $OEFGL$, in which OL, the resultant, is the closing side, and that its direction is opposed to that of all the other sides. This fact is made use of in what is called the **method of the polygon of forces.**

To find the resultant of several forces acting on a body at the same point:

Rule.—*Let the several forces be represented in direction and magnitude by lines, as explained in Art.* **10.** *Through any point draw a line parallel to one of the forces, and having the*

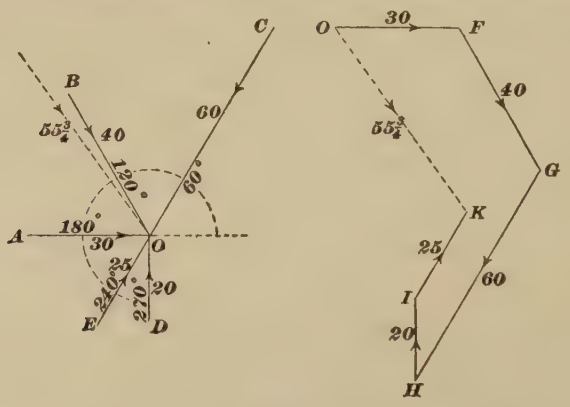


Fig. 7

same direction and magnitude. At the end of this line draw another line, parallel to a second force, and having the same direction and magnitude as this second force; at the end of the second line draw a line parallel and equal in magnitude and direction to a third force. Thus continue until lines have been drawn equal in magnitude, and having the same directions, respectively, as the lines representing the several forces.

The straight line joining the free ends of the first and last lines will be the closing side of the polygon; mark its direction opposite to that of the other forces around the polygon, and it will

represent in magnitude and direction the resultant of all the forces.

EXAMPLE.—If five forces act on a body at angles of 60°, 120°, 180°, 240°, and 270° toward the same point, and their respective magnitudes are 60, 40, 30, 25, and 20 pounds, find the magnitude and direction of their resultant by the method of polygon of forces.*

SOLUTION.—From a common point O, Fig. 7, draw the lines of action of the forces, making the given angles with a horizontal line through O, and mark them as acting toward O, by means of arrowheads, as shown. Now, choose some convenient scale, such that the whole figure may be drawn in a space of the required size on the drawing. Choose any one of the forces, as AO, and draw OF parallel to it, and equal in length to 30 lb. on the scale; it must also act in the same direction as AO. From F, draw FG parallel to BO, and make its length equal to 40 lb. on the assumed scale. In a similar manner, draw GH, HI, and IK parallel to CO, DO, and EO, and equal on the scale to 60, 20, and 25 lb., respectively. Join O and K by OK; then OK will represent in magnitude and direction the resultant of the combined action of the five forces. The direction of the resultant is opposed to that of the other forces around the polygon $OFGHIK$, and its magnitude is $55\frac{3}{4}$ lb. Ans.

18. If the resultant OK, Fig. 7, were to act alone on the body in the direction shown by the arrowhead with a force of $55\frac{3}{4}$ pounds, it would produce exactly the same effect as the combined action of the five forces. If OF, FG, GH, HI, and IK represent the distances and directions that the forces would move the body, if acting separately, OK is the direction and distance of movement of the body when all the forces act together.

From what has been said, it is evident that any number of forces acting on a body at the same point, or having their lines of action pass through the same point, can be replaced by a *single force* (resultant), whose line of action shall pass through that point.

Heretofore, it has been assumed that the forces acted on a single point on the surface of the body, but it will make no difference where they act, so long as the lines of action of all the forces intersect at a single point either within or

* All the angles in the figure are measured from a horizontal line in a direction opposite to the movement of the hands of a watch, from 1° up to 360°.

without the body, only so that the resultant can be drawn through the point of intersection. If two forces act on a body in the same straight line and in the same direction, their resultant is the *sum* of the two forces; but if they act in opposite directions, their resultant is the *difference* of the two forces, and its direction is the same as that of the greater force. If they are equal and opposite, the resultant is zero, for one force just balances the other.

EXAMPLE.—Find the resultant of the forces whose lines of action pass through a single point, as shown in Fig. 8.

SOLUTION.—Take any convenient point G, and draw a line GF parallel to one of the forces, say the one marked *40*, making it equal in

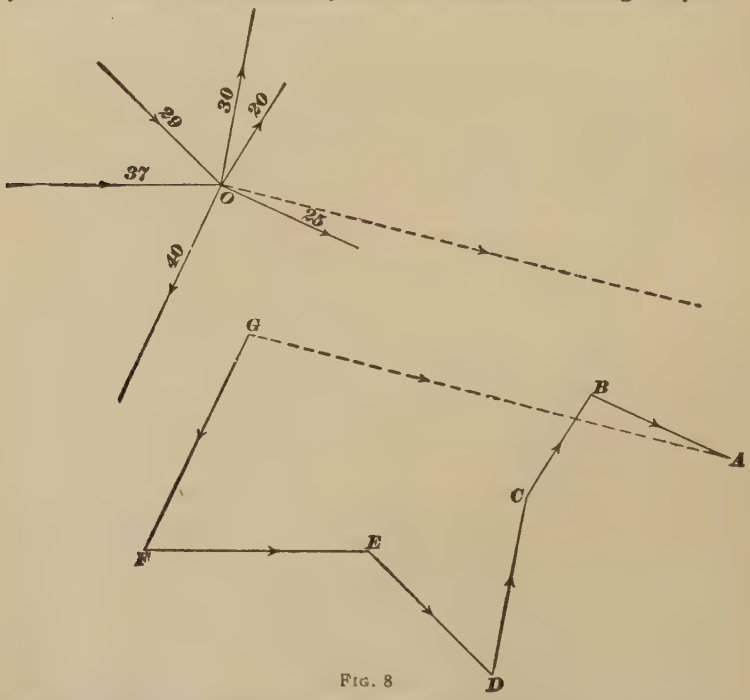


FIG. 8

length to 40 lb. on the scale, and indicate its direction by the arrow-head. Take some other force—the one marked *37* will be convenient; the line FE represents this force. From the point E draw a line parallel to some other force, say the one marked *29*, and make it equal in magnitude and direction to that force. So continue with the

other forces, taking care that the general direction around the polygon is not changed. The last force drawn in the figure is $B\,A$, representing the force marked *25*. Join the points G and A; then, $G\,A$ represents the resultant of all the forces shown in the figure. Its direction is from G to A, opposed to the general direction of the others around the polygon. It does not matter in what order the different forces are taken, the resultant will be the same in magnitude and direction, if the work is done correctly.

The various methods of finding the resultant of several forces are all grouped under the composition of forces.

THE RESOLUTION OF FORCES

19. Since two forces can be combined to form a single resultant force, we may also treat a single force as if it were the resultant of two forces whose joint action on a body will be the same as that of a single force. Thus, in Fig. 9, the force $O\,A$ may be resolved into two forces, $O\,B'$ and $B'\,A$.

It will be noted that one force may have an innumerable number of combinations of components. Instead of $O\,B'$ and $B'\,A$, Fig. 9, we may take $O\,B''$ and $B''\,A$ or $O\,B'''$ and $B'''\,A$ or, in fact, we can place the point B anywhere we wish and draw $O\,B$ and $B\,A$, which will represent in magnitude and direction two components that will together make the force $O\,A$. It is usual, however, for reasons that will appear later, to make $O\,B$ and $A\,B$ perpendicular to each other. That is to say, it is customary to make the angle B a right angle.

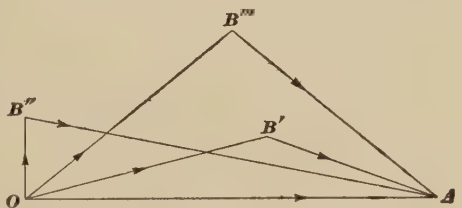


FIG. 9

20. It frequently happens that the position, magnitude, and direction of a certain force are known, and that it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 10, suppose that $O\,A$ represents, to some scale, the magnitude, direction,

and line of action of a force acting on a body at A, and that it is desired to know what effect OA produces in the direction BA. It is necessary, of course, that OB be at right angles to BA, so that all the effect of OA in the required direction may be represented by BA and none of it by OB. Thus, OB' and $B'A$, although components of OA, would not be a correct solution of the problem because, besides $B'A$, OA exerts some more effect in the line BA, namely, a part of OB' is in that direction with an amount BB'. To find the value of the component of OA, which acts in the direction BA, we employ the following rule:

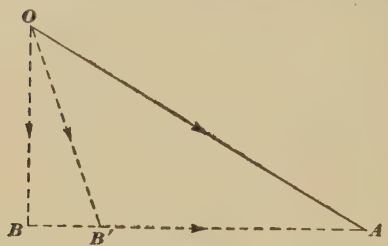


FIG. 10

Rule.—*From one extremity of the line representing the given force draw a line parallel to the direction in which it is desired that the component shall act; from the other extremity of the given force draw a line perpendicular to the component first drawn, and intersecting it. The length of the component, measured from the point of intersection to the intersection of the component with the given force, will give the magnitude of the effect produced by the given force in the required direction.*

Thus, suppose OA, Fig. 10, represents a force acting on a body resting on a horizontal plane, and it is desired to know what vertical pressure OA produces on the body. Here the desired direction is vertical; hence, from one extremity, as O, draw OB parallel to the desired direction (vertical in this case), and from the other extremity draw AB perpendicular to OB, and intersecting OB at B. Then OB, when measured to the same scale as OA, will give the magnitude of the vertical pressure produced by OA.

EXAMPLE.—If a body weighing 200 pounds rests on an inclined plane whose angle of inclination to the horizontal is 18°, what force does it exert perpendicular to the plane, and what force does it exert parallel to the plane, tending to slide it downwards?

SOLUTION.—Let $A\,B\,C$, Fig. 11, be the plane, the angle A being 18°, and let W be the weight. Draw a vertical line $F\,D$ = 200 lb., to represent the magnitude of the weight. Through F draw $F\,E$ parallel to $A\,B$, and through D draw $D\,E$ perpendicular to $E\,F$, the two lines intersecting at E. $F\,D$ is now resolved into two components, one $F\,E$ tending to pull the weight down the incline, and the other $E\,D$ acting as a perpendicular pressure on the plane. On measuring $F\,E$ with the same scale by which the weight $F\,D$ was laid off, it is found to be about 61.8 lb., and the perpendicular pressure $E\,D$ on the plane is found to measure 190.2 lb. Ans.

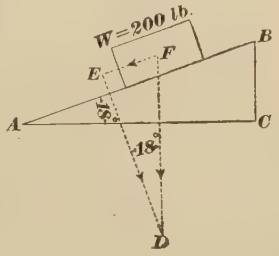


FIG. 11

21. The preceding articles give us another and useful method of finding the resultant of two or more forces acting at one point. In Fig. 12, let the

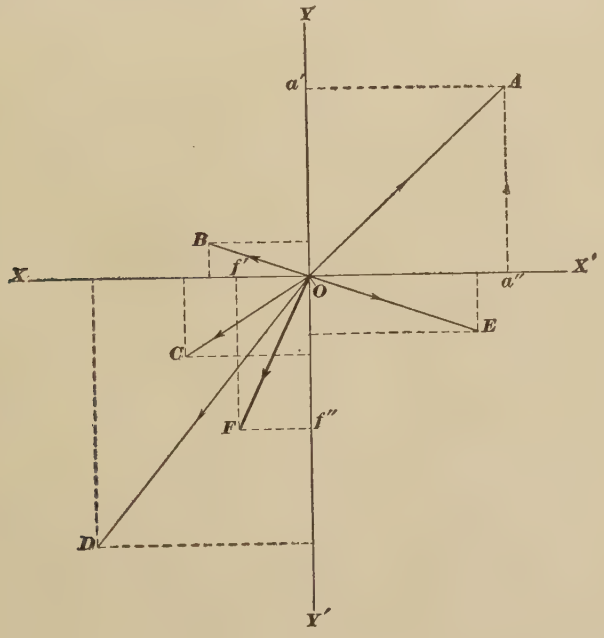


FIG. 12

forces $O\,A$, $O\,B$, $O\,C$, $O\,D$, and $O\,E$ all act on a body at the point O. Through O draw two axes at right angles to each

other, say one horizontal and one vertical, as XX' and YY'. Find the horizontal and vertical components of each of the five forces; thus, OA has a horizontal component of Oa' and a vertical component of Oa''. In like manner, each of the other forces may be divided into horizontal and vertical components along the XX' and YY' axes. Now, the horizontal components of OA and OE act toward the right, and the horizontal components of OB, OC, and OD act in the opposite direction. Hence, the difference of the sum of the horizontal component of OA and OE and the sum of the horizontal components of OB, OC, and OD will give the resultant horizontal component, which is Of'. Likewise, the difference of the sum of the vertical components of OA and OB and the sum of the vertical components of OC, OD, and OE gives Of'', the resultant vertical component. Now, by completing the parallelogram, of which Of' and Of'' are two sides, we get for the diagonal OF, which is the resultant of all the forces OA, OB, OC, OD, and OE.

22. All the previous discussion has assumed that the lines of action of forces intersect. This is not always the case. They may not be in the same plane or they may be

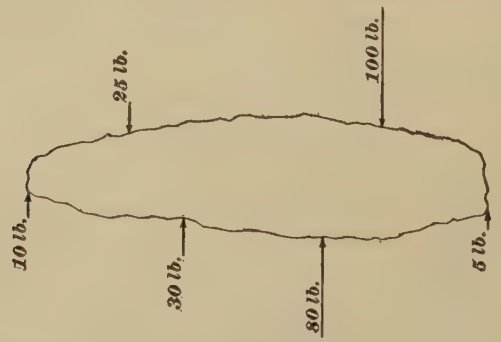


FIG. 13

parallel. The study of forces not in the same plane does not concern us here, but the study of forces parallel does. The condition of equilibrium of parallel forces, so far as motion of translation is concerned, is simply this, that there must be as great a quantity of forces acting one way as there

is the other. Great care must be taken to note that parallel forces may be in equilibrium, so far as moving the body as a whole is concerned, but the body itself on which they act is not in equilibrium. That is to say, the forces may not tend to move the body as a whole through space, but they may tend to rotate it. This tendency to rotate will be discussed later.

Let Fig. 13 represent the forces acting on any body, as, for instance, the one shown. Now, there are four forces acting upwards and their sum is $10 + 30 + 80 + 5 = 125$ pounds. There are two forces acting downwards and their sum is $25 + 100 = 125$ pounds. Therefore, there is the same amount of force acting one way that there is the other, and the body will move neither up nor down. Suppose, however, that the 25-pound force acting downwards is taken away. There will then be a predominance of force acting upwards and the body will move upwards.

23. It will be noted that if to any system of forces a force is added equal in magnitude, but acting in the opposite direction, to the resultant of the forces in the system, this new force will put the system in equilibrium; that is, the body on which they act will not tend to move. This fact is self-evident. By referring to Fig. 12, the forces OA, OB, OC, OD, and OE may be replaced by OF. Now, if we add to this system a force FO acting from F to O, we will just balance OF, which is the same thing as all the forces in the system.

MOMENTS OF FORCES

24. In Fig. 14, W is a weight which tends to fall, that is, acts downwards with a force of 10,000 pounds. If we take some fixed point as a, not in the line along which the weight W acts, and connect the point a with the line of action of W by a rigid arm, so that W pulls on one end of this arm, while the other end is firmly held at a, our daily experience teaches us that the pull of W tends to turn or rotate the arm around the point a.

Experience also teaches that the tendency to rotation is directly proportional to the magnitude of the force, provided that the arm remains at the same length, and directly proportional to the length of the arm, if the force remains constant. In general, therefore, the rotative effect is proportional to the product of the magnitude of the force multiplied by the length of the lever arm. This product is called the **moment** of the force with respect to the point in question. Thus, in Fig. 14, the moment of the force W with respect to the point a, is the product obtained by multiplying the magnitude, 10,000 pounds, by the perpendicular distance, 10 feet, from the point a to the line of action of W.

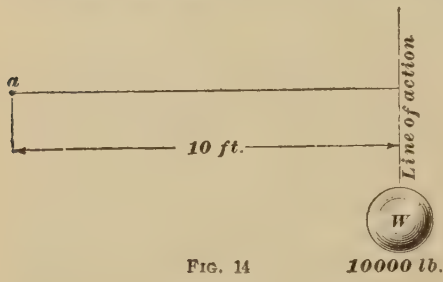


FIG. 14

25. Center of Moments.—The point a that is assumed as the center around which there is a tendency to rotate, is called the **center, or origin, of moments.**

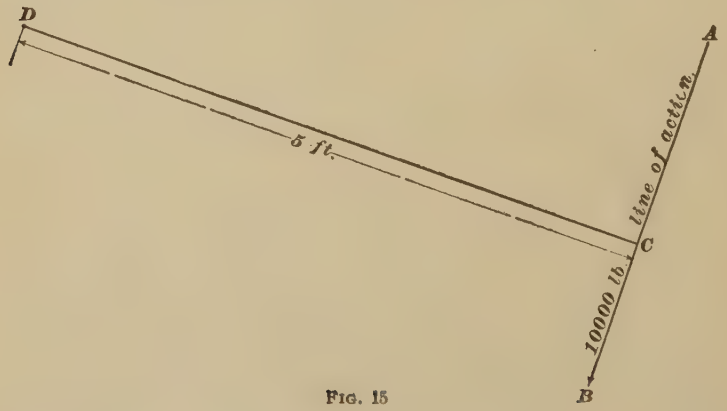


FIG. 15

26. Lever Arm.—The perpendicular distance from the center of moments to the line along which the force acts, is the **lever arm** of the force, also called the **leverage** of the force.

27. Foot-Pound.—Since the unit of force is the pound, and the ordinary unit of length is the foot, the unit of moment will be a derived unit, the **foot-pound**, and moments will usually be expressed in foot-pounds. In Fig. 14, for example, the moment of the force W with respect to the point a is $10{,}000 \times 10 = 100{,}000$ foot-pounds.

EXAMPLE.—What is the moment of the force of 10,000 pounds whose line of action is AB, Fig. 15, the center of the moments being at D?

SOLUTION.—The perpendicular distance CD from the line of action of the force to the center of moments being 5 ft., and the magnitude of the force 10,000 lb., the moment is

$$10{,}000 \times 5 = 50{,}000 \text{ ft.-lb.} \quad \text{Ans.}$$

28. In Fig. 16, the line of action of the force of 10,000 pounds passes directly through the point D; consequently,

FIG. 16

the perpendicular distance from the line of action to the point D is zero, and there is no tendency to rotate around that point; that is, the moment of the force about the point D is zero.

29. The moment of a force may be expressed in *inch-pounds*, *foot-pounds*, or *foot-tons*, depending on the unit of measurement used to designate the magnitude of the force and the length of its lever arm. For instance, if the magnitude of a force is measured in pounds, and the lever arm through which it acts in inches, the moment will be in inch-pounds; again, if a force of 10 tons acts through a lever arm of 20 feet, the moment of the force is $10 \times 20 = 200$ foot-tons.

EXAMPLE.—What is the moment, in inch-pounds, of a force of 8,000 pounds, if the length of the lever arm is 13 feet?

SOLUTION.—Since the moment is to be in inch-pounds, the length of the lever arm must be in inches.

$$13 \text{ ft.} = 13 \times 12 = 156 \text{ in.}$$

and the moment is

$$8{,}000 \times 156 = 1{,}248{,}000 \text{ in.-lb.} \quad \text{Ans.}$$

30. Positive and Negative Moments.—We may distinguish between the directions in which there is a tendency

to produce rotation by the use of the signs + and −. Thus, if a force tends to produce right-hand rotation, its moment is called positive and it has the plus sign, while a force that tends to produce rotation in the opposite direction is called negative, and its moment has the minus sign.

31. Resultant Moments.—In Fig. 17 is shown a lever composed of two arms at right angles to each other, and free to turn about the center C. A force A acts on the horizontal arm in such a manner that it tends to produce left-

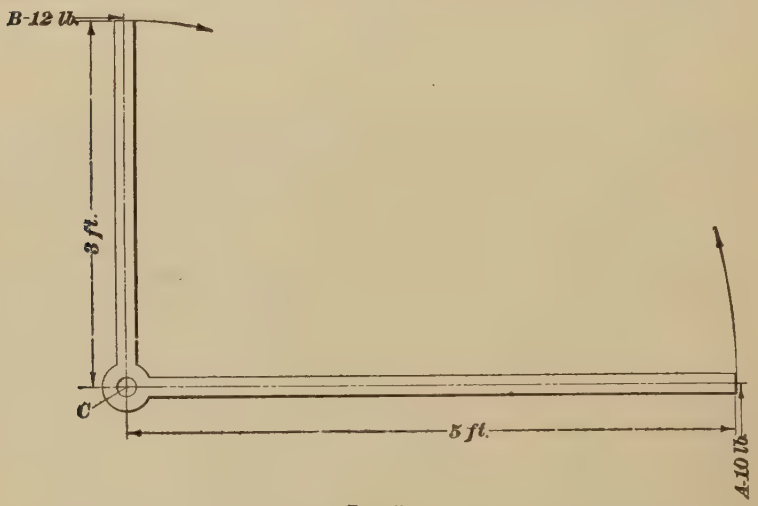


Fig. 17

hand rotation, its moment being $10 \times 5 = 50$ foot-pounds, which, since it tends to produce left-hand rotation, shall be called negative. Another force B, whose moment with respect to the center C is $12 \times 3 = 36$ foot-pounds, tends to produce right-hand rotation. $-50 + 36 = -14$ foot-pounds, which is the **resultant moment**, and has the same turning effect as the two moments already given.

If the length of the lever arm of the force that acts to produce the resultant moment is known, the magnitude of the force may readily be found. Thus, in the present case, the resultant moment is − 14 foot-pounds; let it be required to find the force to produce the same turning effect as the

two forces, when acting with a lever arm 7 feet long. Since the moment is the product of the force multiplied by its lever arm, it follows that the required force may be found by dividing the given moment by the length of the lever arm; consequently, the required force is $14 \div 7 = 2$ pounds.

If, instead of the two forces just considered, we have a body acted on by any number of forces whose moments about a given center are known, the resultant moment of these forces is the algebraic sum of the moments of the given forces; and, further, if the length of the lever arm of the resultant moment is known, the magnitude of the required force can be found by dividing the moment by the length of the lever arm.

32. Equilibrium of Moments.—Let Fig. 18 represent a rod 15 feet long with a weight on each end, one weighing 10 pounds and one 5 pounds. It will be found that if the rod

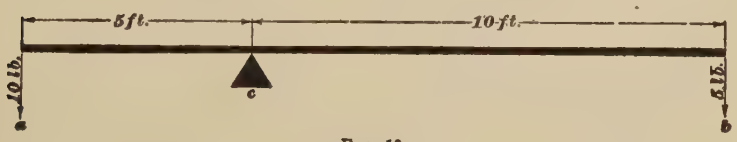


FIG. 18

is supported 5 feet from the 10-pound end, the two weights will just balance each other. Any one who has played "seesaw," or "teeter," in childhood, knows that the lighter playmate was always given the longer end of the board. This is exactly the same principle with the two weights in Fig. 18. When they balance at 5 feet from the 10-pound end, the positive moment about the point of support is 10 feet $\times$ 5 pounds = 50 foot-pounds. The negative moment is 5 feet $\times$ 10 pounds = 50 foot-pounds. It is seen that the moments balance each other. The general rule can be stated that when the values of the moments of two forces are equal at the point around which they tend to turn and when they are in opposite directions, they neutralize each other. Then the moments are said to be in equilibrium.

This subject of the equilibrium of moments will be treated more fully later on.

33. The foregoing principles may be expressed as follows:

Rule.—*To find the force required to produce equilibrium of moments about a fixed point, when the moments of any number of given forces and the lever arm of the required force are given about that point, divide the algebraic sum of the given moments by the length of the given lever arm. If the algebraic sum is positive, the tendency of the required force is to produce left-hand rotation; if negative, the tendency of the force is to produce right-hand rotation.*

EXAMPLE.—In Fig. 19, we have a system of forces, shown by the arrows, acting in various directions and at various distances from the center O. The force F' is 25 pounds and its lever arm Op' is 8 feet, F'' is 16 pounds with a lever arm Op'' of 12 feet, F''' is 40 pounds with a lever arm Op''' of 6 feet, and the force F^{v} is 100 pounds, acting directly through the center O. If the distance Op^{iv} is 12 feet, what must be the magnitude of the force F^{iv} in order to produce equilibrium of moments?

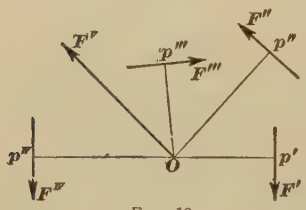


FIG. 19

SOLUTION.—As shown by the arrows, the forces tending to produce right-hand rotation are F' and F''', and their moments, called positive, are, respectively,

$$25 \times 8 = +200 \text{ ft.-lb., and } 40 \times 6 = +240 \text{ ft.-lb.}$$

The lever arm of the force F^{v} is zero; consequently, it has no moment with respect to the center O. The force F'' tends to produce left-hand rotation, and its moment is

$$16 \times 12 = -192 \text{ ft.-lb.}$$

The algebraic sum of the moments of the given forces is

$$+200 + 240 - 192 = +248 \text{ ft.-lb.;}$$

therefore, according to the rule, the force F^{iv} must be $248 \div 12 = 20\frac{2}{3}$ lb., which, since the algebraic sum of the given moments is positive, must tend to produce left-hand rotation, as shown by the arrow. Ans.

34. The principles involved in the theory of moments are among the most simple in mechanics, and at the same time of the greatest practical importance in the solution of problems relating to the strength of beams, girders, and trusses.

EXAMPLE.—In Fig. 20, the lower tie-member in the roof truss has been raised to get a vaulted-ceiling effect in the upper story of the building, which the truss covers. The weight transmitted through

this member to the pier wall is 30,000 pounds; there is, consequently, an equal upward force due to the reaction of the wall. This force of 30,000 pounds tends to break the truss by producing rotation about the point *b*. What is its moment around the point *b*?

SOLUTION.—Since the perpendicular distance from the line of action of the force is 3 ft., the moment of the force *a* around the point *b* is

$30{,}000 \times 3 = 90{,}000$ ft.-lb.

Ans.

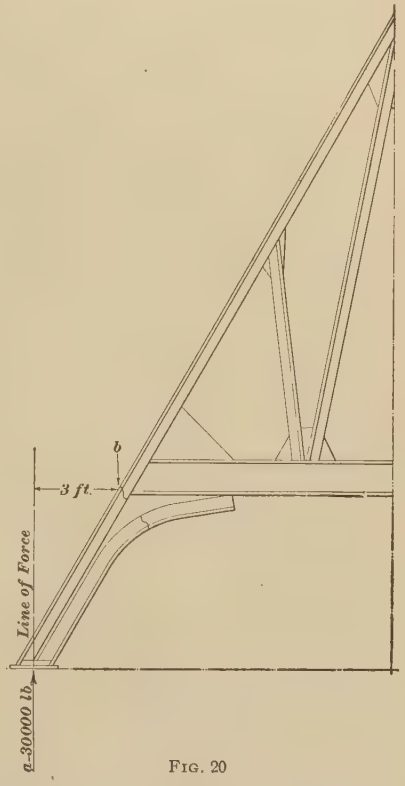


FIG. 20

THE LEVER

35. A **lever** is a bar capable of being turned about a pin, pivot, or point, as in Figs. 21, 22, and 23.

The object W to be lifted is called the **weight;** the force P used is called the **power;** and the point, or pivot, F is called the **fulcrum.**

That part of the lever between the weight and the fulcrum, or Fb, is called the **weight arm,** and the part between the power and the fulcrum, or Fc, is called the **power arm.**

Take the fulcrum, or point F, as the center of moments;

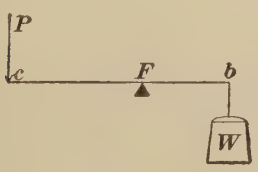


FIG. 21

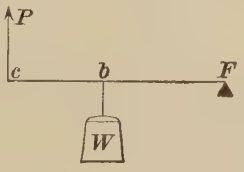


FIG. 22

then, in order that the lever shall be in equilibrium, the moment of P about F, or $P \times Fc$, must be equal to the

moment of W about F, or $W \times Fb$. That is, $P \times Fc = W \times Fb$, or, in other words, *the product of the power and the power arm is equal to the product of the weight and the weight arm.*

If F is taken as a center, and arcs are described through b and c, it will be seen that, if the weight arm is moved through a certain angle, the power arm will move through the same angle. From this, it is seen that the power arm is proportional to the distance through which the power moves, and the weight arm is proportional to the distance through which the weight moves.

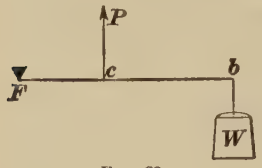


FIG. 23

Hence, instead of writing $P \times Fc = W \times Fb$, we might have written it $P \times$ distance through which P moves $= W \times$ distance through which W moves. This is the general law of all machines, and can be applied to any mechanism, from the simple lever up to the most complicated arrangement. Stated in the form of a rule, it is as follows:

Rule.—*The power multiplied by the distance through which it moves is equal to the weight multiplied by the distance through which it moves.*

EXAMPLE 1.—If the weight arm of a lever is 6 inches long and the power arm is 4 feet long, how great a weight can be raised by a force of 20 pounds at the end of the power arm?

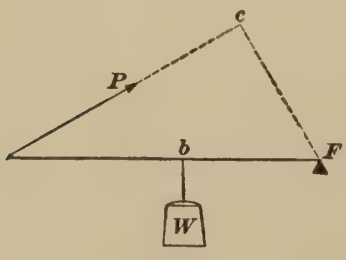


FIG. 24

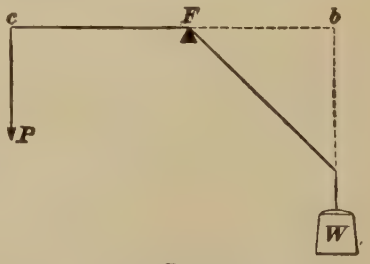


FIG. 25

SOLUTION.— 4 ft. = 48 in. Hence, $20 \times 48 = W \times 6$, or $W = 160$ lb. Ans.

EXAMPLE 2.—(*a*) What is the ratio between the power and the weight in the last example? (*b*) If P moves 24 inches, how far does W move? (*c*) What is the ratio between the two distances?

SOLUTION.—(*a*) 20 : 160 = 1 : 8; that is, the weight moved is 8 times the power. Ans.

(*b*) $20 \times 24 = 160 \times x$. $x = 480 \div 160 = 3$ in., the distance that W moves. Ans.

(*c*) 3 : 24 = 1 : 8, or the ratio is 1 : 8. Ans.

The law that governs the straight lever also governs the bent lever; but care must be taken to determine the true lengths of the lever arms, which are, in every case, *the perpendicular distances from the fulcrum to the line of direction of the weight or power.*

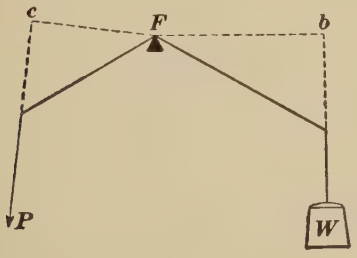


FIG. 26

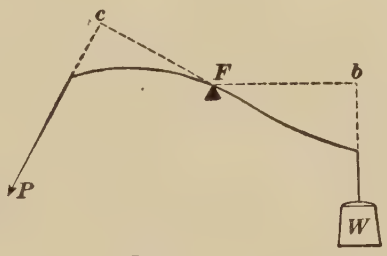


FIG. 27

Thus, in Figs. 24, 25, 26, and 27, Fc in each case represents the power arm, and Fb, the weight arm.

EXAMPLES FOR PRACTICE

1. A lever arm has a length of 10 feet; the load acting on the end of the lever is 6,000 pounds; what is the moment of this load, in inch-pounds? Ans. 720,000

2. A piece of timber 20 feet long is balanced at a point 8 feet from one end, the load at this end being 9,000 pounds; what is the load at the other end? Ans. 6,000 lb.

3. The one support of a beam 20 feet long is 8 feet from the left-hand end; at this end is a load of 25 pounds; at the right of the support, 3 feet distant, is a load of 5 pounds; and at 7 feet to the right is a load of 10 pounds; what load, and at which end should it be placed to produce balance, or equilibrium, in the beam?

Ans. 9.58 lb. at right-hand end

4. A steel I beam, which extends 6 feet outside of the center of a building wall, and 3 feet inside, is required to support a load of 4,000 pounds on the outside end; what load on the inner end will keep the beam from tilting? Ans. 8,000 lb.

CENTER OF GRAVITY

36. The **center of gravity** of a body, or of a system of bodies, is that point from which, if the body or system were suspended, it would be in equilibrium. If the body or system were suspended from any other point than the center of gravity, and in such a manner as to be free to turn about the point of suspension, the body would rotate until the center of gravity reached a position directly under the point of suspension.

37. Center of Gravity of Plane Figures.—If a plane figure has an *axis of symmetry*, this axis passes through its center of gravity. If the figure has *two* axes of symmetry, its center of gravity is at their point of intersection.

The center of gravity of a *triangle* lies on a line drawn from a vertex to the middle point of the opposite side, and at a distance from that side equal to one-third of the length of the line. Or, draw a line from another vertex to the middle point of the side opposite, and the intersection of the two lines will be the center of gravity. The perpendicular distance of the center of gravity of a triangle from the base is equal to one-third of the altitude.

The center of gravity of a *parallelogram* is at the intersection of its two diagonals.

The center of gravity of an *irregular four-sided figure* may be found by first dividing it by a diagonal into two triangles and joining their centers of gravity by a straight line; then, by means of the other diagonal, divide it into two other triangles, and join their centers of gravity by another straight line; the center of gravity of the figure is at the intersection of the lines joining the centers of gravity of the two sets of triangles.

The distance of the center of gravity of the *surface of a half circle* from the center is equal to the product of the radius multiplied by .424.

LOADS CARRIED BY STRUCTURES

DEAD LOAD

WEIGHT OF BUILDING MATERIALS

38. Dead Load.—The materials used in building construction have considerable weight. Every piece of iron, timber, masonry, and brickwork has a tendency to move toward the center of the earth, and this tendency of the parts of the structure to fall to the ground must be resisted by the strength of the various members composing it. The portion of this force to be resisted by any member of the structure is called the **dead load**; it is the sum of the weights of every piece of material in the building that must be supported by that member.

Before the dead load can be computed, the weight of various building materials must be known. The tables that follow give the weights of building materials in common use. The units in which these weights are expressed are those most often used to make estimates of loads in engineering calculations. Thus, Table I gives the weight per cubic foot of the materials usually measured by that unit, together with the weight per cubic inch of a few often measured in inches; while Table II gives the weights of such materials as are used in the construction of floors, roofs, ceilings, etc., where the quantities are generally expressed in square feet. While it is not necessary to memorize all of Table I, it is well to keep in mind the weights of the materials printed in Italics.

TABLE I

WEIGHT OF BUILDING MATERIALS

Name of Material	Average Weight in Pounds	
	Per Cu. In.	Per Cu. Ft.
Aluminum	.096	166
Asphalt pavement composition		130
Bluestone		160
Brass	.302	523
Brickwork, in lime mortar		120
Brickwork, in cement mortar		130
Bronze	.319	552
Cement, Portland		80 to 100
Cement, Rosendale		56 to 60
Concrete		140
Copper, cast	.319	550
Earth, dry and loose		72 to 80
Earth, dry and moderately rammed		90 to 100
Gneiss, common		168
Gneiss, in loose piles		96
Granite		165 to 170
Gravel		117 to 125
Iron, cast	.26	450
Iron, wrought	.277	480
Lead, commercial cast	.412	712
Limestone		170
Marble		164
Masonry, granite or limestone		165
Masonry, granite or limestone rubble		150
Masonry, granite or limestone dry rubble		138
Masonry, sandstone		145
Mortar, hardened		90 to 100
Quartz, common pure		165
Sand, pure quartz, dry		90 to 106
Sandstone, building, dry		144 to 151
Slate		160 to 180
Snow, fresh fallen		5 to 12
Steel, structural	.283	490
Terra cotta		110
Terra-cotta masonry work		112
Tile		110 to 120

TABLE II

WEIGHT OF BUILDING MATERIALS

Name of Material	Average Weight per Square Foot Pounds
Corrugated galvanized iron No. 20, unboarded	$2\frac{1}{4}$
Copper, 16 ounce, standing seam	$1\frac{1}{4}$
Felt and asphalt, without sheathing	2
Glass, $\frac{1}{8}$ inch thick	$1\frac{3}{4}$
Hemlock sheathing, 1 inch thick	$2\frac{1}{2}$
Lead, about $\frac{1}{8}$ inch thick	6 to 8
Lath and plaster ceiling (ordinary)	6 to 8
Mackite, 1 inch thick, with plaster	10
Neponset roofing felt, two layers	$\frac{1}{2}$
Spruce sheathing, 1 inch thick	2
Slate, $\frac{3}{16}$ inch thick, 3 inches double lap	$6\frac{3}{4}$
Slate, $\frac{1}{8}$ inch thick, 3 inches double lap	$4\frac{1}{2}$
Shingles, 6 in. × 18 in., $\frac{1}{3}$ to weather	2
Skylight of glass, $\frac{3}{16}$ inch to $\frac{1}{4}$ inch, including frame	4 to 10
Slag roof, four-ply	4
Tin, IX	$\frac{3}{4}$
Tiles, $10\frac{1}{2}$ in. × $6\frac{1}{4}$ in. × $\frac{5}{8}$ in.; $5\frac{1}{4}$ inches to weather (plain)	18
Tiles, $14\frac{1}{2}$ in. × $10\frac{1}{2}$ in.; $7\frac{1}{4}$ inches to weather (Spanish)	$8\frac{1}{2}$
White-pine sheathing, 1 inch thick	$2\frac{1}{2}$
Yellow-pine sheathing, 1 inch thick	4

EXAMPLE.—In Fig. 28, what is the total dead load on the girder B?

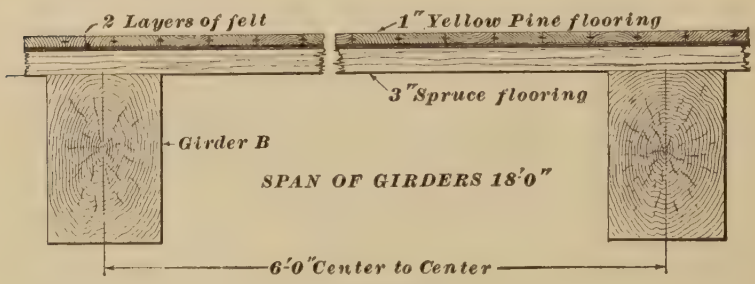


FIG. 28

SOLUTION.—

Yellow-pine flooring, 1 inch thick	4 lb. per sq. ft.
Two layers of felt	$\frac{1}{2}$ lb. per sq. ft.
Rough spruce flooring, 3 inches thick . .	6 lb. per sq. ft.
Assume the weight of the girder	8 lb. per sq. ft.
Total dead load of floor surface	$18\frac{1}{2}$ lb. per sq. ft.

The area of the floor carried by the girder is $6 \times 18 = 108$ sq. ft. Then, $108 \times 18\frac{1}{2} = 1{,}998$ lb. is the entire dead load on the girder B. Ans.

EXAMPLES FOR PRACTICE

1. A $2'' \times 3''$ wrought-iron bar is $36\frac{1}{2}$ inches long; what is its weight? Ans. 60.66 lb.

2. The outside diameter of a cast-iron column is 10 inches, and the thickness of the material composing the column is $\frac{3}{4}$ inch; what is its weight per foot of length? Ans. 68 lb.

3. The wall of a brick building was laid in cement mortar and is 24 inches thick, 36 feet high, and 100 feet long; in it are located 20 window openings, 2 feet 6 inches wide by 6 feet high. What is the weight of this wall? Ans. 858,000 lb.

4. What is the weight of a structural steel angle 6 in. $\times$ 6 in. $\times \frac{1}{2}$ in. $\times$ 20 ft. long? Ans. 390.54 lb.

5. The roof of a building is made of No. 20 corrugated galvanized iron, laid on 1-inch spruce boarding; what is the weight of the roof covering per square foot? Ans. $4\frac{1}{4}$ lb.

6. What will be the difference in weight between a four-ply slag roof, laid on 3-inch tongued-and-grooved yellow-pine planking, and a $\frac{3}{16}$-inch slate roof laid on 2-inch hemlock sheathing, covered with Neponset roofing felt, two layers thick? Ans. $3\frac{3}{4}$ lb.

LIVE LOAD

39. Besides the dead load, which includes the weight of all the material used in the structure itself, there is a load due to the weight of the people and merchandise in the building; this load is called the **live load.** The live load comprises people in the building, furniture, movable stocks of goods, small safes, and varying weights of any character. Large safes and extremely heavy machinery require some special provision, usually embodied in the construction.

TABLE III

LIVE LOADS FOR BUILDING CONSTRUCTION

Type of Structure	Live Loads per Square Foot Pounds
Dwellings	70
Offices	70
Hotels and apartment houses	70
Theaters	120
Churches	120
Ballrooms and drill halls	120
Factories	from 150 up
Warehouses	from 150 to 250 up

Table III gives the live loads per square foot recommended as good practice in conservative building construction.

The load of 70 pounds will probably never be realized in dwellings; but inasmuch as a city house may, at some time, be applied to some purpose other than that of a dwelling, it is not generally advisable to use a lighter load. In the case of a country house, a hotel, or a building of light character, where economy demands it, and its actual use for a long time for some fixed purpose is almost certain, a live load of 40 pounds per square foot of floor surface is ample for all rooms not used for public assembly.

For rooms thus used, a live load of 100 pounds will be sufficient, experience having demonstrated that a floor cannot be crowded to more. If the desks and chairs are fixed, as in a schoolroom or a church, a live load of more than 40 to 50 pounds will never be attained. Retail stores should have floors proportioned for a live load of 100 pounds and upwards. Wholesale stores, machine shops, etc. should have the floors proportioned for a live load of not less than 150 pounds per square foot.

The static load in factories seldom exceeds 40 to 50 pounds per square foot of floor surface, and, therefore, in the majority of cases, a live load of 150 pounds, including the effects of vibrations due to moving machinery, is ample. The conservative rule is, in general, to assume loads not less than the foregoing, and to be sure that the beams are proportioned so as to avoid excessive deflection. Stiffness is a factor as important as strength.

EXAMPLE.—What will be the entire live load coming on a large girder supporting a portion of a church floor, if the floor area to be supported is 600 square feet?

SOLUTION.—From the list given in Table III, 120 lb. is usually considered safe for a live load in a church. Therefore, $600 \times 120 = 72{,}000$ lb. is the total live load on the girder. **Ans.**

EXAMPLES FOR PRACTICE

1. What will be the entire live load on the floor of a church 50 ft. $\times$ 120 ft.? Ans. 720,000 lb.

2. What live load will a joist in a city dwelling be required to bear, the distance between centers being 14 inches, and the span of the joist, 20 feet? Ans. 1,633 lb.

3. A steel beam, supporting a portion of the floor in an office building, sustains an area of 80 square feet; what will be the live load coming on the beam? **Ans. 5,600 lb.**

STRESSES AND STRAINS

DEFINITIONS

40. Stress.—It has been shown that the weight of the materials composing a building and its contents produces forces that must be resisted by the different members of the structure; the action of these forces has a tendency to change the relative position of the particles composing the members, and this tendency is, in turn, resisted by the cohesive force in the materials, which acts to hold the particles together.

The internal resistance with which the force of cohesion opposes the tendency of an external force to change the relative position of the particles of any body subjected to a load is called a **stress.** Or, stress may be defined as the load that produces a fractional alteration in the form of a body, and this fractional alteration of form is called the *strain.*

41. Kinds of Stress.—In accordance with the direction in which the forces act with reference to a body, the stress produced may be either *tensile*, *compressive*, or *shearing.*

42. Tensile stress is the effect produced when the external forces act in such a direction that they tend to stretch a body; that is, to pull the particles away from each other. A rope by which a weight is suspended is an example of a body subjected to a tensile stress.

43. Compressive stress is the effect produced when the tendency of the forces is to compress the body or to push the particles closer together. A post or the column of a building is an example of a body subjected to a compressive stress.

44. Shearing stress is the effect produced when the forces act as in a shear, so as to produce a tendency for the particles in one section of a body to slide over the particles of the adjacent section. When a steel plate is acted on by a

punch or the knives of a shear, or where a load acts on a beam, as shown in Fig. 29, the plate or beam is subjected to a shearing stress, as at the sections *a b* and *c d*, where the beam is shown to have already sheared or failed.

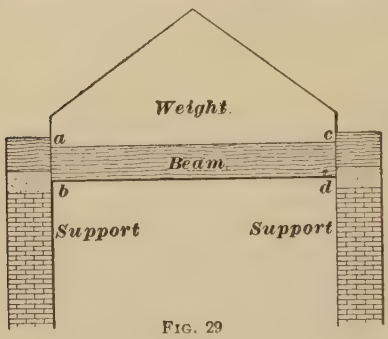


FIG. 29

45. When a beam is loaded in such a manner that there is in it a tendency to bend, as shown in Fig. 30, it is subjected to a **transverse**, or **bending, stress.** In this case, there is a combination of the three stresses

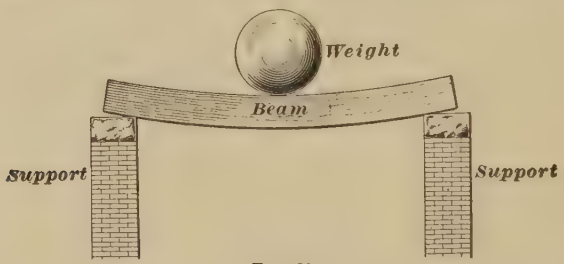


FIG. 30

already mentioned (tension, compression, and shear) in different parts of the beam.

46. The **unit stress** (called, also, the **intensity** of stress) is the name given to the stress per unit of area; or, it is the total stress in a tie-rod, column or the like divided by the area of the cross-section. Thus, if a weight of 1,000 pounds is supported by an iron rod whose area is 4 square inches, the unit stress is $1{,}000 \div 4 = 250$ pounds per square inch. If, with the same load, the area is $\frac{1}{2}$ square inch, the unit stress is $1{,}000 \div .5 = 2{,}000$ pounds per square inch.

Let P = total stress, in pounds;
A = area of cross-section, in square inches;
S = unit stress, in pounds per square inch.

Then, $$S = \frac{P}{A}, \text{ or } P = AS$$

That is, *the total stress is equal to the area of the section multiplied by the unit stress.*

EXAMPLE.—An iron rod 2 inches in diameter sustains a load of 90,000 pounds; what is the unit stress?

SOLUTION.—Using the formula,

$$S = \frac{P}{A} = \frac{90{,}000}{2^2 \times .7854} = 28{,}647.8 \text{ lb. per sq. in. Ans.}$$

47. When a body is stretched, shortened, or in any way deformed through the action of a force, the deformation is called a **strain.** Thus, if the rod just mentioned had been elongated $\frac{1}{10}$ inch by the load of 1,000 pounds, the strain would have been $\frac{1}{10}$ inch. Within certain limits, to be given hereafter, strains are proportional to the stresses producing them.

48. The **unit strain** is the strain per unit of length or of area, but is usually taken per unit of length and called for tension the *elongation* per unit of length. If we consider the unit of length as 1 inch, the unit strain is equal to the total strain divided by the length of the body in inches.

Let l = length of body, in inches;
e = elongation, in inches;
s = unit strain.

Then, $$s = \frac{e}{l}, \text{ or } e = ls$$

EXAMPLES FOR PRACTICE

1. A wrought-iron tension member in a roof truss has a load on it of 27,000 pounds; if it has been designed to sustain a unit stress of 6,000 pounds, what will be its diameter? Ans. $2\frac{1}{2}$ in., nearly

2. The sectional dimensions of a wooden compression member are 8 in. × 10 in.; if the load on it is 64,000 pounds, what is the unit stress on the material? Ans. 800 lb.

3. What is the total load on a hollow cast-iron column 10 inches outside diameter; the thickness of the metal is 1 inch, and the unit stress on the column is 20,000 pounds? Ans. 565,600 lb.

4. Steel angles form the tension member in a roof truss; they are subjected to excessive stress, and are stretched $\frac{1}{4}$ inch. What is the unit strain in them if they are 10 feet long? Ans. .00208 in.

49. Hooke's Law.—When a certain unit stress is created in a substance, a certain unit strain is developed. If the unit stress is doubled, it will be found that the unit strain has also been doubled. That is to say, that the alteration of shape, or the strain in a body is proportional to the force applied to that body.

50. Elastic Limit.—When a certain stress is created in a body a certain strain is produced. When the stress is removed, the body returns to its original shape, provided that the unit stress has not been too great in the first place. There is a certain unit stress, however, for each substance, that is the maximum stress that the substance will stand and then return to its original shape after the external forces are removed. This unit stress is called the **elastic limit** of the material. If a body is strained beyond the elastic limit, it will maintain a permanent distortion, or *set*, even after the straining forces are removed. Hooke's law, which is quite exact for most materials below the elastic limit, does not hold good above it, as the strain increases much more rapidly than the stress. Thus, if we double the unit stress beyond the elastic limit, we much more than double the unit strain.

51. Modulus of Elasticity.—To restate Hooke's law, the ratio of the unit stress to the unit strain for any substance is constant below the elastic limit. This ratio of unit stress to unit strain is called the **modulus of elasticity,** which will be represented by the symbol E. It is

$$E = \frac{S}{s}$$

From this equation, when S and E are known, s may be found; that is to say, if the modulus of elasticity of a certain substance and the unit stress are known, the unit strain can readily be found.

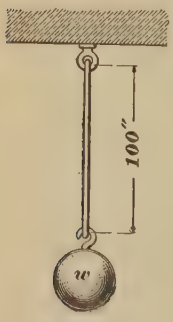


FIG. 31

EXAMPLE 1.—The unit stress at the elastic limit of medium structural steel is 33,000 pounds. The modulus of elasticity is 29,000,000. How much would a bar 100 inches long stretch when loaded to its elastic limit, as shown in Fig. 31?

SOLUTION.— $S = 33{,}000$, $E = 29{,}000{,}000$; therefore,

$$s = \frac{S}{E} = \frac{33{,}000}{29{,}000{,}000}$$

which is the unit strain or the elongation for 1 in. of length. Now, the bar in question is 100 in. long. Therefore, the total elongation is

$$100 \times \frac{33{,}000}{29{,}000{,}000} = 0.113 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 2.—A steel rod of 2 square inches cross-section is loaded with 58,000 pounds; how long will the rod have to be to stretch $\frac{11}{100}$ inch?

SOLUTION.—Here $E = 29{,}000{,}000$; $S = 58{,}000 \div 2 = 29{,}000$. Therefore,

$$s = \frac{29{,}000}{29{,}000{,}000} = \frac{1}{1000}$$

which is the stretch for 1 in. of the rod. Now, the length of the rod to stretch $\frac{11}{100}$ in. is as many inches long as $\frac{1}{1000}$ will go into $\frac{11}{100}$, or

$$\frac{11}{100} \div \frac{1}{1000} = 110 \text{ in.} \quad \text{Ans.}$$

STRENGTH OF BUILDING MATERIALS

52. The **ultimate strength** of any material is that unit stress which is just sufficient to break it.

53. The **ultimate elongation** is the total elongation produced in a unit of length of the material by a unit stress equal to the ultimate strength of the material.

54. The **factor of safety,** or, as some call it, the **safety factor,** is the ratio of the breaking strength of the structure to the load that, under usual conditions, it is called on to carry. Suppose the load required to break, dismember, or crush a structure is 5,000 pounds, and the load it is called on to carry is 1,000 pounds, then the factor of safety may be obtained by dividing the 5,000 pounds by the 1,000 pounds, or $5{,}000 \div 1{,}000 = 5$, the factor of safety in this structure.

The safety factor depends on the conditions, circumstances, and materials used; in other words, it is the *factor of ignorance.* When a piece of steel, wood, or cast iron is used in a building, the engineer does not know the exact strength of that particular piece of steel, wood, or cast iron. From his own experience, and that of others, he knows the approximate tensile strength of structural steel to be 60,000 pounds per square inch, and that it varies more or less from this

value. In regard to timber, the uncertainty is much greater, because of knots, shakes, and interior rot, not always evident on the surface. Cast iron is even more unreliable, on account of almost indeterminable blowholes, flaws, and imperfections in the castings.

55. Deterioration.—Another factor to be considered is deterioration in the material, due to various causes. In metals, there is corrosion on account of moisture and gases in the atmosphere, especially noticeable in the steel trusses over railroad sheds, where the sulphur fumes from the stacks of the locomotives unite with the moisture in the air, forming free sulphuric acid, which attacks the steel vigorously, and demands constant painting, to prevent its entire destruction. Wood is subject to decay from either dry or wet rot, caused by local conditions; it may, like iron and steel, be subjected to *fatigue*, produced by constant stress due to the load it may have to sustain. Cast iron does not deteriorate to any great extent, its corrosion not being as rapid as that of steel or wrought iron. There are, however, internal strains produced in cast iron by the irregular cooling of the metal in the mold. Castings under the slightest blow will sometimes, owing to these internal stresses, snap and break in a number of places.

These reasons are, in truth, sufficiently cogent to require the factor of safety now adopted in all engineering work. Table IV gives factors often employed by conservative constructors.

TABLE IV

FACTORS OF SAFETY

Materials	Factor of Safety
Structural steel and wrought iron	3 to 4
Cast iron	6 to 10
Stone	10 at least

56. Table V gives values of the strength of building timber commonly used, when subjected to different stresses.

TABLE V

AVERAGE ULTIMATE BREAKING UNIT STRESSES IN POUNDS PER SQUARE INCH

Kind of Timber	Tension		Compression			Transverse		Shearing	
			With Grain						
	With Grain	Across Grain	End Bearing	Columns Under 15 Diams.	Across Grain	Modulus of Rupture	Modulus of Elasticity	With Grain	Across Grain
White oak	10,000	2,000	7,000	4,500	2,000	6,000	1,100,000	800	4,000
White pine	7,000	500	5,500	3,500	800	4,000	1,000,000	400	2,000
Southern long-leaf or Georgia yellow pine	12,000	600	8,000	5,000	1,400	7,000	1,700,000	600	5,000
Douglas, Oregon, and yellow fir	12,000		8,000	6,000	1,200	6,500	1,400,000	600	
Washington fir or pine (red fir)	10,000					5,000			
Northern or short-leaf yellow pine	9,000	500	6,000	4,000	1,000	6,000	1,200,000	400	4,000
Red pine	9,000	500	6,000	4,000	800	5,000	1,200,000		
Norway pine	8,000		6,000	4,000	800	4,000	1,200,000		
Canadian (Ottawa) white pine	10,000			5,000				350	
Canadian (Ontario) red pine	10,000			5,000		5,000	1,400,000	400	
Spruce and eastern fir	8,000	500	6,000	4,000	700	4,000	1,200,000	400	3,000
Hemlock	6,000			4,000	600	3,500	900,000	350	2,500
Cypress	6,000		6,000	4,000	700	5,000	900,000		
Cedar	8,000		6,000	4,000	700	5,000	700,000		1,500
Chestnut	9,000			5,000	900	5,000	1,000,000	600	1,500
California redwood	7,000			4,000	800	4,500	700,000	400	
California spruce				4,000		5,000	1,200,000		
Factor of safety	10	10	5	5	4	6	2	4	4

57. The values given in Table V, under their several headings, being conservative, are on the safe side. The column headed Compression With Grain, Columns Under 15 Diameters will be found useful in computing the strength of long columns in a method to be explained elsewhere.

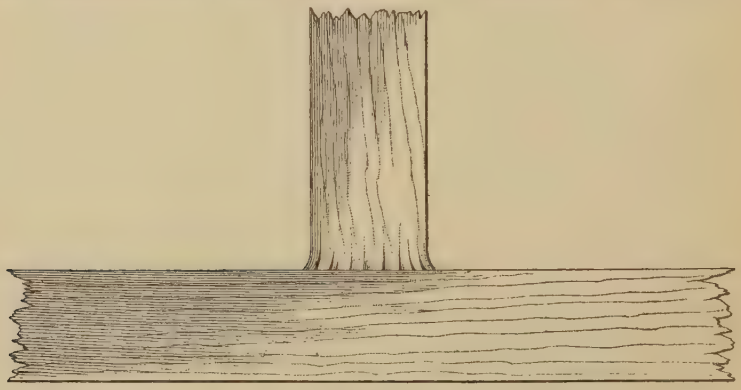


FIG. 32

The column headed Compression With Grain, End Bearing gives the value of crushing at the end of a timber in such a case as shown in Fig. 32. The values in the column headed Compression Across Grain are used in all cases similar to the condition of the lower timber in Fig. 32. The values

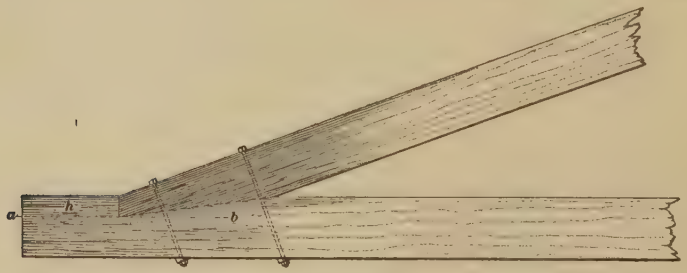


FIG. 33

under the heading Shearing With Grain are used in computing the strength of the end of the tie-beam at the heel of the main rafter in a roof truss, as shown in Fig. 33. The tendency is to shear off the piece *h* parallel to the grain along the line *a b*. The figures in the column headed

Modulus of Rupture are the constants, or values, used when computing the strength of beams. When a simple beam breaks, the fibers at the top side are in compression, and those at the bottom side in tension, as shown in Fig. 34. By actual tests, it has been found that though some of the fibers of materials under transverse stresses are in compression and some in tension, the ultimate resistance of the material does not agree with the ultimate resistances of the fibers to either tension or compression; hence, it becomes necessary to obtain some constant, or value, more closely agreeing with the actual strength of materials under transverse stresses. It is usual, therefore, where the cross-section of the beam is uniform, to obtain, by actual tests, the constants, or values, for each material, and these values are called the *modulus of rupture* and are generally expressed in pounds per square inch. The use of the modulus of rupture will be fully explained later.

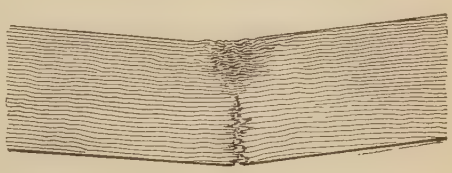


FIG. 34

At the foot of each column is given the factor of safety that it is customary to use for timber under the special kind of stress given in that column.

EXAMPLE.—In Fig. 35 is shown the splice of a tie-beam in a wooden roof truss composed of yellow pine; what is the strength of the splice, disregarding the bolts *a*, *a* entirely?

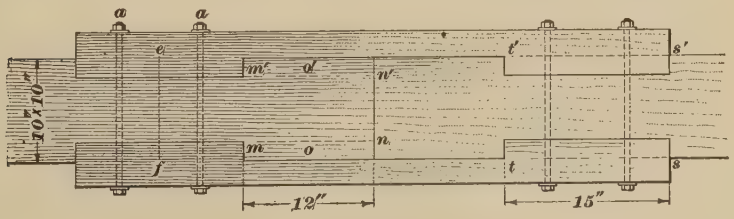


FIG. 35

SOLUTION.—The strength of the splice depends on the tensile strength of the wood at the net section *e f* and on the tensile strength of the net section of the two splice plates. It also depends on the tendency of

the splice plates to shear along the lines $s\,t$ and $s'\,t'$, and on the tendency of the tie to shear along the line $m\,n$ and $m'\,n'$. Assume the areas of the net sections to be sufficient to make their strength greater than that of the sections which will fail by shearing; then, referring to Fig. 35, it will be seen that the line of shear on the splice plate at $s\,t$ and $s'\,t'$ is longer than that of the tie-member at $m\,n$ and $m'\,n'$; therefore, the strength of the tie along the line $m\,n$ and $m'\,n'$ only need be considered in computing the strength of the splice. The pieces o and o' tend to slide or shear off of the main tie along the lines $m\,n$ and $m'\,n'$. The area along these lines is

$$12 \times 10 \times 2 = 240 \text{ sq. in.}$$

Referring to Table V under the column headed Shearing With Grain, the value for yellow pine is found to be 600 lb. per sq. in. Hence, $240 \times 600 = 144{,}000$ lb., the ultimate strength of the splice, disregarding the bolts a, a. Ans.

58. Table VI gives the strength and other properties of various metals. The use of the different columns will be readily understood.

EXAMPLE 1.—What pull will be required to break a rod of wrought iron 2 inches in diameter?

SOLUTION.—The area of the rod is equal to the area of a 2-in. circle, which is $2^2 \times .7854 = 3.14$ sq. in.; the ultimate tensile, or breaking, strength of wrought iron, according to Table VI, is 48,000 lb. per sq. in. Therefore, the ultimate strength of the rod in question is about

$$3.14 \times 48{,}000 = 150{,}720 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—What length of wrought-iron bar, if hung by one end, will break of its own weight?

SOLUTION.—Assume any size of bar; say $1\frac{1}{8}$ in. in diameter. The area of this bar is .99 sq. in., which may, for convenience, be called 1 sq. in. Wrought iron, according to Table I, weighs .277 lb. per cu. in. Now, as there is just 1 cu. in. in each lineal inch in the rod, a length of 1 ft. weighs $.277 \times 12 = 3.32$ lb. The tensile strength of wrought iron being 48,000 lb. per sq. in., and 1 ft. of its length weighing 3.32 lb., the length of rod required is

$$48{,}000 \div 3.32 = 14{,}458 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Provided a factor of safety of 4 is adopted, what will be the safe working stress on a 2-inch diameter tension rod of structural steel?

Ans. 50,266 lb.

2. A short block of Georgia yellow pine 10 in. × 10 in. in section, standing on end, supports 50,000 pounds; what is its factor of safety?

Ans. 10

TABLE VI

AVERAGE ULTIMATE STRENGTHS OF MATERIALS IN POUNDS PER SQUARE INCH

	Compression	Tension	Elastic Limit	Shearing	Modulus of Rupture	Modulus of Elasticity
BRASS, BRONZE, AND COPPER						
Brass, cast	(30,000)	24,000	6,000	36,000	20,000	9,000,000
Brass wire, annealed		50,000				
Bronze, manganese	120,000	60,000	30,000			
Bronze, phosphor		50,000	24,000			14,000,000
Copper, bolts	30,000	30,000				
Copper, cast	(40,000)	24,000	6,000	30,000	22,000	10,000,000
Copper wire, annealed		36,000				15,000,000
CAST AND WROUGHT IRON						
Iron, cast	80,000	15,000	6,000	18,000	30,000	12,000,000
Iron, wrought, shapes	46,000	48,000	26,000	40,000	44,000	27,000,000
CAST AND STRUCTURAL STEEL						
Steel, castings	70,000	70,000	40,000	60,000	70,000	30,000,000
Steel, structural, soft	56,000	56,000	30,000	48,000	54,000	29,000,000
Steel, structural, medium	64,000	64,000	33,000	50,000	60,000	29,000,000
Steel wire, annealed		80,000	40,000			29,000,000
Steel wire, crucible		180,000	80,000			30,000,000
Steel wire, for suspension bridges		200,000	90,000			30,000,000

NOTE.—Compression values enclosed in parentheses indicate loads producing 10-per-cent. reduction in original lengths.

BEAMS

DEFINITIONS

59. Any bar resting on supports, and liable to transverse stresses, is called a **beam.**

A **simple beam** is a beam resting on two supports very near its ends.

A **cantilever** is a beam resting on one support near its middle, or one that has one end fixed (as in a wall) and the other end free.

A **fixed beam** is one that has both ends firmly secured (as in a wall).

A **continuous beam** is one that rests on more than two supports.

The **span** of a simple beam is the distance between its supports.

Fig. 36 represents a simple beam. It presses on its supports at the ends. The supports resist this pressure and

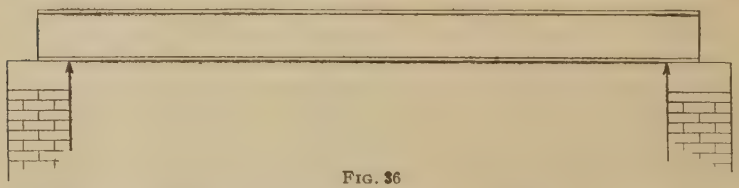


FIG. 36

prevent the beam from falling. This upward force exerted by each support is called the **reaction.**

60. Art. 5 states that all bodies at rest are in equilibrium. A beam, as in Fig. 36, is at rest and must therefore be in equilibrium. The rule is that when a beam or any other body is in equilibrium, the forces acting on the beam must first balance each other, and, second, the resultant moment of the forces about any point must be zero. This first

condition of equilibrium has already been briefly discussed. The second condition, however, has not been mentioned, but it is of equal importance when compared with the first. The first condition is the one that prevents the beam from moving off through space, and the second condition is the one that prevents it from revolving.

Now, what are the forces acting on a beam? First, there are the forces due to loads on the beam and the weight of the beam itself, if that is considered; then, there are the reactions of the supports on the beam. Now, all the loads act vertically downwards and the reactions act vertically upwards. They are, therefore, parallel, and it follows from Art. 22 that the sum of the loads must equal the sum of the reactions. Let us consider a beam with two supports. We know the sum of the reactions from the loads, but we do not know how much

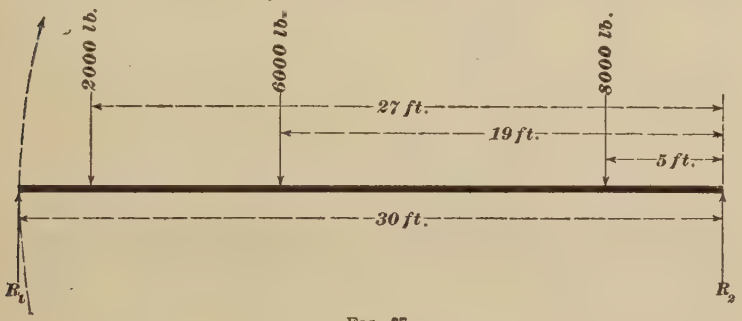


FIG. 37

each reaction is. To find out this, it is necessary to resort to the second condition of equilibrium, namely, that the resultant moment of all the forces about any point must be zero. Any point can be assumed, but it is found convenient to choose one reaction as the point about which to take moments. The moment of one reaction about the point is zero, since the arm of the moment is zero. It is therefore necessary that the sum of the moments of the loads about one reaction and the moment of the other reaction about that same point equal zero. In this way, by solving this equation, one reaction is found, and by subtracting this reaction from the sum of the loads, the other reaction is found.

61. Take a practical example. In Fig. 37, let it be required to find the reactions R_1 and R_2. (In all the subjoined problems, R_1 and R_2 represent the reactions.) The center of moments may be taken at either R_1 or R_2. Taking R_2 as the center in this case, construct a diagram as in Fig. 37. The three loads are forces acting in a downward direction; the sum of their moments, in foot-pounds, with respect to the assumed center may be computed as follows:

$$\begin{aligned} 8{,}000 \times 5 &= 40000 \\ 6{,}000 \times 19 &= 114000 \\ 2{,}000 \times 27 &= 54000 \\ \text{Total,} \quad & 208000 \end{aligned}$$

The magnitude of the reaction R_1 acting in an upward direction about R_2 with a lever arm of 30 feet is, therefore, $208{,}000 \div 30 = 6{,}933\frac{1}{3}$ pounds. The sum of all the loads is $2{,}000 + 6{,}000 + 8{,}000 = 16{,}000$ pounds. Then the reaction at R_2 is $16{,}000 - 6{,}933\frac{1}{3} = 9{,}066\frac{2}{3}$ pounds.

EXAMPLE 1.—What is the reaction at R_2 in Fig. 38?

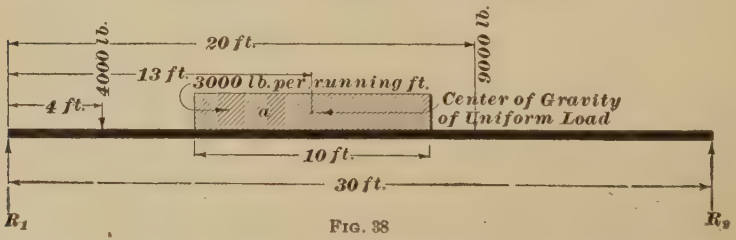


FIG. 38

SOLUTION.—In computing the moment due to a uniform, or evenly distributed, load, as at a, the lever arm is always considered as the distance from the center of moments to the center of gravity of the load. The amount of the uniform load a is $3{,}000 \times 10 = 30{,}000$ lb., and the distance of its center of gravity from R_1 is 13 ft. The moments of the loads on this beam about R_1, in foot-pounds, are as follows:

$$\begin{aligned} 30{,}000 \times 13 &= 390000 \\ 4{,}000 \times 4 &= 16000 \\ 9{,}000 \times 20 &= 180000 \\ \text{Total,} \quad & 586000 \end{aligned}$$

This is the sum of the moments of all the loads about R_1, as a center. The leverage of the reaction R_2 is 30 ft. Hence, the reaction at R_2 is $586{,}000 \div 30 = 19{,}533\frac{1}{3}$ lb. Ans.

EXAMPLE 2.—A beam is loaded as shown in Fig. 39; compute the reactions R_1 and R_2.

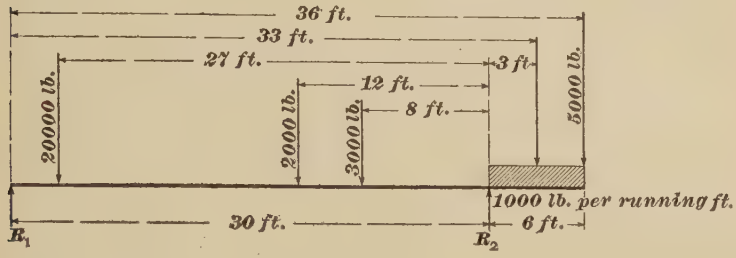


FIG. 39

SOLUTION.—Consider, say, R_2 as the center of moments. Then, the negative moments of the loads about R_2 (see Arts. **30** and **31**) are, in foot-pounds:

$$\begin{aligned} 20{,}000 \times 27 &= 540000 \\ 2{,}000 \times 12 &= 24000 \\ 3{,}000 \times 8 &= 24000 \\ \text{Total,} \quad & 588000 \end{aligned}$$

Now, the positive moments of the loads about R_2 are, in foot-pounds:

$$\begin{aligned} 1{,}000 \times 6 \times 3 &= 18000 \\ 5{,}000 \times 6 &= 30000 \\ \text{Total,} \quad & 48000 \end{aligned}$$

The resultant moment of the loads about R_2 is negative and is $-588{,}000 + 48{,}000 = -540{,}000$ ft.-lb. $540{,}000 \div 30 = 18{,}000$ lb., which is the value of R_1.

The sum of the loads is:

$$\begin{aligned} & 20000 \\ & 2000 \\ & 3000 \\ 6 \times 1{,}000 = \quad & 6000 \\ & 5000 \\ & 36000 \end{aligned}$$

$$R_2 = 36{,}000 - 18{,}000 = 18{,}000 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 3.—Compute the reactions at the supports R_1 and R_2 in a beam loaded as shown in Fig. 40.

SOLUTION.—Letting R_1 be the center of moments, the moments of the loads, in foot-pounds, are:

$$\begin{aligned} 5{,}000 \times 10 &= 50000 \\ 10{,}000 \times 20 &= 200000 \\ 30{,}000 \times 40 &= 1200000 \\ \text{Total,} \quad & 1450000 \end{aligned}$$

Now, 1,450,000 ÷ 30, the distance between the supports = 48,333$\frac{1}{3}$ lb., the required reaction of R_2. The sum of the loads is

$$5{,}000 + 10{,}000 + 30{,}000 = 45{,}000 \text{ lb.};$$

therefore, the reaction R_2 is greater than the sum of the loads. This shows that the force at R_1 must act in a downward direction in order that the sum of the downward forces may equal the upward force at R_2. Since this is opposite to the usual direction, the reaction at R_1 is called negative, or minus. In other words, instead of an upward reaction at R_1, there must be a downward force at this point, or the

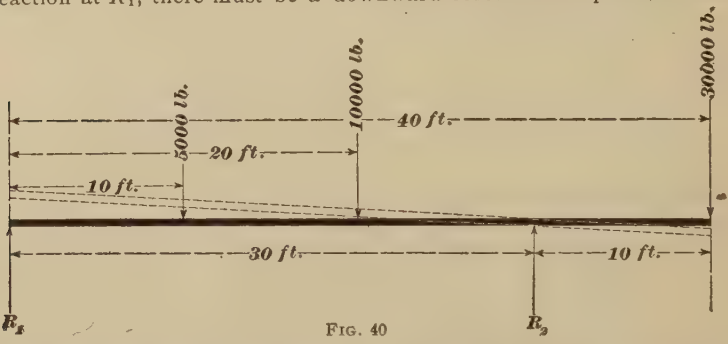


FIG. 40

beam will, as shown by the dotted lines, rotate around the support R_2. The magnitude of this downward force R_1 is the difference between the upward reaction at R_2 and the sum of the downward pressures due to the loads; that is,

$$48{,}333\tfrac{1}{3} - 45{,}000 = 3{,}333\tfrac{1}{3} \text{ lb.}$$

We will now as a check compute the reaction at R_1 by taking the center of moments at R_2, and applying the principles given in Arts. **30** and **31** to find the magnitude and direction of action of the force at R_1 whose moment is equal and opposite to the resultant of the moments of the loads on the beam. The load of 30,000 lb. tends to produce right-hand rotation around the center R_2; hence, its moment, 30,000 × 10 = 300,000 ft.-lb., is positive. The 10,000-lb. load is 10 ft. to the left of R_2, and its tendency is to produce left-hand rotation about R_2; consequently, its moment is negative and equal to 10,000 × 10 = 100,000 ft.-lb. We find, in a similar manner, the moment of the 5,000-lb. load to be negative and equal to 5,000 × 20 = 100,000 ft.-lb. These results may be collected thus:

Positive moment:

30,000 × 10 = +300000 ft.-lb.

Negative moments:

10,000 × 10 = 100000

5,000 × 20 = 100000

−200000 ft.-lb.

Algebraic sum, +100000 ft.-lb.

the resultant of the moments of the three loads. Since the positive moment is greater than the sum of the negative moments, we see that to produce equilibrium the force at R_1 must tend to produce left-hand rotation; that is, it must act downwards; its lever arm being 30 ft. long, its magnitude must be $100{,}000 \div 30 = 3{,}333\frac{1}{3}$ lb., the same result as was obtained before.

EXAMPLES FOR PRACTICE

1. The span of a simple beam is 25 feet; at distances of 9 feet, 16 feet, and 18 feet from the left-hand end are placed concentrated loads of 8,000, 4,000, and 16,000 pounds, respectively. What is the magnitude of the left-hand reaction? Ans. 11,040 lb.

2. The two reactions supporting a beam are 2,500 and 3,000 pounds; what is the single concentrated load necessary to produce these reactions? Ans. 5,500 lb.

3. The length of a beam is 30 feet, and it overhangs the right-hand support 6 feet. From the overhanging end, there is a weight of 6,000 pounds; 10 feet, 12 feet, and 18 feet from the left-hand support are loads of 8,000, 6,200, and 7,800 pounds, respectively. What is the magnitude of the right-hand reaction? Ans. $19{,}783\frac{1}{3}$ lb.

4. If for a distance of 10 feet from the left-hand end of a beam is distributed a load of 1,000 pounds per running foot, and at the center of the beam is located the concentrated load of 16,500 pounds, what is the magnitude of the left-hand reaction, provided the beam is supported at both ends and is 30 feet long? Ans. $16{,}583\frac{1}{3}$ lb.

STRESSES IN BEAMS

62. It has been seen that a beam is acted on by various external forces so related as to be in a condition of equilibrium; so far, however, the effect of these forces on the beam itself has not been considered.

In a body subjected to a direct pull or thrust, as a rope or a column, the external forces are directly opposed to each other, and the resultant stresses in all sections are of the same kind, tension or compression. In a beam, however, the external forces, while they generally act in parallel lines, are not directly opposed to each other, and it is the function of the beam to transfer these forces from one line of action to another. Take, for example, the case of a weight

suspended from a pin driven into a wall, as shown in Fig. 41. The downward force of 20 pounds due to the action of the weight is balanced by the upward pressure or reaction of the pin on the rope; the rope is thus subjected to the action of two directly opposing forces, the result being a tensile stress that is the same for each section of the rope between the weight and pin. The pin acts as a cantilever beam that transfers horizontally to the wall, where it is balanced by an equal upward pressure or reaction, the downward pressure due to the pull of the rope. The pin is thus subjected to two opposing forces, which, however, act in different lines; these forces produce a set of opposing forces, or stresses, in the pin itself, which are different in kind for different parts of the pin, and vary in magnitude for each section between the rope and the wall.

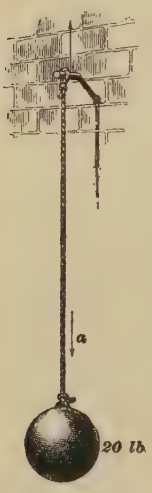


Fig. 41

SHEAR

63. By an inspection of Fig. 41, it will be seen that if a vertical plane is passed through any point between the rope and wall, the part of the pin between this plane and the rope will be acted on by a downward force due to the pull of the rope, while the other is subjected to an equal upward force due to the reaction of the wall; the action of these two forces tends to slide the two parts of the pin past each other, along the section formed by the cutting plane. The pin is thus subjected to a stress that, from its similarity to a shearing action, is called **shear.**

64. Shear in a Simple Beam.—Consider now the simple beam shown in Fig. 42. Since the loads are symmetrically applied, each reaction is equal to 40 pounds, one-half of the total load on the beam. Beginning at the left reaction R_1, there is an upward force of 40 pounds acting on the beam; since the forces are in equilibrium, this upward force is balanced by an equal downward force, which is the vertical resultant of the loads and the reaction R_2. Considering,

therefore, any section of the beam between R_1 and the point of application of the load n, we see that the part of the beam at the left of this section is subjected to an upward thrust of 40 pounds, while the part at the right is subjected to an equal downward thrust; the result is a shearing stress on this section, whose magnitude is equal to the reaction R_1.

When the point of application of n is reached, the effect of the upward force R_1 is partly balanced by the downward force of 10 pounds due to the load n; considering, therefore, any section of the beam, as $a\,b$, between the points of application of the loads n and m, we see that the part of the beam at the left is acted on by the vertical resultant of the reaction R_1 and the load n, that is, by an upward force of $40 - 10 = 30$ pounds, while the part at the right is acted on by an equal downward force, the vertical resultant of the

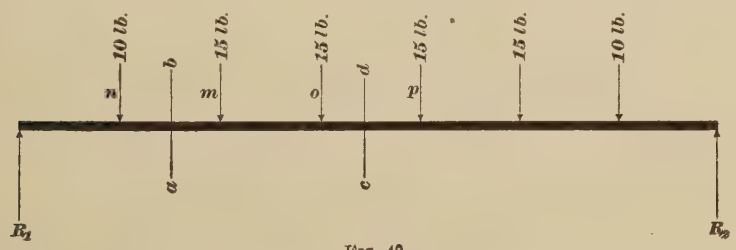


FIG. 42

remaining loads and the reaction R_2. Any section between the points of application of n and m is, therefore, subject to a shearing stress equal to the difference between the reaction R_1 and the load n, that is, to $40 - 10 = 30$ pounds. In the same way, it follows that the shearing stress for any section between m and o is $40 - (10 + 15) = 15$ pounds. For any section, as $c\,d$, between the points of application of o and p, the shearing stress is $40 - (10 + 15 + 15) = 0$; in other words, on each side of this section the downward forces and the reactions are equal, and their resultant is zero; it is therefore a section in which there is no shear.

For convenience, it is customary to call the reactions, or forces, acting in an upward direction, positive, and the loads, or downward forces, negative; since the difference between

the sums of the positive and negative numbers representing a given set of values is called their algebraic sum, it follows that *the shear for any section of a beam is equal to the algebraic sum of either reaction and the loads between this reaction and the section under consideration.*

In nearly all cases, the external forces—loads and reactions—act on a beam along vertical lines; the shearing stress just considered being the resultant of these forces along a section formed by an imaginary vertical cutting plane, is often called the **vertical shear.**

65. Maximum Shear.—From what has been said, it is evident that the shear in any simple beam is always greatest between the reactions and the nearest loads, and that in any case the maximum shear is equal to the greater reaction.

66. Positive and Negative Shear.—If we take a section of the beam near the left reaction and consider the forces acting on the part of the beam at the left, it will be seen that their resultant is positive; the shear at this section is, therefore, called **positive shear.** If, however, we take

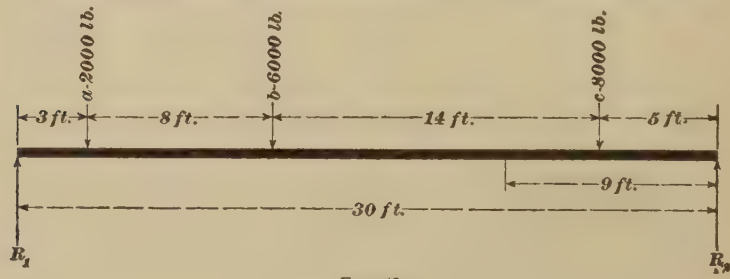


FIG. 43

a section near the right reaction, the resultant of the forces at the left of this section is found to be negative, and in consequence the shear is called **negative.** It is also evident that there is a section between the two, where the resultant of the forces changes from positive to negative; at such a section the shear is said to **change sign.**

EXAMPLE 1.—(*a*) What is the maximum shear on the beam shown in Fig. 43? (*b*) What is the shear at a point 9 feet from the right support? (*c*) What is the shear at a point 18 feet from the right support?

SOLUTION.—(*a*) First estimate the reactions as follows: Taking the center of moments at the left support, the moments of the loads, in foot-pounds, are:

$$\begin{aligned} 2{,}000 \times 3 &= 6000 \\ 6{,}000 \times 11 &= 66000 \\ 8{,}000 \times 25 &= 200000 \\ \text{Total,} \quad & 272000 \end{aligned}$$

$272{,}000 \div 30 = 9{,}066\frac{2}{3}$ lb., the reaction at R_2. The sum of the loads equals

$$2{,}000 + 6{,}000 + 8{,}000 = 16{,}000 \text{ lb.}$$

$16{,}000 - 9{,}066\frac{2}{3} = 6{,}933\frac{1}{3}$ lb., the reaction at R_1. The maximum shear is therefore $9{,}066\frac{2}{3}$ lb. Ans.

(*b*) As the reaction R_2 at the right support is equal to $9{,}066\frac{2}{3}$ lb., and as there is only the one load *c* of 8,000 lb. between R_2 and a point 9 ft. away, the shear at this point must equal

$$9{,}066\frac{2}{3} - 8{,}000 = 1{,}066\frac{2}{3} \text{ lb.} \quad \text{Ans.}$$

(*c*) The shear at 18 ft. from the reaction R_2 is also $1{,}066\frac{2}{3}$ lb., because there is no other weight occurring between this point and R_2.

EXAMPLE 2.—At what point in the beam loaded as shown in Fig. 44 does the shear change sign?

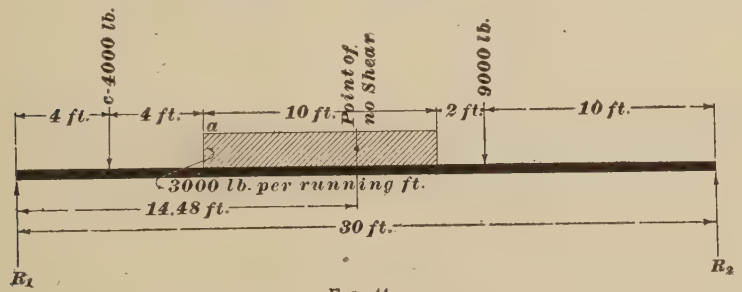


FIG. 44

SOLUTION.—Compute the reaction R_1 as follows: With the center of moments at R_2 the moments of the loads in foot-pounds are:

$$\begin{aligned} 9{,}000 \times 10 &= 90000 \\ 4{,}000 \times 26 &= 104000 \\ 3{,}000 \times 10 \times 17 &= 510000 \\ \text{Total,} \quad & 704000 \end{aligned}$$

$704{,}000 \div 30 = 23{,}466\frac{2}{3}$ lb., the reaction at R_1. The first load that occurs, working out on the beam from R_1, is *c* of 4,000 lb. Then, $23{,}466\frac{2}{3} - 4{,}000 = 19{,}466\frac{2}{3}$ lb. The next load that occurs on the beam is the uniform load of 3,000 lb. per running ft. There being altogether 30,000 lb. in this load, it is evident that it will more than absorb the

remaining amount of the reaction R_1; the point where the change of sign occurs must consequently be somewhere in that part of the beam covered by the uniform load. The load being 3,000 lb. per running ft., if the remaining part of the reaction, 19,466⅔ lb., is divided by the 3,000 lb., the result will be the number of feet of the uniform load required to absorb the remaining part of the reaction, and this will give the distance of the section, beyond which the resultant of the forces at the left becomes negative, from the edge of the uniform load at a; thus, $19{,}466\frac{2}{3} \div 3{,}000 = 6.48$ ft. The distance from R_1 to the edge of the uniform load is 8 ft. The entire distance to the section of change of sign of the shear is, therefore,

$$8 + 6.48 = 14.48 \text{ ft. from } R_1. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The uniformly distributed load on a beam supported at both ends is 40,000 pounds; what is the maximum shear on the beam?

Ans. 20,000 lb.

2. A beam is loaded with three concentrated loads: A of 2,000 pounds, B of 6,000 pounds, and C of 8,000 pounds; they are located 10 feet, 12 feet, and 18 feet, respectively, from the left-hand end of the beam, the span of which is 40 feet. What is the shear between the loads C and B? Ans. 2,100 lb.

3. The span of a beam is 20 feet, and there is a uniformly distributed load on three-quarters of this distance from the left-hand support of 9,000 pounds. At distances of 8 feet and 12 feet from the right-hand support are located concentrated loads of 5,000 pounds and 6,000 pounds, respectively. At what distance from the left-hand end of the beam does the shear change sign? Ans. 8 ft. 8½ in.

BENDING STRESSES

67. Bending Moment.—If, in a cantilever loaded as in Fig. 45, we take any point x on the center line $a\,b$ as a center of moments, and consider a section made by a vertical plane $c\,d$ through this center, it is evident the moment of the force due to the downward thrust of the load tends to turn the end of the beam to the right of $c\,d$, around the center x; the measure of this tendency is the product of the weight W multiplied by its distance from $c\,d$, and, since it is the moment of a force that tends to bend the beam, it is called the **bending moment.**

68. Resisting Moment.—A further inspection of Fig. 45 shows that if the end of the beam turns around the center x until it takes the position shown by the dotted lines, the parts of the two surfaces formed by the cutting plane $c\,d$, which are above the center x, must be pulled away from each other, while those below are pushed closer together. It is thus seen that, if we consider a vertical section through any point on the center line $a\,b$, between the load and the point of support, the tendency of the load is to separate the particles in this section above the center line, and to push those below the center line closer together; in other words, through the bending action of the load, the upper part of the beam is subjected to a tensile stress, while the lower is subjected to a compressive stress.

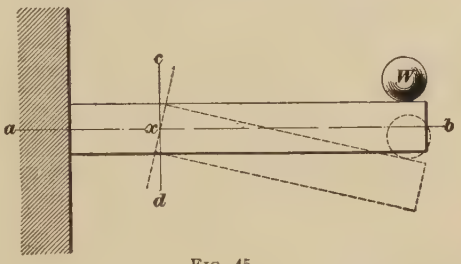


FIG. 45

Fig. 45 also shows that the greater the distance of the particles in the assumed section above or below the center x, the greater will be their displacement; since the stress in a loaded body is directly proportional to the strain, or relative displacement of the particles, it follows that the stress in a particle of any section is proportional to its distance from the center line, and that the greatest stress is in the particles composing the upper and lower surfaces of the beam.

In accordance with the conditions of equilibrium, the algebraic sum of the moments of all the forces tending to produce rotation around a given center must be zero; we have seen that the weight of the load is a force that tends to produce right-hand rotation around the center x; therefore, if the beam does not break under the action of the load, there must be forces acting whose moments, with respect to the center x, balance the moment of the load. These forces are the resistances with which the particles of the beam oppose any effort to change their relative position. The tensile

stresses in the particles above the center x, and the compressive stresses in those below it, are a set of forces that resist the tendency of the load to turn the end of the beam, and, when the effect of the load is just balanced by the effect of these forces, it is evident that the sum of the moments of these resisting stresses is equal to the moment of the load. *The sum of the moments of the stresses of all the particles composing any section of a beam is called the* **resisting moment** *or* **moment of resistance** *of that section.*

69. Neutral Axis.—In Fig. 46, let $A\,B\,C\,D$ represent a cantilever. A force F acts on it at its extremity A; the principles developed in Art. **68** show that the force will tend to bend the beam into something like the shape shown by $A'\,B\,C\,D'$. It is evident from what has preceded that the upper part $A'\,B$ is now longer than it was before the force was applied; that is, $A'\,B$ is longer than $A\,B$. It is also evident that $D'\,C$ is shorter than $D\,C$. Hence, the effect of the force F in bending the beam is to lengthen the upper fibers and to shorten the lower ones. Further consideration will show that there must be a fiber, $S\,S''$, which is neither lengthened nor shortened when the beam is bent, that is, $S\,S'' = S'\,S''$. When the beam is straight, the line through the fiber $S\,S''$, which is neither lengthened nor shortened when the beam is bent, is called the **neutral line.**

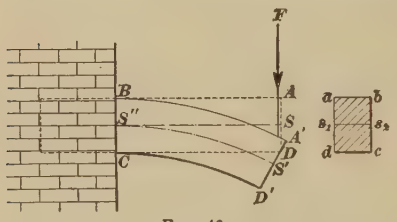


FIG. 46

The neutral line corresponds to the center line $a\,b$, Fig. 45, on which the center of moments x was taken.

70. The relations between the effect of a load and the resulting stresses in a beam have been thoroughly proved, both by mathematical investigations and numerous experiments. The results of these experiments on beams may be briefly expressed by the following:

Experimental Law.—*When a beam is bent, the horizontal elongation (or compression) of any fiber is directly proportional*

to its distance from the neutral surface, and, since the strains are directly proportional to the horizontal stresses in each fiber, they are also directly proportional to their distances from the neutral surface, provided the elastic limit is not exceeded.

The line $s_1 s_2$, which passes through any section, as *a b c d*, Fig. 46, at the neutral line, perpendicular to the direction in which the beam bends, is called the **neutral axis.** It is shown in works on mechanics that *the neutral axis always passes through the center of gravity of the cross-section of the beam.*

71. Bending Moments in Simple Beams.—Referring to the simple beam shown in Fig. 43, let us take the center of moments on the neutral axis directly under the load *a*, and consider the effect produced on a vertical section of the beam through this center, by the reaction R_1. It was shown in Art. **66** that the reaction R_1 is an upward force of $6{,}933\frac{1}{3}$ pounds; it therefore has a tendency to turn the end of the beam upwards around the assumed center with a moment of $6{,}933\frac{1}{3} \times 3 = 20{,}800$ foot-pounds. It is evident that, to prevent this turning from actually taking place, the positive moment of the reaction must be balanced by a negative moment, which can be produced only by a set of internal stresses. The condition that the moment of the stresses must be negative makes it plain that the upper fibers must be in compression and the lower fibers in tension, a result exactly opposite to the effect produced by the bending moment on the fibers in the cantilever.

72. Effect of the Moments Due to Loads.—The only force acting on the beam at the left of the section considered in Art. **71** was the reaction R_1. The load *a* acted downwards directly through this section, but its lever arm, and consequently its moment, with respect to the assumed center, was zero. Take now a point on the center line of the beam directly under the positive load *b*. The reaction has a moment with respect to this center of $6{,}933\frac{1}{3} \times 11 = 76{,}266\frac{2}{3}$ foot-pounds, while the load *a*, that acts downwards with a lever arm of 8 feet, has a negative moment of $2{,}000 \times 8 = 16{,}000$ foot-pounds. The bending moment at the assumed

section is the algebraic sum of these moments, that is, $76{,}266\frac{2}{3} - 16{,}000 = 60{,}266\frac{2}{3}$ foot-pounds. Again, taking the center of moments on a section 9 feet from the right reaction R_2, the moments are as follows:

Positive moment:

Reaction R_1, $6{,}933\frac{1}{3} \times 21 =$ 145600 foot-pounds

Negative moments:

Load a, $2{,}000 \times 18 =$ 36000

Load b, $6{,}000 \times 10 =$ 60000 96000 foot-pounds

Difference, 49600 foot-pounds

This resultant moment is the bending moment at the given section. It will be noted that the bending moment at a certain section can be figured from either end of the beam. Thus, for the section last considered, suppose it is figured from the right-hand reaction.

Negative moment:

Reaction R_2, $9{,}066\frac{2}{3} \times 9 =$ 81600 foot-pounds

Positive moment:

Load c, $8{,}000 \times 4$ = 32000 foot-pounds

Difference, 49600 foot-pounds

This is the same result as was obtained before.

73. The illustrations show that the bending moment varies from point to point in a beam, and depends on the length of the beam and on the magnitude and position of the loads. Since the stresses in the beam, and consequently its ability to carry its loads, depend directly on the bending moment, it follows that it is important to find, not only the bending moment for any assumed section, but also the section where the bending moment is greatest. It is, in this connection, useful to note the relation between the bending moment and the shear.

74. The shear in a simple beam is always greatest at the greater reaction, being equal to that reaction. In passing along the beam from either reaction, there is no change in the shear until a load is reached. At each point where a load is added, the shear is diminished by an amount equal to the load. At the point where the sum of the added loads equals or exceeds the reaction, the shear is said to change

sign. The section where the change in sign in the shear takes place, depends on the method of loading. With a uniformly distributed load, the shear diminishes uniformly from each reaction, and the section where the sign changes is the section of the beam midway between the supports. With a single concentrated load, the shear is equal to each reaction at all sections between that reaction and the point where the load is applied, and the section where the shear changes sign is directly under the load. With any system of loading, the section where the shear changes sign can be found by adding together the successive loads from either reaction toward the center of the beam, until a sum is obtained that equals or exceeds the reaction; the section where the shear changes sign is under the point of application of the last load added.

75. The bending moment in a simple beam increases as the shear decreases; it is zero at either reaction and increases toward the center, *becoming greatest at the section where the shear changes sign*. With a uniformly distributed load, the

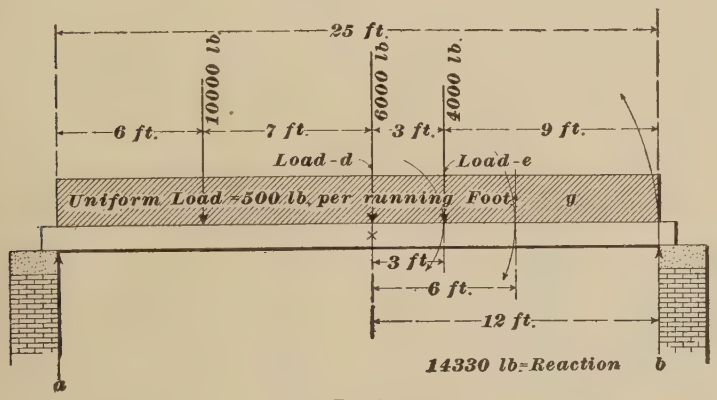


FIG. 47

greatest bending moment is at the section of the beam midway between the supports; with a single concentrated load, the greatest bending moment is directly under the load; and with any system of loading, the greatest bending moment occurs at the section where the shear changes sign. Having

located the section of the greatest bending moment, the magnitude of this moment can be readily computed by taking the center of moment on the section of greatest bending moment and computing the resultant moment of either reaction and all the loads between it and the center in question.

EXAMPLE.—A wooden beam, supported on two brick piers, is loaded as shown in Fig. 47. (*a*) What is the greatest shear? (*b*) Where does the shear change sign? (*c*) What is the greatest bending moment, in inch-pounds?

SOLUTION.—(*a*) Since the greatest shear is equal to the greater reaction, first compute the reactions. Taking the center of moments at the edge of pier *a*, and remembering that the moment of the uniform load is the same as the moment of an equal concentrated load acting at the center of gravity of the uniform load, the moments of the loads, in foot-pounds, are:

10,000-lb. load	$10{,}000 \times 6$	= 60000
Load *d*	$6{,}000 \times 13$	= 78000
Load *e*	$4{,}000 \times 16$	= 64000
Uniformly distributed load . . .	$12{,}500 \times 12\frac{1}{2}$	= 156250
Total		358250

This also equals the moment of the reaction of the pier *b*. The reaction at *b* is, therefore, $358{,}250 \div 25 = 14{,}330$ lb. The total load is $10{,}000 + 6{,}000 + 4{,}000 + 12{,}500 = 32{,}500$ lb.; the reaction at *a* is, therefore, $32{,}500 - 14{,}330 = 18{,}170$ lb. This, being the greater reaction, is the greatest shear.

(*b*) Beginning at the left reaction and adding the loads in succession toward the right, it will be found that the load of 10,000 lb. plus the uniformly distributed load between the reaction and the point of application of the load *d*, is $10{,}000 + 500 \times 13 = 16{,}500$ lb. This is less than the left reaction, but when the load *d* is added, the sum of the loads is greater than the reaction; consequently, the shear changes sign under the load *d*. Ans.

(*c*) Taking the center of moments, under the load *d*, and considering the forces at the left, we have

Positive moment:

$18{,}170 \times 13 =$ 236210 ft.-lb.

Negative moments:

$6{,}500 \times 6\frac{1}{2} =$ 42250

$10{,}000 \times 7 =$ 70000

112250 ft.-lb.

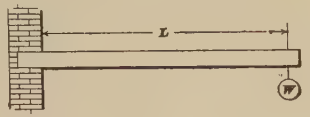
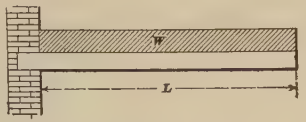
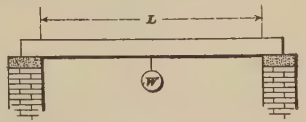
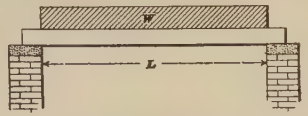
Difference (bending moment, in ft.-lb.) 123960 ft.-lb.

The greatest bending moment, in inch-pounds, is, therefore,

$123{,}960 \times 12 = 1{,}487{,}520$ in.-lb. Ans.

TABLE VII

MAXIMUM BENDING MOMENTS FOR BEAM LOADING

Case	Method of Loading	Rule	Formula
I		To obtain the maximum bending moment, in foot-pounds, on this beam, multiply the weight W, in pounds, by the distance L, in feet.	$M' = WL$ (1)
II		To obtain the maximum bending moment, in foot-pounds, on this beam, multiply the weight W, in pounds, by the distance L, in feet, and divide by 2.	$M' = \frac{WL}{2}$ (2)
III		To obtain the maximum bending moment, in foot-pounds, on this beam, multiply the weight W, in pounds, by the distance L, in feet, and divide by 4.	$M' = \frac{WL}{4}$ (3)
IV		To obtain the maximum bending moment, in foot-pounds, on this beam, multiply the weight W, in pounds, by the distance L, in feet, and divide by 8.	$M' = \frac{WL}{8}$ (4)
V		To obtain the maximum bending moment, in foot-pounds, on this beam, multiply the weight W, in pounds, by the distance L, in feet, and divide by 6.	$M' = \frac{WL}{6}$ (5)

76. General Formulas for Bending Moments. Where a cantilever or a simple beam is symmetrically loaded, it is not necessary to calculate the moments of the forces acting on the beam in order to find the reactions and bending moments. The rules and formulas of Table VII are available in computing the maximum bending moment M' in the five simple cases of beam loading set forth.

77. The rule given in Case IV is the one most used, as it applies to a beam uniformly loaded, such as floor joists, girders, and, in some cases, the rafters of a roof. The rule in Case V is convenient in calculating the bending moment on lintels supporting brickwork or masonry over openings. If, in Case III, the beam is firmly fixed or fastened at both ends, the bending moment under the same load will be only one-half as much. If, in Case IV, the ends of the beam are firmly fixed, instead of dividing by 8 a constant of 12 is to be used. It is seldom advisable, in ordinary building practice, to consider the ends of a beam to be firmly fixed, it being good practice to assume the ends of the beams as simply bearing on the wall, using the rules and formulas in Table VII. It should be understood, however, that all these rules and formulas apply to static loads. The same load suddenly applied produces a stress in the beam twice as great as that of a static load. The safe sudden load should be therefore only one-half as much at the static load the beam is designed for.

EXAMPLE.—What will be the bending moment, in inch-pounds, on a wooden girder supporting a floor area of 150 square feet, the dead and live load being 100 pounds per square foot, and the span of the girder 20 feet?

SOLUTION.—The total uniformly distributed load is $150 \times 100 = 15{,}000$ lb.; therefore, by applying the formula in Case IV, Table VII, we have the bending moment

$$M = \frac{WL}{8} = \frac{15{,}000 \times 20 \times 12}{8} = 450{,}000 \text{ in.-lb.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A beam has a span of 20 feet, and is loaded with a uniformly distributed load of 2,500 pounds per lineal foot; what is the greatest bending moment in inch-pounds on the beam? Ans. 1,500,000 in.-lb.

2. What is the bending moment, in foot-pounds, on a cantilever beam securely fastened into a wall, extending from the point of support 10 feet and loaded with a uniformly distributed load of 1,000 pounds per lineal foot? Ans. 50,000 ft.-lb.

3. What is the bending moment, in inch-pounds, on a girder having a span of 30 feet, if there is a uniformly distributed load of 1,500 pounds per lineal foot, and a load of 20,000 pounds concentrated at the center? Ans. 3,825,000 in.-lb.

4. A plate girder in a building is required to support a uniformly distributed load of 2,000 pounds per lineal foot, extending 20 feet each side of the center of the girder; in addition, it is required to support a load of 30,600 pounds, concentrated 10 feet from one end of the girder, and another load of 43,000 pounds, located 22 feet from the same end. What will be the greatest bending moment on the girder, in foot-pounds, if the span is 60 feet? Ans. 1,528,933 ft.-lb

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STRENGTH OF BEAMS

78. Moment of Inertia and Section Modulus. It has already been explained that the external bending moment at any section is equal to the resisting moment of the fibers of the beam at that section. It now remains, however, to express the resisting moment in terms of the stress produced in the beam. Let the resisting moment be measured in *inch-pounds* and designated by the letter M. Then,

$$M = \frac{SI}{c},$$

where S is the unit stress in a fiber (tension or compression per square inch, as the case may be) in the section, at a vertical distance up or down, equal to c inches from the neutral axis, and I is the moment of inertia of the section about the neutral axis. Now, moment of inertia as here used is simply the name for a certain constant depending on the shape of the cross-section of a beam. It is not a moment as is generally understood and it might have been called any thing so far as the structural engineer is concerned, but it has received its name due to some relations in higher mathematics where it is derived. This quantity must be considered simply as a constant that has been found suitable

to use in the equation. The proof of this formula is rather complicated, so it has been omitted here and the result only is given.

The moment of inertia for the section of a rectangular beam about its neutral axis is $\frac{b\,d^3}{12}$, where b is the breadth, in inches, and d is the depth, in inches. If this value, namely, $\frac{b\,d^3}{12}$, is substituted in the foregoing equation, when the external bending moment at any section is known, the unit stress in various parts of the beam at that section can be found at any desired distance from the neutral axis.

Now, the unit stress at a given section in a beam due to bending is greater the farther it is away from the neutral axis, as explained in Art. **68.** Therefore, the greatest unit stress in the section under consideration is at the point farthest from the neutral axis, which, in the case of a rectangular beam, is the top and bottom surface. Then, if S is the maximum stress, c must be equal to $\frac{d}{2}$, and we have for a rectangular beam

$$M = \frac{S\,I}{c} = \frac{S = \frac{b\,d^3}{12}}{\frac{d}{2}} = S\frac{b\,d^2}{6}$$

If the beam is square, the equation further reduces to

$$M = \frac{S\,d^3}{6}$$

Now, it can, therefore, be seen that the bending moment at a certain section, due to external loads, equals the maximum fiber stress produced in this section times a constant depending on the shape of the cross-section of the beam. This constant is $\frac{I}{c}$, where c is the distance from the neutral axis to the most extreme fiber. This constant is called the **section modulus.** In the text, the section modulus will be designated by the letter K. As we have seen, if the section is rectangular, the section modulus is equal to $\frac{b\,d^2}{6}$.

Now, beams have been carefully loaded and broken by various experimenters, who all found that when a beam broke at a certain section, the unit stress in the fibers at that section due to the external bending moment calculated by the formula was not exactly the same as the ultimate unit compressive or tensile stress of the material. This was due to a slight inaccuracy in some of the assumptions made in deriving the formula. The values they actually did find for the unit stress called out when the beam would break by bending was the moduli of rupture as given in Tables V and VI.

79. Radius of Gyration.—Table VIII gives values for the moment of inertia and section modulus of various common sections. This table would hardly be complete unless it gave values for the **radius of gyration** also. The radius of gyration is the square root of the quotient obtained by dividing the moment of inertia of a section by the area of the section. The least radius of gyration is obtained by using an axis about which the moment of inertia is least. The results are given in Table VIII. It is not necessary to understand how the radius of gyration is obtained, although it may be very simple. Take, for instance, the circular section given in the third row of figures: $R = \sqrt{\frac{I}{A}}$; $I = \frac{A D^2}{16}$. Therefore,

$$R = \sqrt{\frac{\frac{A D^2}{16}}{A}} = \sqrt{\frac{D^2}{16}} = \frac{D}{4}$$

which is the value given. The radius of gyration is not of much use in the study of beams, but is of importance in the study of columns and will be discussed in another Section.

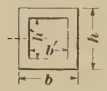
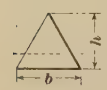
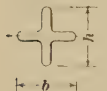
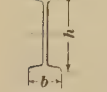
EXAMPLE 1.—(*a*) What will be the section modulus of a yellow-pine beam 10 inches wide by 12 inches in depth? (*b*) What will be the ultimate resisting moment of this beam?

SOLUTION.—(*a*) Here $d = 12$ in., $b = 10$ in. Therefore, by applying the formula, we have

$$K = \frac{b d^2}{6} = \frac{10 \times 12^2}{6} = \frac{1{,}440}{6} = 240. \quad \text{Ans.}$$

TABLE VIII

ELEMENTS OF USUAL SECTIONS

Section	Moment of Inertia I	Section Modulus K	Distance of Base From Center of Gravity	Square of Least Radius of Gyration R^2	Least Radius of Gyration R
	$\frac{b\,h^3}{12}$	$\frac{b\,h^2}{6}$	$\frac{h}{2}$	$\frac{(Least\ side)^2}{12}$	$\frac{Least\ side}{3.46}$
	$\frac{b\,h^3 - b'\,h'^3}{12}$	$\frac{b\,h^3 - b'\,h'^3}{6h}$	$\frac{h}{2}$	$\frac{h^2 + h'^2}{12}$ *	$\frac{h + h'}{4.89}$ *
	$\frac{A\,D^2}{16}$	$\frac{A\,D}{8}$	$\frac{D}{2}$	$\frac{D^2}{16}$	$\frac{D}{4}$
	$\frac{A\,D^2 - a\,d^2}{16}$	$\frac{A\,D^2 - a\,d^2}{8\,D}$	$\frac{D}{2}$	$\frac{D^2 + d^2}{16}$	$\frac{D + d}{5.64}$ *
	$\frac{b\,h^3}{36}$	$\frac{b\,h^2}{24}$	$\frac{h}{3}$	The smaller: $\frac{h^2}{18}$ or $\frac{b^2}{24}$	The smaller: $\frac{h}{4.24}$ or $\frac{b}{4.9}$
	$\frac{A\,h^2}{10.2}$	$\frac{A\,h}{7.2}$	$\frac{h}{3.3}$	$\frac{b^2}{25}$	$\frac{b}{5}$
	$\frac{A\,h^2}{9.5}$	$\frac{A\,h}{6.5}$	$\frac{h}{3.5}$	$\frac{(h\,b)^2}{13\,(h^2 + b^2)}$	$\frac{h\,b}{2.6\,(h + b)}$
	$\frac{A\,h^2}{19}$	$\frac{A\,h}{9.5}$	$\frac{h}{2}$	$\frac{h^2}{22.5}$	$\frac{h}{4.74}$
	$\frac{A\,h^2}{11.1}$	$\frac{A\,h}{8}$	$\frac{h}{3.3}$	$\frac{b^2}{22.5}$	$\frac{b}{4.74}$
	$\frac{A\,h^2}{6.66}$	$\frac{A\,h}{3.33}$	$\frac{h}{2}$	$\frac{b^2}{21}$	$\frac{b}{4.58}$
	$\frac{A\,h^2}{7.34}$	$\frac{A\,h}{3.67}$	$\frac{h}{2}$	$\frac{b^2}{12.5}$	$\frac{b}{3.54}$
	$\frac{A\,h^2}{6.9}$	$\frac{A\,h}{4}$	$\frac{h}{2.3}$	$\frac{b^2}{36.5}$	$\frac{b}{6}$

NOTE.—Moments refer to horizontal axis through center of gravity. This table is intended for convenient application where extreme accuracy is not important. The values for the last seven sections and those marked * are approximate. A = area of section; in case of hollow section, a = area of interior space.

(*b*) According to Table V, the modulus of rupture for yellow pine is 7,000 lb. per sq. in.; hence, the resisting moment of the 10″ × 12″ yellow-pine beam is

$$240 \times 7{,}000 = 1{,}680{,}000 \text{ in.-lb. Ans.}$$

EXAMPLE 2.—What size of spruce girder is required to support a uniformly distributed load of 500 pounds per lineal foot, the span of the girder being 22 feet, and the factor of safety 6?

SOLUTION.—First, find the maximum bending moment, in inch-pounds. The total load on the girder is $500 \times 22 = 11{,}000$ lb. The bending moment due to a uniformly distributed load, according to Table VII, is $\frac{WL}{8}$. Then,

$$\frac{11{,}000 \times 22}{8} = 30{,}250 \text{ ft.-lb.}$$

which multiplied by 12 gives 363,000, the bending moment, in inch-pounds. To find the size of a spruce beam having a safe resisting moment equal to this bending moment, assume a depth of say 18 in. The modulus of rupture, from Table V, is 4,000 lb. and the safe working stress using a factor of safety of 6 is $\frac{4000}{6}$.

$$K = \frac{b\,d^2}{6} = \frac{b \times 18^2}{6}$$

Therefore, $$363{,}000 = \frac{b \times 18^2}{6} \times \frac{4{,}000}{6}$$

Therefore, $b = 10+$ in. Ans.

80. To determine the uniformly distributed load that will break a beam whose size is known, take the formula

$$B = \frac{\left(\frac{K \times S}{12}\right)8}{L} = \frac{2KS}{3L}$$

in which K = section modulus of beam;
S = modulus of rupture of material;
L = span of beam, in feet;
B = breaking load of rectangular beam.

This formula may be stated by the following rule:

Rule.—*To determine the breaking load, in pounds, of any rectangular beam, multiply the sectional modulus of the beam by the modulus of rupture of the material; multiply this product by 2 and divide the result by three times the span of the beam, in feet.*

EXAMPLE.—(*a*) What uniformly distributed load will break a hemlock joist, 3 inches by 14 inches, the span being 25 feet? (*b*) What will be the safe load if a factor of safety of 6 is used?

SOLUTION.—(*a*) Section modulus is

$$\frac{b\,d^2}{6} = \frac{3 \times 14^2}{6} = 98$$

The modulus of rupture for hemlock, according to Table V, is 3,500 lb. per sq. in. Substituting these values in the formula, we have

$$B = \frac{2 \times 98 \times 3{,}500}{3 \times 25} = 9{,}146 \text{ lb.} \quad \text{Ans.}$$

(*b*) If a factor of safety of 6 is used, the safe uniformly distributed load that this beam will carry is

$$9{,}146 \text{ lb.} \div 6 = 1{,}524 \text{ lb.} \quad \text{Ans.}$$

81. Steel Beams.—Steel beams used in building construction are usually either **channels, I beams, angles,** or **tees.** For floorbeams, channels and I beams are principally used, while angles and tees are used in roof trusses and like construction. The strength of channels and I beams only are here to be considered.

In engineering, the two rolled shapes, channels and I beams, are generally called channels and beams. When the word *beam* is used, it is understood that an I beam is meant. For instance a 12-inch, 40-pound beam would mean an I beam 12 inches in depth, weighing 40 pounds to the lineal foot; while a channel of the same size would be expressed as a 12-inch 40-pound channel. In designating rolled shapes on working drawings, various systems of abbreviations are used. A 12-inch 40-pound beam may be expressed as 12″ I 40♯, or a channel as 12″ ⊏ 40♯. This is entirely a matter of judgment with the draftsman, or is governed by the practice followed in the particular drafting room. So long as the size, character, and weight of the beam are given, it matters little how these abbreviations are expressed, if intelligibly written.

82. Strength of Steel Beams.—In calculating the strength of steel beams, it is first necessary to find the bending moment, using the methods and rules already laid down. Then the section modulus required in the beam may be

obtained by dividing the bending moment, in inch-pounds, by the quotient obtained by dividing the modulus of rupture by the factor of safety. Assume, for example, the bending moment on a beam to be 50,000 foot-pounds. Reduce it to inch-pounds by multiplying it by 12, which gives 600,000 inch-pounds. The modulus of rupture for structural steel is 60,000 pounds. If a factor of safety of 4 is used, the safe working value of this material will be $60{,}000 \div 4 = 15{,}000$ per square inch. Then $600{,}000 \div 15{,}000 = 40$, the section modulus required.

83. Tables IX and X give the principal elements of I beams and channels, and obviate the necessity of calculating the section modulus, this being given in the column headed Section Modulus on Axis AB.

The moment of inertia on the neutral axis AB is also given in these tables. The section modulus of any beam may be obtained by dividing the moment of inertia of the beam section by one-half the depth of the beam, in inches. In Table IX, for instance, Elements of Beams, a 15-inch 60-pound beam has a moment of inertia of 538.6. If this is divided by one-half the depth of the beam, in inches, in this case $7\frac{1}{2}$, the result obtained will be 71.8, which, as may be seen by referring to the column headed Section Modulus on Axis AB, is the correct section modulus of this beam.

84. Calculating Dimensions of Steel Beams.—To illustrate the method of calculating the dimensions of a steel beam, let it be required to find what size of steel I beams is necessary to support the floor of an office building, this floor resting on brick arches sprung between the beams and weighing complete 110 pounds per square foot. The building is designed to carry a live load of 40 pounds per square foot. The span of the beams is 20 feet, spaced 5 feet on centers. The owner requires that the building have a large factor of safety, and suggests that for the floorbeams a safety factor of 5 be used. The total dead and live load on the floor is $110 + 40 = 150$ pounds per square foot. The floor area supported by one beam is $20 \times 5 = 100$

TABLE IX

ELEMENTS OF BEAMS

Size Inches	Area Square Inches	Weight per Foot Pounds	Moments of Inertia on Axis *A B*	Section Modulus on Axis *A B*
3	1.63	5.50	2.5	1.7
3	1.91	6.50	2.7	1.8
3	2.21	7.50	2.9	1.9
4	2.21	7.50	6.0	3.0
4	2.50	8.50	6.4	3.2
4	2.79	9.50	6.7	3.4
4	3.09	10.50	7.1	3.6
5	2.87	9.75	12.1	4.8
5	3.60	12.25	13.6	5.4
5	4.34	14.75	15.1	6.1
6	3 61	12.25	21.8	7.3
6	4.34	14.75	24.0	8.0
6	5.07	17.25	26.2	8.7
7	4.42	15.00	36.2	10.4
7	5.15	17.50	39.2	11.2
7	5.88	20.00	42.2	12.1
8	5.33	18.00	56.9	14.2
8	5.96	20.25	60.2	15.0
8	6.69	22.75	64.1	16.0
8	7.43	25.25	68.0	17.0
9	6.31	21.00	84.9	18.9
9	7.35	25.00	91 5	20.4
9	8.82	30.00	10)	22.6
9	10.29	35.00	111.8	24.8
10	7.37	25.00	122.1	24.4
10	8.82	30.00	134.2	26.8
10	10.29	35.00	146.4	29.3
10	11.76	40.00	158.7	31.7
12	9.26	31.50	215.8	36.0
12	10.29	35.00	228.3	38.0
12	11.76	40.00	245.9	41.0
15	12.48	42.00	441.8	58.9
15	13.24	45.00	455.8	60.8
15	14.71	50.00	483.4	64.5
15	16.18	55.00	511.0	68.1
15	17.65	60.00	538.6	71.8
18	15.93	55.00	795.6	88.4
18	17.65	60.00	841.8	93.5
18	19.12	65.00	881.5	97.9
18	20.59	70.00	921.2	102.4
20	19.08	65.00	1,169.5	117.0
20	20.59	70.00	1,219.8	122.0
20	22.06	75.00	1,268.8	126.9
24	23.32	80.00	2,087 2	173.9
24	25.00	85.00	2,167.8	180.7
24	26.47	90.00	2,238.4	186.5
24	27.94	95.00	2,309.0	192.4
24	29.41	100.00	2,379.6	198.3

TABLE X

ELEMENTS OF CHANNELS

Size Inches	Area Square Inches	Weight per Foot Pounds	Moments of Inertia on Axis *A B*	Section Modulus on Axis *A B*
3	1.19	4.00	1.6	1.1
3	1.47	5.00	1.8	1.2
3	1.76	6.00	2.1	1.4
4	1.55	5.25	3.8	1.9
4	1.84	6.25	4.2	2.1
4	2.13	7.25	4.6	2.3
5	1.95	6.50	7.4	3.0
5	2.65	9.00	8.9	3.5
5	3.38	11.50	10.4	4.2
6	2.38	8.00	13.0	4.3
6	3.09	10.50	15.1	5.0
6	3.82	13.00	17.3	5.8
6	4.56	15.50	19.5	6.5
7	2.85	9.75	21.1	6.0
7	3.60	12.25	24.2	6.9
7	4.34	14.75	27.2	7.8
7	5.07	17.25	30.2	8.6
7	5.81	19.75	33.2	9.5
8	3.35	11.25	32.3	8.1
8	4.04	13.75	36.0	9.0
8	4.78	16.25	39.9	10.0
8	5.51	18.75	43.8	11.0
8	6.25	21.25	47.8	11.9
9	3.89	13.25	47.3	10.5
9	4.41	15.00	50.9	11.3
9	5.88	20.00	60.8	13.5
9	7.35	25.00	70.7	15.7
10	4.46	15.00	66.9	13.4
10	5.88	20.00	78.7	15.7
10	7.35	25.00	71.0	18.2
10	8.82	30.00	103.2	20.6
10	10.29	35.00	115.5	23.1
12	6.03	20.50	128.1	21.4
12	7.35	25.00	144.0	24.0
12	8.82	30.00	161.6	26.9
12	10.29	35.00	179.3	29.9
12	11.76	40.00	196.9	32.8
15	9.90	33.00	312.6	41.7
15	10.29	35.00	319.9	42.7
15	11.76	40.00	347.5	46.3
15	13.24	45.00	375.1	50.0
15	14.71	50.00	402.7	53.7
15	16.18	55.00	430.2	57.4

square feet. Then the total load on one beam is $100 \times 150 = 15{,}000$ pounds. The load being uniformly distributed, the formula for the bending moment, according to Table VII, is $M = \frac{WL}{8}$; substituting the values for W and L,

$$M = \frac{15{,}000 \times 20}{8} = 37{,}500$$

the bending moment, in foot-pounds, which, being multiplied by 12, gives 450,000 inch-pounds.

The modulus of rupture for structural steel being 60,000 pounds per square inch, and, since a factor of safety of 5 is required, the safe working value will be $60{,}000 \div 5$

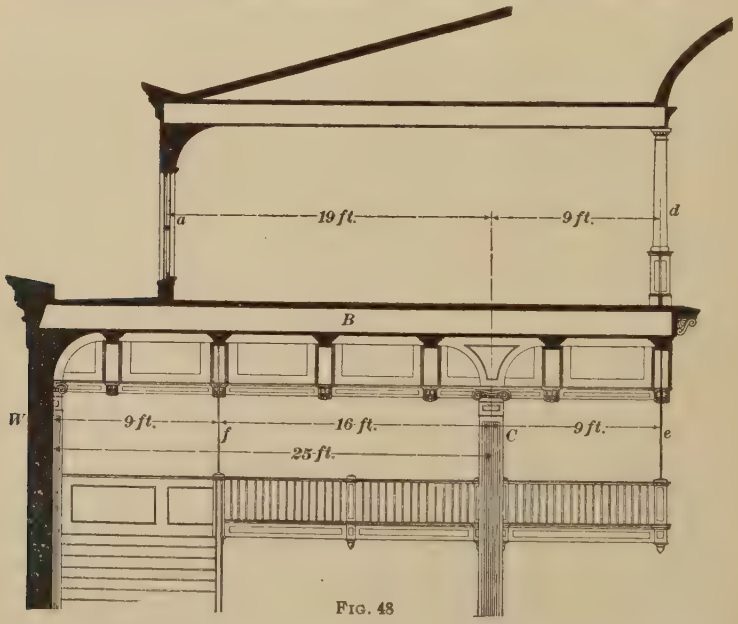


FIG. 48

$= 12{,}000$ pounds per square inch. The bending moment, in inch-pounds, is 450,000, which divided by 12,000 gives a section modulus of 37.5.

Referring to Table IX, it is seen that the section modulus on the axis $A\,B$ of a 12-inch 40-pound beam is 41, which is ample, for a modulus of 37.5 only is required.

Referring, also, to the 15-inch beam in the same table, it is seen that the section modulus of a 15-inch beam, 42 pounds, is 58.9, and, as it weighs only 2 pounds more per foot, and therefore costs but little more, it might be advisable to use this beam, especially as the owner requires a large factor of strength.

In selecting beams from the table, care should be taken to obtain the deepest beams of the least weight with the required section modulus. Thus, by referring to Table IX, it is seen that the section modulus of a 10-inch beam, 30 pounds, is 26.8, while a 12-inch beam of 31.5 pounds, nearly the same weight as the 10-inch beam, has a modulus of 36, and, in consequence, possesses nearly one-third more strength, making it, therefore, the more economical beam to use.

85. By way of general review of the subject of beams, we present the following practical example: Fig. 48 shows the transverse sectional elevation of a large department store. It will require two I beams to form the girder *B*. What will be the size and weight of these steel beams?

Before commencing the calculations, draw the outline diagram as shown in Fig. 49. These are called frame diagrams.

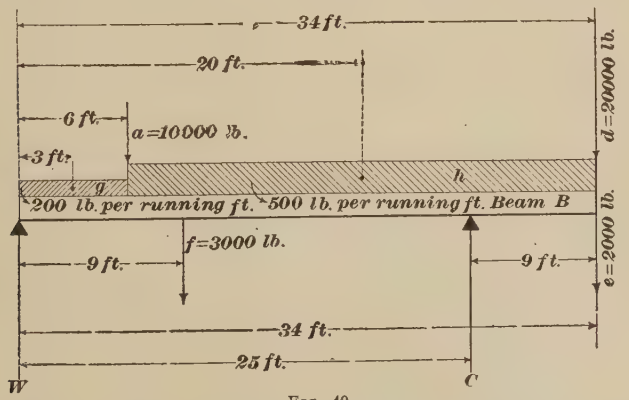


FIG. 49

The two supports for the girder are the wall *W* and the column *C*. The loads on the girder are the two uniform loads *g* and *h*. The load *h* is due to the weight of the floor,

girder, and the ceiling, together with the live load on the floor, due to the people, furniture, etc. This load has been assumed to amount to 500 pounds per running foot of the girder. The load g being due only to the ceiling and a portion of the roof, and there being no floor load on it, has been considered as amounting to 200 pounds per running foot.

The girder is also loaded with four concentrated loads: a of 10,000 pounds, due to the weight of the light wall and a portion of the roof; d of 20,000 pounds, due to the load coming down the small column from a portion of the roof; and two hanging loads f and e, of 3,000 and 2,000 pounds, respectively, from the weight of the stair landing or hall.

Let us now calculate the reactions. The moments about W, due to the various loads, are as follows:

		Foot-Pounds
Load g (200 × 6 = 1,200 lb.)	1,200 × 3 =	3600
Load h (500 × 28 = 14,000 lb.)	14,000 × 20 =	280000
Load a	10,000 × 6 =	60000
Load f	3,000 × 9 =	27000
Load d	20,000 × 34 =	680000
Load e	2,000 × 34 =	68000
Total		1118600

This, divided by the distance between the supports, or the span, 25 feet, gives 44,744, the load, in pounds, coming on the column C; or, in other words, the reaction at C. The loads, in pounds, are as follows:

Load g =	1200
Load h =	14000
Load a =	10000
Load f =	3000
Load d =	20000
Load e =	2000
Total load =	50200

Then, the reaction at W is 50,200 − 44,744 = 5,456 pounds.

Next, find the point between the two supports W and C, where the shear changes sign. Starting from W, the first

load encountered and to be deducted from the reaction W is the uniform load g, equal to $200 \times 6 = 1{,}200$ pounds. Then, 5,456 (reaction at W) $-$ 1,200 (load g) = 4,256 pounds. The next load on the beam is the concentrated load a of 10,000 pounds, which is much more than the remaining portion of the reaction W. The greatest bending moment occurring between the columns and the wall is therefore at the point a, and is equal to 5,456 (reaction at W) $\times$ 6 = 32,736 foot-pounds, less the moment of the load g of $1{,}200 \times 3$ = 3,600 foot-pounds, or 32,736 $-$ 3,600 = 29,136 foot-pounds.

Again referring to the diagram, Fig. 49, it is seen that there is a large bending moment directly over the column C, due to the two concentrated loads d and e on the end of the beam and the portion of the uniform load h overhanging the support C. It may be seen by calculation that the shear also changes sign at this section and it must therefore be considered also. This portion of the beam may be considered as a cantilever; the bending moment at C is equal to the sum of the moments of all the loads on the overhanging portion of the beam, which are:

Load d, $20{,}000 \times 9 = 180{,}000$ foot-pounds.

Load e, $2{,}000 \times 9 = 18{,}000$ foot-pounds.

Load h (overhanging portion = $500 \times 9 = 4{,}500$ pounds) = $4{,}500 \times 4.5 = 20{,}250$ foot-pounds.

Total: 218,250 foot-pounds, or $218{,}250 \times 12 = 2{,}619{,}000$ inch-pounds.

Since this bending moment is greater than that under the load a, it is used in determining the size of the beam. This bending moment divided by 20,000, the safe working value of structural steel (using the modulus of rupture of 60,000 pounds $\div$ 3, the safety factor used in this case), gives 131, the required section modulus in the two beams. Then, the section modulus required in one of the beams, since two are to be used side by side, is $131 \div 2 = 65.5$.

Referring to Table IX, it is seen that the section modulus of a 15-inch beam, 55 pounds, is 68.1. While this is in excess of the required amount, it is, in this case, the most economical beam to use, and two of this kind are required.

86. Relation Between Span and Depth of Beam. In order to select beams that will not deflect too much under the load they are required to sustain, the depth of the beam, in inches, should never be less than one-half the span of the beam, in feet. Thus, if the span of the beam is 20 feet, a beam not less than 10 inches in depth should be used to avoid excessive deflection, because although a shallower beam might carry safely the required load yet it would sag so much that it would crack the plaster on the ceiling underneath.

87. Separators for I Beams.—In building construction, it frequently happens that a single I beam is insufficient to carry the imposed load. Where heavy loads, such as brick walls, vaults, etc., are to be supported, a single I beam is inadequate, and two or more beams are applied side by side, bolted together with cast-iron or steel **separators,** as shown in Fig. 50. These separators hold the compression flanges

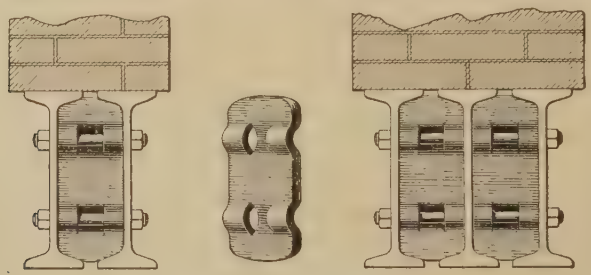


FIG. 50

of the beams in position, preventing deflection sidewise, and also, in a measure, cause the beams to act together, distributing the load uniformly on both. Separators should be spaced from 6 to 7 feet throughout the length of the beam; they should also be provided at the supports and at points where heavy loads are concentrated.

88. Beam Girders.—In designing floors of buildings, it is desirable to have a minimum number of interior supporting columns, consistent with economy. A beam girder

consisting of a pair of I beams is frequently advantageous for supporting the steel floorbeams, as shown at *a*, Fig. 51.

Girders composed of two or more I beams are commonly used to span openings in brick walls. If the wall to be supported is thoroughly seasoned and without openings, the weight carried by the girder can safely be assumed as the weight of a triangular piece of brickwork whose altitude is one-third of the span of the girder. If the wall is newly

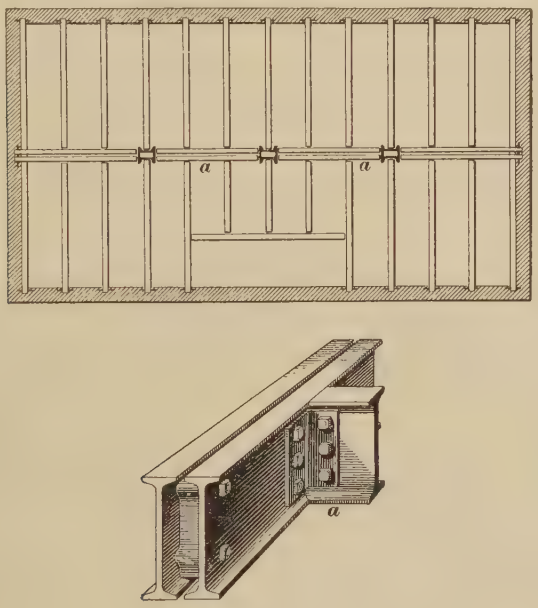


Fig. 51

built, or has openings for windows or other purposes, the girder must be designed to carry the entire wall above the girder between the supports.

Example.—Required, the size of a steel I-beam girder to carry a wall 12 inches thick, made of hard brick laid in lime mortar; there are no openings in the wall above the girder, nor does the wall support floor joists or roof beams, while the span of the opening is 24 feet.

Solution.—Draw a diagram as shown in Fig. 52. The area of the triangular piece of brickwork is $24 \times 4 = 96$ sq. ft. The area of a triangle is equal to one-half of the product of the base and altitude. As

the wall is 1 ft. thick, there are 96 cu. ft. in this triangular piece. The weight of brickwork in lime mortar per cubic foot, according to Table I, is 120 lb. Then, the load on the girder is $96 \times 120 = 11{,}520$ lb.

The bending moment may be determined by the formula or rule given in Table VII, for a beam carrying a triangular load, or it may be determined by calculating the moments, as follows: The reactions at the two supports are each equal to one-half the load, or $11{,}520 \div 2 = 5{,}760$ lb. The greatest bending moment is at the center of the beam. Then, the moment of the reaction about the point x is $5{,}760 \times 12 = 69{,}120$ ft.-lb. But counterbalancing this, and to be deducted from it, is the moment of the load at the left of x, equal to one-half of the triangular piece of brickwork. The moment of this load about the point x is equal to the product of its weight multiplied

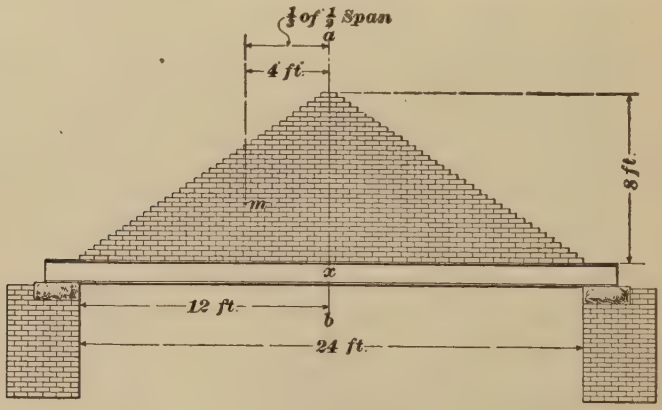
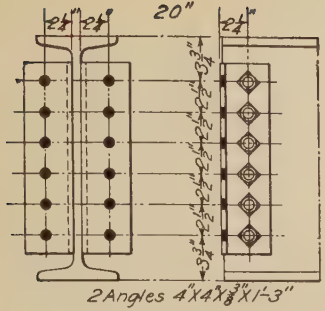
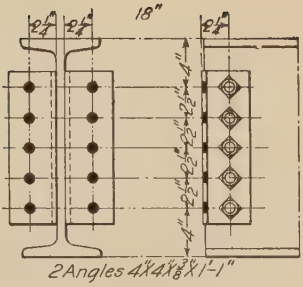
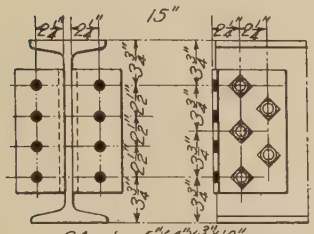


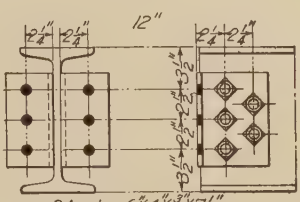
FIG. 52

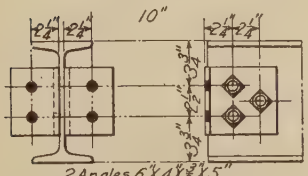
by the horizontal distance from a vertical line through its center of gravity to the point x. Take the line $a\,b$ as the base of a triangle, remembering that a line drawn parallel to the base line of a triangle, at a distance of one-third of the altitude from it, always passes through its center of gravity. Now, the distance from the point x to the vertical line through the center of gravity m of the triangle is 4 ft., and the moment due to the triangular piece of brickwork to the left of the center is $5{,}760 \times 4 = 23{,}040$ ft.-lb. Deducting this from the moment of the reaction already found, the moment at the center is: $69{,}120 - 23{,}040 = 46{,}080$ ft.-lb., the bending moment on this beam or girder; or, $46{,}080 \times 12 = 552{,}960$ in.-lb. This calculation may be checked by applying the formula $\frac{WL}{6}$. The bending moment, in inch-pounds, being 552,960, using a safe unit working value, or fiber stress, of 15,000 lb., the section modulus required is $552{,}960 \div 15{,}000 = 36.8$.

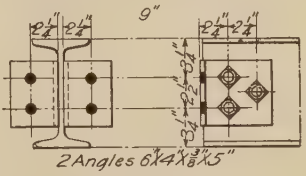


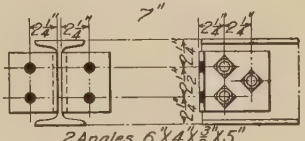


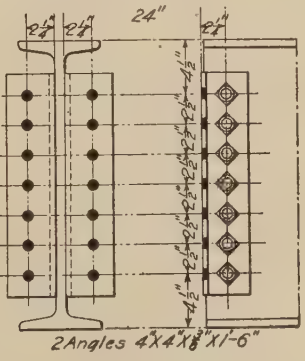


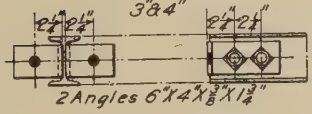












All Rivets and Bolts ¾"

FIG. 53

Referring to Table IX, it is seen that the section modulus of a 12-in. beam, 35 lb., is 38, which gives the required strength in this case. It may be found to be better practice to use two channels instead of one I beam, for the top flange of the I beam may be too narrow to properly support the brick wall, while the two channels placed side by side, with separators between, could be made of the same thickness as the wall.

TABLE XI

MINIMUM SPANS FOR I BEAMS, WITH STANDARD CONNECTIONS, THAT ARE UNIFORMLY LOADED TO REALIZE A UNIT STRESS OF 16,000 POUNDS

Depth of Beam	Weight per Foot	Minimum Safe Span	Depth of Beam	Weight per Foot	Minimum Safe Span	Depth of Beam	Weight per Foot	Minimum Safe Span
Inches	Pounds	Feet	Inches	Pounds	Feet	Inches	Pounds	Feet
3	5.50	1.7	9	30.0	6.8			
3	6.50	1.2	9	35.0	7.5			
3	7.50	1.2	10	25.0	9.3			
4	7.50	2.8	10	30.0	8.1			
4	8.50	2.2	10	35.0	8.8			
4	9.50	2.0	10	40.0	9.6	18	55.0	13.7
4	10.50	2.2	12	31.5	7.3	18	60.0	11.9
5	9.75	4.1	12	35.0	7.7	18	65.0	11.8
5	12.25	3.3	12	40.0	8.2	18	70.0	12.4
5	14.75	3.7				20	65.0	13.9
6	12.25	5.6				20	70.0	12.5
6	14.75	4.8				20	75.0	12.8
6	17.25	5.3	15	42.0	10.2			
7	15.00	4.9	15	45.0	9.4			
7	17.50	3.8	15	50.0	9.7			
7	20.00	3.6	15	55.0	10.3			
8	18.00	6.2	15	60.0	10.8			
8	20.25	5.1				24	80.0	17.7
8	22.75	4.8				24	85.0	16.1
8	25.25	5.1				24	90.0	16.1
9	21.00	7.7				24	95.0	16.6
9	25.00	6.2				24	100.0	17.1

89. Connection Angles.—The standard connection angles for the principal sizes and weights of steel I beams are illustrated in Fig. 53. These connections are based on shearing stresses of 10,000 pounds per square inch, bearing stresses of 20,000 pounds per square inch, and an extreme fiber stress of 16,000 pounds. The connections are properly designed for beams whose spans are not less than those given in Table XI.

The connections in Fig. 53 are designed for $\frac{13}{16}$-inch holes and $\frac{3}{4}$-inch diameter rivets or bolts. Connection angles may, if so specified, be riveted instead of bolted to the beams; but, unless otherwise ordered, bolted connections are generally used.

These connections are so proportioned as to cover most cases occurring in ordinary practice; but where beams have short spans and are loaded to their full capacity, connections having a greater number of bolts than used in the standard connections may be found necessary.

90. Stone Beams.—The strength of stone beams, or lintels, in which form stone beams are usually found, may be calculated in the same manner as rectangular beams of any material, except that it is usual to use a factor of safety of 10. First, compute the bending moment on the beam, in inch-pounds; then, estimate the resisting moment of the beam, using the modulus of rupture given in Table XII, which, however, should be divided by a safety factor of at least 10.

Another method, which may be found more convenient, is to use the formula

$$W = \frac{b\,d^2}{l} \times c,$$

in which W = safe uniformly distributed load, in tons of 2,000 pounds;

b = breadth of beam, in inches;

d = depth of beam, in inches;

l = span of beam, in inches;

c = a coefficient taken from the following table:

	Coefficient c		Coefficient c
Bluestone	.125	Sandstone	.060
Granite	.100	Slate	.275
Limestone	.083		

The formula may be expressed as follows:

Rule.—*To obtain the safe uniformly distributed load, in tons, that a stone lintel will support, square the depth of the beam, in inches, and multiply by the breadth, in inches. Divide this product by the span of the lintel, in inches; then, multiply*

TABLE XII

MODULI OF RUPTURE OF VARIOUS STONES, ETC.

Material	Modulus of Rupture Pounds
Slate	5,000
Glass	3,000
Bluestone	2,250
Marble, white Italian	2,090
Marble, white Vermont	1,850
Marble, gray Vermont	1,260
Granite, Quincy	1,800
Freestone, Little Falls	1,730
Freestone, Belleville, N. J.	1,440
Freestone, Conn.	936
Freestone, Dorchester	792
Freestone, Hubeginy	648
Freestone, Caen	450
Limestone	1,500
Sandstone	1,080
Brick, American pressed	720
Brick, common	360
Concrete, Portland cement 1-2-4 mixture, 1 year old	590
Concrete, Portland cement, 1-2-4 mixture, 1 month old	540

this last result by the coefficient given in the table for the particular stone used.

EXAMPLE.—A limestone lintel is 20 inches wide and 14 inches thick, spanning an opening of 42 inches; what distributed load will this beam safely carry?

SOLUTION.—The safe distributed load, in tons, is $\frac{b\,d^2}{l} \times$ coefficient.

Then, $$W = \frac{20 \times 14 \times 14}{42} \times .083 = 7.747 \text{ T.}$$

or . $$7.747 \times 2{,}000 = 15{,}494 \text{ lb.}$$ Ans.

If the load is concentrated at the center of the span, the safe load will be one-half the safe uniform load given by the rule.

EXAMPLES FOR PRACTICE

1. What is the section modulus of a rectangular section 10 inches wide by 10 inches deep? Ans. $166\frac{2}{3}$

2. Calculate the section modulus of a 10-inch I beam, the sectional area of which is 11.8 square inches. Ans. 36.87

3. What must be the width of a yellow-pine beam to support a uniformly distributed load of 10,380 pounds, the span of the beam being 18 feet, its depth 12 inches, and the safety factor 5? Ans. 8 in.

4. A uniformly distributed load on a steel I beam having a span of 24 feet is 25,000 pounds; what size of beam will be the most economical to use, the allowable unit stress on the material being 18,000 pounds? Ans. 15 in., 42 lb.

5. It is desired to span an opening 20 feet wide in an 18-inch wall, laid in lime mortar, with two steel channels to support a solid brick wall over the opening. What will be the size and weight of the channels if a safe fiber stress of 15,000 pounds is adopted?
Ans. 12 in., 20.5 lb.

6. A builder wishes to determine which will be the cheaper: to span a 20-foot opening in a 12-inch brick wall laid in cement mortar with two steel channels, or with yellow-pine timber; the steel beams being worth $1\frac{3}{4}$ cents per pound and the yellow-pine timber being worth $25 per M. The load to be supported is the solid brick wall above; a factor of safety of 4 will be required if the steel channels are used; if the wooden beam is used, a factor of safety of 6 will be adopted. If the builder adopts the cheaper, which will be used?
Ans. Yellow-pine timber

7. What safe uniformly distributed load will a granite lintel, 16 inches deep and 24 inches wide, sustain, its span being 6 feet?
Ans. 8.53 T.

8. A sandstone beam is required to support a uniformly distributed load of 4 tons; it is necessary that it should be 18 inches wide; what should be the depth of this beam, if the span is 8 feet?

Ans. About 19 in.

9. A heavy bluestone flag, 6 inches thick and 5 feet wide, spans a culvert that is 7 feet wide; what safe uniformly distributed load will this flag support? Ans. 3.2 T.

DEFLECTION OF BEAMS

91. Deflection is the name applied to the distortion or bending produced in a beam when subjected to transverse stresses. If, on the removal of the transverse stresses or loads on the beam, it returns to the straight or original form, the material in the beam has not been strained beyond the elastic limit. On the other hand, if the internal stresses exceed the elastic limit of the material, a permanent set will be given the beam.

92. Stiffness is a measure of the ability of a body to resist bending; this property is very different from the strength of the material or its power to resist rupture.

The stiffness of a structure does not depend so much on the elasticity of the material of which it is composed as on its arrangement and form; for example, a floor may be built of shallow and wide joists that will be sufficiently strong to carry a given load, but it will not be nearly so stiff as a floor of equal strength built of narrow and deep ones. This property of stiffness is as important in building construction as mere strength, and the two should be considered together; thus, the floor joists of a building may be strong enough to resist breaking, but so shallow as to lack stiffness, in which case the floor will be springy and vibrate under the footsteps of people walking on it. If there is a plastered ceiling on the under side of the joists of such a floor, the deflection of the joists may cause the plaster to crack and fall into the room below. Where stiffness is lacking in the rafters of a roof, they will be liable to sag, thereby causing unsightly hollows in the surface of the roof, in which moisture and

snow may lodge, which will be very detrimental to the roof covering.

93. From the foregoing, it is evident that not only must the strength of the beams composing a structure be calculated to withstand rupture, but the beams must be stiff, or rigid, enough to resist bending. It is therefore important to be able to calculate the deflection of any beam under its load, and if found excessive, the size of the beam may be increased and the deflection reduced to practical limits.

The amount of deflection that exists in beams loaded and supported in different ways may be calculated by the formulas given in Table XIII. In using these formulas, all the loads should be expressed in pounds and the length in inches. The modulus of elasticity is denoted by E, and the moment of inertia of the section by I.

EXAMPLE 1.—A 10-inch special steel **I** beam, supported at the ends, must support a uniformly distributed load of 10,000 pounds. The span of the beam is 20 feet, and its moment of inertia is 178; there is to be a plastered ceiling on its under side, the allowable deflection of which is $\frac{1}{360}$ inch for each foot of span. Will the deflection of the beam be excessive?

SOLUTION.—The formula for the deflection of a beam of this character from Table XIII is $D = \frac{5\,WL^3}{384\,EI}$. From Table VI, the modulus of elasticity of structural steel is found to be 29,000,000, and this, with the values given in the example, substituted in the formula, gives the deflection

$$D = \frac{5 \times 10{,}000 \times 240^3}{384 \times 29{,}000{,}000 \times 178} = .34$$

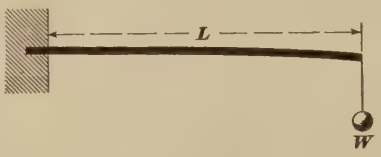
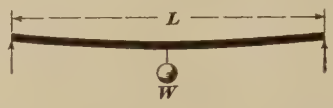
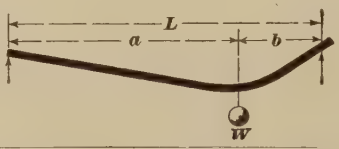
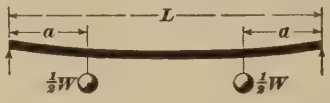
about $\frac{1}{3}$ in. Since the allowable deflection for each foot of span is $\frac{1}{360}$ in., the total allowable deflection is one-thirtieth of 20 or $\frac{2}{3}$ in. This is twice as much as the calculated deflection, and the beam therefore satisfies the required conditions. Ans.

EXAMPLE 2.— A $12'' \times 16''$ Northern short-leaf, yellow-pine girder must support a symmetrically placed triangular piece of brickwork, which weighs about 12,000 pounds; what will be the deflection of the timber if the span is 20 feet?

SOLUTION.—The formula for the deflection in this case from Table XIII is $D = \frac{WL^3}{60\,EI}$. From Table V, the value of the modulus

TABLE XIII

DEFLECTION OF BEAMS

Description	Mode of Loading Lengths, in Inches — Loads, in Pounds	Greatest Deflection, in Inches
One end firmly fixed, the other end loaded.		$\frac{W L^3}{3 E I}$
Supported at both ends, loaded at the center.		$\frac{W L^3}{48 E I}$
Supported at both ends, loaded any place.		$\frac{W a b \sqrt{3 a (2 L - a)^3}}{27 L E I}$
Both ends supported, two symmetrical loads.		$\frac{W a}{48 E I}(3 L^2 - 4 a^2)$
One end fixed, load uniformly distributed.		$\frac{W L^3}{8 E I}$
Both ends supported, load uniformly distributed.		$\frac{5 W L^3}{384 E I}$
Both ends supported, load distributed, increasing uniformly toward the center.		$\frac{W L^3}{60 E I}$

of elasticity is found to be 1,200,000. The moment of inertia of the section, from the formula $I = \frac{b\,d^3}{12}$, is

$$I = \frac{12 \times 16^3}{12} = 4{,}096$$

Then, by substituting the given values, the deflection is

$$D = \frac{12{,}000 \times 240^3}{60 \times 1{,}200{,}000 \times 4{,}096} = .56, \text{ about } \tfrac{9}{16} \text{ in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A 12-inch 35-pound steel I beam is supported at the ends over a span of 25 feet; what will be its deflection under a concentrated load of 10,000 pounds suspended from its center? Ans. .85 in.

2. A cantilever beam of $12'' \times 16''$ Georgia yellow pine extends from a building wall 10 feet, and is loaded on the end with a concentrated load of 18,000 pounds; what will be the greatest deflection of the beam? Ans. 1.49 in.

FLITCH-PLATE GIRDERS

94. The name **flitch-plate girder** is applied to a beam composed of two timbers bolted together with a flat plate of iron or steel sandwiched between them; hence, it is often called a **sandwiched girder.** This girder is used where it is desired to impose a greater weight than the wooden beams will support.

The great advantage a beam of this character possesses over a steel rolled section probably lies in the fact that it is partially fireproof; the wood may be charred considerably without losing all its strength, and it will protect the iron or steel plate from the intensity of the heat, thus acting as a non-conductor and fireproofing material for the metal.

It is difficult to so proportion a girder of this character as to be economical in the use of both the wood and the metal. Theoretically, the iron or steel plate should be so proportioned that, when carrying its share of the load, it deflects equally with the wooden beams, otherwise there is a tendency for the bolts that secure the parts together either to shear off or to crush and tear the wood.

The two timbers should be considered as a single beam, and the calculation made for the safe load they will sustain.

The plate will then be proportioned to support the remainder of the load coming on the girder. The timber beams and the plate should be so proportioned that they will both deflect equally under their respective loads. On account of the wood and steel being bolted so that they must deflect together, it may be proved that the stress due to the transverse load in the wood and steel are in the ratio of the moduli of elasticity of these two materials. Therefore, the stress in either material may be assumed, and the stress that will be produced in the other found, by the ratio stated. If this stress is above the safe allowable stress, both stresses must be proportionally reduced. The design of such a girder will perhaps be better understood by the following examples.

EXAMPLE 1.—If a flitch-plate girder 12 inches in depth is required to resist a bending moment of 300,000 inch-pounds, what thickness of spruce planks will be required, the steel plate being $\frac{1}{2}$ inch thick? The modulus of elasticity for structural steel is 29,000,000 and for spruce 1,200,000, while the safe unit stress for steel may be taken at 15,000 pounds.

SOLUTION.—The ratio of the moduli of elasticity for the two materials is $\frac{1200000}{29000000} = \frac{6}{145}$. Then, the safe unit stress that may be employed for spruce is $\frac{6}{145} \times 15{,}000 = 620$ lb. per sq. in., which is below the safe allowable stress for spruce and is therefore all right. The resisting moment exerted by the steel plate is

$$15{,}000 \times \frac{b\,d^2}{6} = 15{,}000 \times \frac{1 \times 12 \times 12}{2 \times 6} = 180{,}000 \text{ in.-lb.}$$

Now, $300{,}000 - 180{,}000 = 120{,}000$ in.-lb., which must be resisted by the spruce.

$$120{,}000 = 620 \times \frac{b\,d^2}{6} = \frac{620 \times b \times 12 \times 12}{6}$$

since the depth of the spruce will be the same as the depth of the steel. Therefore,

$$b = \frac{120{,}000 \times 6}{620 \times 12 \times 12} = 8.06, \text{ say, } 8 \text{ in.}$$

This is the width of the entire spruce needed. Therefore, the spruce on each side of the steel plate is $8 \div 2 = 4$ in. Ans.

EXAMPLE 2.—Design a flitch-plate girder made of yellow pine and steel, to carry a load of 1,000 pounds per foot on a 20-foot span.

SOLUTION.—The total load to be carried is $1{,}000 \times 20 = 20{,}000$ lb. From Table VII,

$$M = \frac{W\,L}{2} = \frac{20{,}000 \times 20}{2} = 200{,}000 \text{ ft.-lb.} = 2{,}400{,}000 \text{ in.-lb.}$$

Assume a depth of girder of, say, 18 in., a thickness of steel plate of 2 in., and an allowable stress in the steel of 15,000 lb. From Tables V and VI, the stress in the pine will be

$$15{,}000 \times \frac{1{,}700{,}000}{29{,}000{,}000} = 879 \text{ lb.}$$

which is below the safe allowable unit stress for pine and therefore all right. The resisting moment offered by the steel is

$$\frac{15{,}000 \times 2 \times 18 \times 18}{6} = 1{,}620{,}000 \text{ in.-lb.}$$

This leaves to be resisted by the wood $2{,}400{,}000 - 1{,}620{,}000 = 780{,}000$ in.-lb.

$$780{,}000 = 879 \times \frac{b\,d^2}{6} = \frac{879 \times b \times 18 \times 18}{6}$$

Therefore,

$$b = \frac{780{,}000 \times 6}{879 \times 18 \times 18} = 16.4, \text{ say, } 16\tfrac{1}{2} \text{ in.}$$

Therefore, each side will be

$$16\tfrac{1}{2} \div 2 = 8\tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

The bolts holding the timbers and the steel plate together are best located along the neutral axis, for at this point there would practically be no stress on them. But to space them sufficiently near together along this line would tend to weaken the timber too much, and would be likely to cause the destruction of the beam from longitudinal shearing along this line. Therefore, it is best to alternate them above

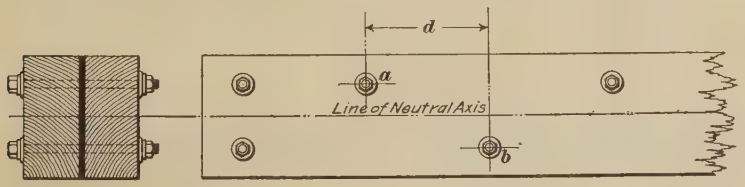


FIG. 54

and below the line of the neutral axis, thus forming two rows of bolts as shown in Fig. 54. The end bolts are doubled, and the horizontal distance d between the bolts a and b should be about equal to the depth of the girder. The bolts in a girder of this character should be about 1 inch in diameter.

TRUSSED BEAMS

95. Camber Rods.—When wooden girders of great span are heavily loaded, it becomes necessary to strengthen them with iron or steel camber rods, as shown in Figs. 55

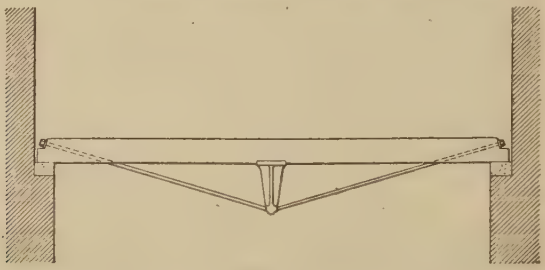


FIG. 55

and 56. Fig. 55 shows a girder with one support in the center, and Fig. 56 shows a girder with two supports. The

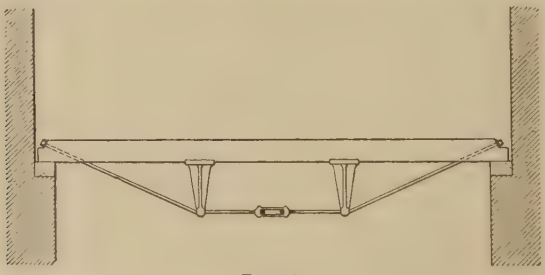


FIG. 56

span of the beam or girder may be considered, in each case, as the distance between the supports, the strength of the girder being thereby materially increased.

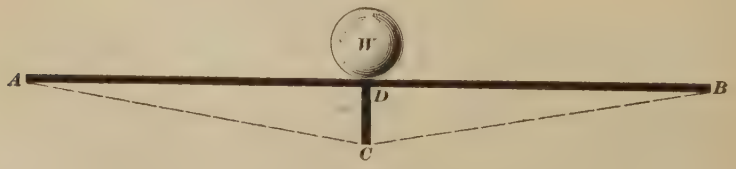


FIG. 57

96. Stresses in a Beam With One Strut.—In Fig. 57, let W represent the load concentrated at D. Then the stress

in the member DC is equal to W. The stress in the other members may be found by applying the following:

Rule.—I. *To find the stress in AC or BC, divide the length of line AC by the length of the line DC, and then multiply this result by one-half of the load W.*

II. *To find the stress in the beam AB, divide the length of the line AD by the length of the line DC, and multiply by one-half of the load W.*

In the diagram, Fig. 57, the members represented by solid lines are in compression, and those shown dotted are in tension.

The length of the members in the rules just stated may be taken in feet or inches, but all lengths should be taken in the same unit of measurement. The rules may be expressed by the formulas:

$$\text{Stress in } DC = +W \qquad (1)$$

$$\text{Stress in } AC \text{ or } BC = -\frac{AC}{DC} \times \frac{W}{2} \qquad (2)$$

$$\text{Stress in } AB = +\frac{AD}{DC} \times \frac{W}{2} \qquad (3)$$

The + and − signs in the formulas indicate compression and tension, respectively. The + sign denotes that the

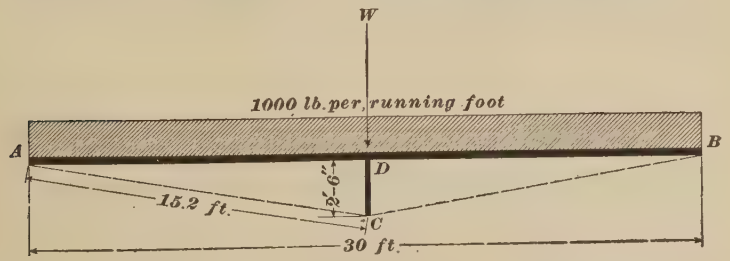


Fig. 58

result obtained is a *compressive stress* and the − sign means that the result is a *tensile stress.*

When a beam with one support at the center has a uniform load, as in Fig. 58, the load W on the center strut to be used in formulas **1** to **3** is about one-half of the entire load.

EXAMPLE.—What is (*a*) the tension in the camber rod, and (*b*) the compression on the trussed beam of the dimensions and loads shown in Fig. 58.

SOLUTION.—(*a*) First, compute the load W coming on the strut $D\,C$. The load is, in this case, usually considered equal to one-half of the entire load on the beam. The entire load on the beam is equal to $30 \times 1{,}000 = 30{,}000$ lb.; one-half of 30,000 is 15,000 lb., the load W acting on the beam directly over the strut $D\,C$. Then, the tension in the camber rod $A\,C$ is equal to the length $A\,C \div D\,C$ multiplied by one-half of W, or, substituting the given dimensions, $15.2 \div 2.5 = 6.08$; and

$$6.08 \times \frac{15{,}000}{2} = 45{,}600 \text{ lb.},$$

the tensile stress on rod $A\,C$. Ans.

(*b*) To determine the direct compressive stress on the beam $A\,B$, divide the length $A\,D$ by $D\,C$ and multiply by one-half of W. Thus, $15 \div 2.5 = 6$;

$$6 \times \frac{15{,}000}{2} = 45{,}000 \text{ lb.},$$

the compressive stress in the beam $A\,B$. Ans.

97. Stresses in Beams With Two Struts.—In Fig. 59, the calculations for the stresses in the various members are

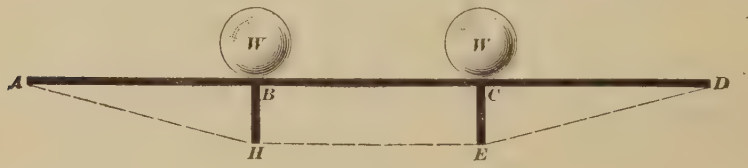


FIG. 59

somewhat similar to those given for the trussed beam with one support. In double trussed beams, the stress in $B\,H$ or $C\,E = W$. The stresses in the other members may be expressed by rule, as follows:

Rule.—I. *To obtain the stress in $A\,H$ or $D\,E$, divide the length of $A\,H$ by the length of $B\,H$, and multiply this result by the load W.*

II. *To find the stress in $A\,D$ or $H\,E$, divide the length of the line $A\,B$ by the length of the line $B\,H$, and multiply this result by the load W.*

The rules may be expressed by the formulas:

$$\text{Stress in } BH \text{ or } CE = +W \qquad (1)$$

$$\text{Stress in } AH \text{ or } DE = -\frac{AH}{BH} \times W \qquad (2)$$

$$\text{Stress in } HE = -\frac{AB}{BH} \times W \qquad (3)$$

$$\text{Stress in } AD = +\frac{AB}{BH} \times W \qquad (4)$$

Compression is, as previously noted, indicated by the + sign and tension by the − sign.

When a beam has two supports one-third of its length from each end, and is uniformly loaded, as in Fig. 60, the load W on each support is about one-third of the total load.

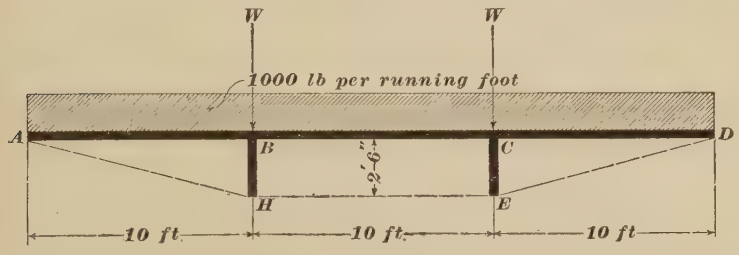


FIG. 60

EXAMPLE.—A beam is trussed, as shown in Fig. 60; what is: (*a*) the stress in camber rod HE? (*b*) the compression in the beam AD?

SOLUTION.—(*a*) The entire load on the beam AD is $30 \times 1{,}000 = 30{,}000$ lb. The loads are each taken as about one-third of the entire load. Hence,

$$W = \tfrac{1}{3} \times 30{,}000 = 10{,}000 \text{ lb.}$$

Applying formula **3**, we have,

$$\text{stress in } HE = \frac{10}{2.5} \times 10{,}000 = 40{,}000 \text{ lb.} \quad \text{Ans.}$$

(*b*) Applying formula **4**, we have,

$$\text{stress in } AD = \frac{10}{2.5} \times 10{,}000 = 40{,}000 \text{ lb.} \quad \text{Ans.}$$

DESIGN OF TRUSSED BEAMS

98. A floor is to be supported by yellow-pine girders, each composed of two $4'' \times 12''$ beams, trussed with a steel rod, as shown in Fig. 61. The span of the girders is 24 feet,

and they are spaced 8 feet from center to center. The load is light, amounting to only 40 pounds per square foot of floor surface. Required, to determine whether the two yellow-pine beams are sufficiently strong and the size of the steel camber rod; also, to design the detail construction for the parts A and B.

The floor area supported by each girder is $24 \times 8 = 192$ square feet; therefore, the total load on a girder is $192 \times 40 = 7{,}680$ pounds. The load carried by the strut may be taken

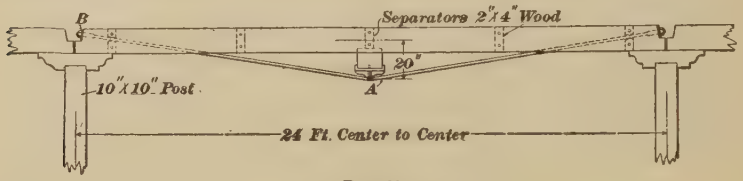


FIG. 61

as one-half of the total load, or $\frac{1}{2} \times 7{,}680 = 3{,}840$ pounds. The tension in the rod is found from formula **2**, Art. **96**,

$$\frac{\sqrt{20^2 \times 144^2}}{20} \times \frac{3{,}840}{2} = 13{,}920 \text{ pounds, about}$$

The compression in the timber from formula **3**, Art. **96**,

$$\frac{144}{20} \times \frac{3{,}840}{2} = 13{,}824 \text{ pounds}$$

From Table V, it will be found that the ultimate resistance to compression of yellow pine per square inch is 5,000 pounds; and as wood is not so reliable as steel, it is considered advisable to use a factor of safety of 5, as against a factor of safety of 4 for the camber rods. Since the trussed girder is secured against lateral deflection by the floor joist, and as it is secured from deflection in a vertical direction at the center and by the camber rod and strut, the length of the wooden girder may be considered under direct compressive stress; that is, it is not necessary to apply a column formula, and its strength may be considered as its resistance to direct compression. Hence, $5{,}000 \div 5 = 1{,}000$ pounds, which is the allowable compression strength of the girder per square inch of section. Then, 13,824 pounds (the compression)

$\div$ 1,000 pounds (the allowable unit stress) = 13.8 square inches required to take care of the compressive stress. As the girder is known to be 12 inches in depth, it is readily seen that this compressive stress will require a section of the timber girder equal to about $1\frac{1}{8}$ in. $\times$ 12 in.

There is, in addition to this, a transverse stress on one-half of the girder produced by the uniformly distributed load. To find the amount of this bending stress, consider the left-hand half of the girder as a simple beam, sustaining a uniformly distributed load equal to one-half of the total load on the girder, that is, a load of 7,680 $\div$ 2 = 3,840 pounds.

Applying the correct formula in Art. **76**, the bending moment due to this load is

$$M' = \frac{3{,}840 \times 12}{8} = 5{,}760 \text{ foot-pounds}$$

and $5{,}760 \times 12 = 69{,}120$ inch-pounds. Then, the section modulus required may be obtained by the formula

$$K = \frac{M}{S}$$

in which K = section modulus;
M = moment resisting bending, in inch-pounds;
S = allowable unit fiber stress of the material, which is equal, in this case, to 7,000 (the modulus of rupture of yellow pine) $\div$ 6 (the factor of safety) = 1,167 pounds.

Substituting the values in this formula, the calculation will be

$$K = \frac{69{,}120}{1{,}167} = 59.2$$

the section modulus required in the girder to successfully resist the transverse stress.

Since the section modulus of a rectangular beam may be obtained by the formula given in Art. **79**, which is $K = \frac{b\,d^2}{6}$, b being the width of the beam in inches, and d the depth, and as K is already known to be 59.2 and the depth of the beam to be 12 inches, the width of the beam required to resist the transverse stress may be obtained by transposing

the formula to $b = \frac{K \times 6}{d^2}$; the values substituted will give

$$b = \frac{59.2 \times 6}{12 \times 12} = 2.47 \text{ inches}$$

which is the width of the required beam. Then, adding the size of the timber required to resist compression and the size of timber required to resist the transverse stress, we have a timber $1\frac{1}{8}$ inches wide by 12 inches deep, added to a timber 2.47 or, say, $2\frac{1}{2}$ in. $\times$ 12 in., which gives a piece $3\frac{5}{8}$ in. $\times$ 12 in. In the girder there are two $4'' \times 12''$ timbers, and as only a single $3\frac{5}{8}'' \times 12''$ timber is required, it is evident the girder is nearly twice as strong as is necessary. It must, however, be borne in mind that the theoretical dimensions of timbers do not always agree with those required in practical rules; for instance, in this case it would not be good practice to make the combined sectional area of the girder equivalent to that of a $3\frac{5}{8}'' \times 12''$ timber, as obtained by the calculation as being correct, because this would make each timber a little less than 2 in. $\times$ 12 in., and no timber or girder, especially where rafter or flooring is spiked to it, should be less than 3 inches wide.

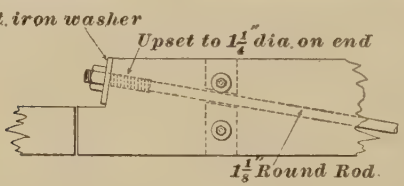


FIG. 62

From Table VI, the ultimate tensile strength of steel is found to be 64,000 pounds per square inch; hence, if a factor of safety of 4 is used, the safe working fiber stress in the rod must be 64,000 ÷ 4 = 16,000 pounds per square inch. According to the results given by the diagram, the total stress in the rod is 13,920 pounds; therefore, the rod must have a section of 13,920 ÷ 16,000 = .87 square inch. The area of a $1\frac{1}{8}$-inch round rod is .9940 square inch, and as this is the nearest standard size having the required sectional area, it will be used. As the area at the bottom of the thread of a $1\frac{1}{8}$-inch bolt is, however, only .693 square inch, it will be necessary to *upset*, or enlarge, the ends of the rod to a diameter of $1\frac{1}{4}$ inches, in order to get the requisite strength

in the threaded portion. The washer at B, Fig. 61, must be large enough to distribute the pressure due to the pull of the rod over a sufficient area of the end of the beams, to prevent danger of crushing the wood. The allowable compressive strength of yellow pine, parallel to the grain, may be taken as $8{,}000 \div 5 = 1{,}600$ pounds per square inch; this requires a washer whose area is $13{,}920 \div 1{,}600 = 8.7$ square inches. Using a washer 6 inches wide, extending across the ends of the two beams, we get a bearing area of $2 \times 4 \times 6 = 48$ square inches. In order to resist the bending stress due to the pull of the rod, the washer should be from $\frac{3}{4}$ inch to 1 inch in thickness. Figs. 61, 62, and 63 show details for the different parts of the trussed stringer under consideration.

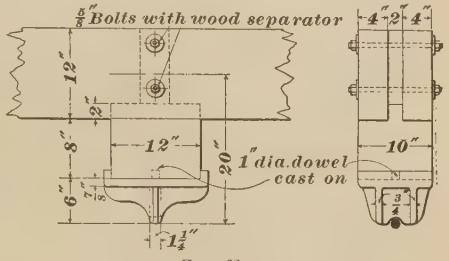


FIG. 63

REINFORCED CONCRETE

99. Reinforced concrete is becoming of more importance every day. It is therefore necessary that the architect be familiar with the methods of design used for beams and floor slabs made of this material. As has been explained in a previous Section, concrete is a poor material to withstand tension, but is reliable in compression. For this reason, in reinforced-concrete beams, it is customary to place the steel near the bottom of the concrete, where the stresses in tension occur, while the stresses in compression near the top of the beam are resisted by the concrete alone.

100. Calculating the Strength of Reinforced Concrete.—The method of calculating the strength of reinforced concrete here given, was devised by Edwin Thacher. Let Fig. 64 represent the cross-section of a beam. The

beam is reinforced by steel bars shown at *a*. Now, above the neutral axis, as shown in the figure, the concrete is in compression, and below the neutral axis the concrete and the steel are in tension. On account of the unreliability of the concrete in tension, Mr. Thacher assumes that it is unable to withstand any such strains without breaking. Therefore, according to his method of calculation, the concrete below the neutral axis simply holds the steel in place. The value of *d* is consequently of vital importance, while the value of *c* is not; *c*, which is the difference between *h* and *d*,

FIG. 64

TABLE XIV

VALUE OF *h*, *d*, AND *c*, IN INCHES

h	*d*	*c*	*h*	*d*	*c*
1.5	1.00	0.50	14.0	12.50	1.50
2.0	1.50	0.50	15.0	13.50	1.50
2.5	1.75	0.75	16.0	14.50	1.50
3.0	2.25	0.75	17.0	15.50	1.50
3.5	2.75	0.75	18.0	16.50	1.50
4.0	3.25	0.75	19.0	17.25	1.75
4.5	3.50	1.00	20.0	18.25	1.75
5.0	4.00	1.00	22.0	20.00	2.00
5.5	4.50	1.00	24.0	22.00	2.00
6.0	5.00	1.00	26.0	24.00	2.00
6.5	5.50	1.00	28.0	26.00	2.00
7.0	6.00	1.00	30.0	28.00	2.00
7.5	6.50	1.00	32.0	30.00	2.00
8.0	7.00	1.00	34.0	32.00	2.00
8.5	7.50	1.00	36.0	33.75	2.25
9.0	7.75	1.25	42.0	39.50	2.50
10.0	8.75	1.25	48.0	45.50	2.50
11.0	9.75	1.25	54.0	51.00	3.00
12.0	10.75	1.25	60.0	57.00	3.00
13.0	11.50	1.50			

is governed by the gripping qualities of the concrete on the steel and on the degree of protection from fire desired. Mr. Thacher recommends the values of h, d, and c given in Table XIV, which are the results of experience. It is therefore well to follow these figures.

101. As has been explained in a previous Section, concrete becomes stronger as it gets older. The strength of concrete also changes with the proportion of cement used in its mixture. It will, therefore, be noticed that Table XV is

TABLE XV

DIFFERENCES IN STRENGTH OF CONCRETE

Mixture	Age	Area of Steel Required for 1 Inch Width of Beam Square Inches	Ultimate Bending Moment for 1 Inch Width of Beam Foot-Pounds
1-2-4	1 month	$\frac{d}{142}$	$35.62\ d^2$
1-2-4	6 months	$\frac{d}{100}$	$51.25\ d^2$
1-3-6	1 month	$\frac{d}{165}$	$30.62\ d^2$
1-3-6	6 months	$\frac{d}{109}$	$46.25\ d^2$

arranged to show these differences in the strength of the concrete.

In designing a beam, the first and most important consideration is to so proportion it that it will be safe; if of reinforced concrete, it must contain sufficient steel and sufficient concrete. There is another consideration, however, in the design of reinforced-concrete beams, which is one of the principal factors in the economy of the construction, and that is the proportion of the concrete to the steel. To obtain an economical design, it is necessary that the concrete shall crush just at the time that the steel is breaking

in the lower part of the beam. If the concrete shows no signs of failure when the steel is giving way, it is proof that there is more concrete in the beam than is necessary. On the other hand, if the concrete crushes while the steel holds fast, the designer has been too extravagant with the metal. In Table XV these considerations have been borne in mind by Mr. Thacher, and by its use economical and safe beams and slabs of reinforced concrete can be designed.

In obtaining these formulas, it will be noticed that the ultimate and not the safe working strength of the concrete has been used and this is given in foot-pounds. It is therefore necessary that the bending moment due to the imposed loads be multiplied by the factor of safety. If a factor of safety is used in the calculation based on the strength of concrete 1 month old, there will not be sufficient steel to develop the full strength of the concrete when the latter is 6 months old. It is better practice, therefore, to use such a factor of safety for concrete 6 months old as will give a minimum factor of safety for concrete 1 month old. That is to say, a factor of safety of 5 for 6-month concrete will give a factor of safety of about 3.5 in 1 month, which will usually be ample. A factor of safety of 4 in 6-month concrete, will give a factor of safety of about 2.75 in 1 month, which will be sufficient for uniform loads without impact, such as earth fills, water pressure, etc. The methods of design of reinforced concrete will be best understood, perhaps, from the following examples:

EXAMPLE 1.—A certain floor in an office building is to be supported by means of reinforced-concrete beams 1-2-4 mixture. The span of these beams is 30 feet and they are on 10-foot centers. The dead load is 150 pounds per square foot and the live load is 50 pounds per square foot. Design the beam.

SOLUTION.—The total load on each beam is

$$200 \times 10 \times 30 = 60{,}000 \text{ lb.}$$

From Table VII, the maximum bending moment is

$$\frac{60{,}000 \times 30}{8} = 225{,}000 \text{ ft.-lb.}$$

Using a factor of safety of 5, the beam must be designed to break under a moment of $225{,}000 \times 5 = 1{,}125{,}000$ ft.-lb.

Assume a value for h of 36 in. This will make d 33.75 in., according to Table XIV. Therefore, the bending moment for 1 in. width of beam is

$$51.25\ d^2 = 51.25 \times 33.75^2 = 58{,}380 \text{ lb.}$$

Therefore,

The width of the beam $= 1{,}125{,}000 \div 58{,}380 = 19.27$ in., say, 20 in.

Call the area of the steel required per inch of width of beam A,

$$A = \frac{d}{100} = \frac{33.75}{100} = .3375 \text{ sq. in. per in. of width}$$

Therefore, the total area of steel required is $.3375 \times 20 = 6.75$ sq. in. This is about the area of sixteen $\frac{3}{4}$-in. round rods. The beam required is, therefore, 36 in. deep and 20 in. wide, and is reinforced by sixteen $\frac{3}{4}$-in. steel bars placed $2\frac{1}{4}$ in. from the bottom of the beam. Ans.

EXAMPLE 2.—Design a concrete-steel floor that has an 8-foot span to carry a safe live load of 175 pounds per square foot and a dead load of 75 pounds per square foot, the concrete to be a 1-3-6 mixture.

SOLUTION.—The maximum moment on a strip of floor 1 ft. wide, in foot-pounds, is

$$\frac{250 \times 8 \times 8}{8} = 2{,}000 \text{ lb.}$$

Using a factor of safety of 5, the beam must be designed to break under a moment of $2{,}000 \times 5 = 10{,}000$ ft.-lb. Now, since the section of the floor under consideration is 12 in. wide instead of 1 in., it must take twelve times the moment given in Table XV. That is,

$$46.25 \times d^2 \times 12 = 10{,}000$$

Solving this equation for d, $d = 4.245$, say, 4.25 in. Therefore, from Table XIV, $h = 4.25 + 1.00 = 5\frac{1}{4}$ in.

$A = 4.35 \div 109 = .03899$, say, .04 sq. in. per in. of width

Now, a $\frac{5}{8}$-in. round bar has an area of .3068 sq. in. Therefore, if $\frac{5}{8}$-in. bars are used, they will be

$.3068 \div .03899 = 7.869$, say, $7\frac{5}{8}$ in. apart. Ans.

EXAMPLES FOR PRACTICE

1. A floor slab is to be made of 1-3-6 concrete. It is to be placed on 9-foot centers and must carry a total load of 275 pounds with a factor of safety of 5 in 6 months. Find h and d, and the correct spacing of $\frac{5}{8}$-inch rods to economically reinforce it.

Ans. $h = 6$ in.; $d = 5$ in.; c to c rods $= 6.2$ in.

2. A beam is to be made of 1-2-4 concrete. It is on a 16-foot span, and must carry a total load of 1,550 pounds per foot of length with a

factor of safety of 5 in 6 months. Assume that the beam is 24 inches deep. Design the beam.

Ans. The beam is 24 in. × 10 in. with 2.2 sq. in. of steel 2 in. from the bottom.

In all the foregoing examples, the weight of the beams and the floor itself has been considered. In very short spans and in unimportant work the dead load is sometimes neglected by some engineers. In heavy, important structures, however, the designer must subtract from the safe load the beams will carry, the dead load of the beam itself, in order to get the allowable live load the structure should sustain.

102. T-Shaped Beam Construction.—There is a style of concrete-beam construction that has not been discussed, and that is when the beam is built in one piece with the concrete floor slab. This style of construction is shown in section in Fig. 65, where *a* is the beam; *b*, the floor slab; and *f*, the reinforcing steel. As a rule, when this monolithic construction is used, the concrete is placed in the molds for the beams and allowed to partially set before the floor slab is laid. In such a case, there is liable to be a crack along the line *c d*, because concrete freshly laid does not adhere well to concrete partly set. Then the beam *a* would be calculated as an ordinary concrete beam supporting the floor *b*, just as if they were in separate pieces. This method of calculation is probably the safest, no matter how well the beams and the slab are bonded together.

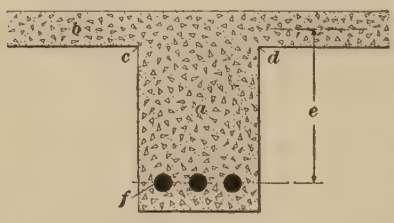


FIG. 65

There is another method of design, however, used by some engineers when the beam and slab are monolithic and made at the same time, and when the sectional area of the steel reinforcement in the beam does not exceed 2 per cent. of the sectional area of the beam. This method is that the safe working resisting moment of the beam, in inch-pounds, is equal to the allowable working stress in the steel, in

pounds per square inch, multiplied by the area of the steel, in square inches, multiplied by the vertical distance, in inches, between the center of the steel and the center line of the slab. For instance, suppose in Fig. 65 that the area of the steel bars shown at f is 2 square inches, that an allowable stress of 16,000 pounds per square inch is used, and that e is equal to 10 inches. Then, the safe resisting moment of the beam is $2 \times 16{,}000 \times 10 = 320{,}000$ inch-pounds, provided that the sectional area of the beam proper is greater than $2 \times \frac{100}{2}$ square inches. The slab must always be thick enough to carry its compression, usually one-fourth the depth of the beam for 1–2–4 concrete.

EXAMPLE.—A T-shaped beam has $3\frac{1}{2}$ square inches of reinforcing steel and the distance from the center of the reinforcement to the center of the top slab of the beam is 9 inches; what bending moment will this beam safely exert?

SOLUTION.—Assume that the safe allowable working stress for the steel is 16,000 lb. per sq. in. Then, the bending moment allowable is

$$16{,}000 \times 3.5 \times 9 = 504{,}000 \text{ in.-lb.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A concrete floor is supported by girders placed on 10-foot centers. The girders and the floor slab are cast in one piece. The girders are on 40-foot spans and each girder contains 8 square inches of steel reinforcement. The floor is to carry a load of 150 pounds per square foot. What is the depth from the center of the floor slab to the center of the steel reinforcement, allowing a unit working stress of 16,000 pounds in the steel? Ans. About 28 in.

2. A T-shaped beam has exerted on it a bending moment of 2,000,000 inch-pounds. The distance from the center of the steel reinforcement to the center of the top slab is 30 inches, and the area of the steel is 4 square inches. What is the unit stress in the steel? Ans. 16,667 lb.

DESIGN OF COLUMNS

WOODEN, CAST-IRON, STEEL, AND CONCRETE COLUMNS

WOODEN COLUMNS

SHORT COLUMNS

1. The column may, in its first stage of development, be considered a cubical or rectangular block, shown in Fig. 1. So long as the column does not exceed in length from ten to about fifteen times the width of the least side, the load it can safely carry may be estimated by multiplying its sectional area, in square inches, by the safe unit resistance to compression of the material parallel to the grain. To get the safe unit resistance of a short column or block, divide the ultimate resistance to compression of the material parallel to the grain, by the factor of safety, which, for wooden columns, according to *Design of Beams*, is 5, though it may be, in

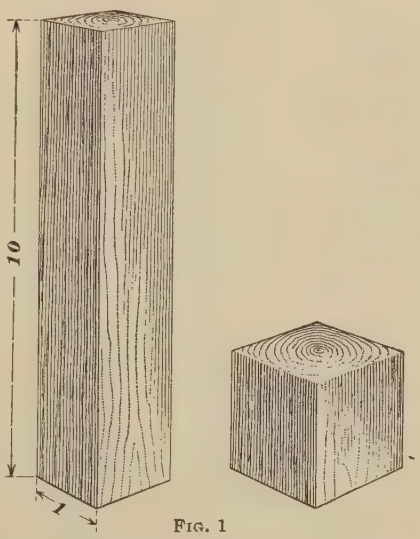


FIG. 1

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some instances, good practice to use 6. It is all a matter of judgment, governed by the conditions to be met. After having obtained the safe resistance of the material to compression, multiply it by the sectional area of the column, in square inches, and the result will be the safe resistance of the short column to compression.

EXAMPLE.—What safe load will a short yellow-pine block, 12 inches square and 6 feet long, standing on end support, using a safety factor of 5?

SOLUTION.—According to *Design of Beams*, the ultimate compression of yellow pine parallel to the grain, per square inch, is 5,000 lb.; using a factor of safety of 5, the safe load per square inch on this short column would be $5{,}000 \div 5 = 1{,}000$ lb. per sq. in. The area of the column is 12 in. × 12 in. = 144 sq. in., and, therefore, $144 \times 1{,}000 = 144{,}000$ lb., the safe resistance to compression of the short column. Ans.

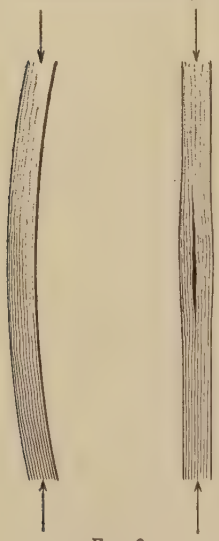


FIG. 2

LONG COLUMNS

2. The statements of the preceding paragraphs do not apply to columns whose length is over fifteen times the width of the least side. When long columns are under compression, and not secured against yielding sidewise, it is evident they are liable to bend before breaking. To ascertain the exact stress in such pieces is sometimes quite difficult. Hence, we must have a formula making due allowance for this tendency in the column to bend, or to split and spread from the center, as shown in Fig. 2.

3. Formula for Wooden Columns.—The formula mostly used for long, square, or rectangular wooden columns, with square ends, is the following, deduced from elaborate tests made on full-length columns at the Watertown arsenal:

$$S = U - \left(\frac{U \times L}{100\,D}\right)$$

in which S = ultimate compressive strength per square inch of sectional area of column;
U = ultimate compressive strength of material per square inch parallel to grain;
L = length of column, in inches;
D = length of least side of column, in inches.

EXAMPLE.—What safe load will a white-pine column, 10 inches square and 20 feet long, support, using a factor of safety of 6?

SOLUTION.—The ultimate compressive strength of white pine parallel to the grain is, according to *Design of Beams*, 3,500 lb. per sq. in. Therefore, by substituting in the formula, we have

$$S = 3{,}500 - \left(\frac{3{,}500 \times 240}{100 \times 10}\right) = 2{,}660 \text{ lb.},$$

the ultimate bearing value of the column per square inch. As the factor of safety required is 6, the safe bearing value per square inch of sectional area is $2{,}660 \div 6 = 443$ lb. The area of the column being 100 sq. in., the safe load is

$$100 \times 443 = 44{,}300 \text{ lb.} \quad \text{Ans.}$$

The column formulas in general use do not give a direct method for calculating the dimensions of a column to carry a given load. The usual method of procedure, when it is required to find the dimensions of a column of a given form that will safely support a given load, is to assume a value for the size of the column, substitute this value in the formula, together with the other values given, and solve for the compressive stress S. If the assumed size gives a value of S that is satisfactory for the given conditions, it is correct; if, however, the resulting value of S is too great, the assumed size is too small and it will be necessary to assume a larger size and try again; if, on the contrary, the value of S is less than the allowable stress, a smaller size of column may be assumed and a new value of S obtained. After a few trials, a size should be found that will give a satisfactory unit stress for the given conditions.

EXAMPLES FOR PRACTICE

1. A yellow-pine column 20 feet long is required to sustain a load of 11,500 pounds; provided a factor of safety of 5 is used, what must be the size of this column? Ans. 12 in. × 12 in.

2. What will be the allowable load on a $4'' \times 10''$ spruce column, 12 feet long, the factor of safety being 6? Ans. 17,066 lb.

3. It is required that a short, round, yellow-pine column shall carry 173,000 pounds; what must be the diameter of the column, the safe unit compressive stress on the material being 1,000 pounds? Ans. 15 in.

---

4. Importance of Method of Securing Ends of Columns.—Materials in compression develop more strength if the end bearing surfaces are true and level, for the tendency is then for all the material in the piece, or member, to resist compression equally, and not crush in one place, before the balance of the material can be brought under compression, as would be the case if the bearing surfaces were uneven and rough.

The manner of securing the ends of columns has, therefore, an appreciable effect on their strength. Columns fixed so firmly at the ends as to be liable to fail in the body before rupturing their end connections, develop greater strength than columns connected by means of pins through the ends. Columns with square ends develop less ultimate strength than if the ends are firmly fixed, but greater than if the ends are pin-connected, that is, fastened by pins which permit them to swing freely.

The formula for wooden columns, Art. **3**, and the formula for cast-iron columns, Art. **5**, apply only to those columns having flat ends, the usual condition met with in building construction.

CAST-IRON COLUMNS

5. Cast-iron columns are most frequently used in buildings of moderate height, but have been, in some cases, used in buildings of sixteen stories and even more. In recent years, however, steel columns have come into such general use for the more important structures that cast iron is now generally confined to buildings not exceeding five stories in height. Cast iron is also used in special cases where advantages are to be gained;

for instance, where a number of ornamental columns are necessary.

The uncertain strength of cast iron has compelled the adoption of a very low unit stress, in other words, a very high factor of safety. The uniform strength of structural steel, on the other hand, is so well understood that cast iron for columns, and especially for girders, is falling into disuse. Considerations of economy may, however, in some cases, still justify its employment.

One disadvantage in the use of cast-iron columns is that when fracture occurs, it comes without warning. In high buildings, erected entirely on cast-iron columns, the danger from wind pressure is very much increased on account of lack of stiffness in the joints of the connections at the several floors. In fact, buildings have been blown out of plumb owing to this lack of rigidity in the connections.

6. Formula for Cast-Iron Columns.—The strength of a cast-iron column with square ends may be calculated by the following rule:

Rule.—*The safe load per square inch of a cast-iron column may be obtained by subtracting thirty times the ratio of the length, in inches, to the radius of gyration, in inches, from 11,300.*

This rule is expressed by the formula

$$S_s = 11{,}300 - 30\frac{L}{R}$$

in which S_s = safe strength, in pounds, per square inch of sectional area;

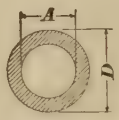
L = length of column, in inches;

R = radius of gyration, in inches.

It will be noted that this formula embodies its own factor of safety. The term radius of gyration, explained in *Design of Beams*, is a mathematical expression much used in calculating the strength of columns. Table I is a repetition of some of the formulas for finding the value of R^2, taken from *Design of Beams*, for the forms of cross-section most often used for cast-iron columns.

TABLE I

THE SQUARE OF THE LEAST RADIUS OF GYRATION FOR DIFFERENT SECTIONS OF CAST-IRON COLUMNS

Solid square		$R^2 = \frac{D^2}{12}$
Solid rectangle		$R^2 = \frac{D^2}{12}$
Solid circular		$R^2 = \frac{D^2}{16}$
Hollow square		$R^2 = \frac{D^2 + A^2}{12}$
Hollow circular		$R^2 = \frac{D^2 + A^2}{16}$

EXAMPLE 1.—What is the square of the radius of gyration of a hollow circular column of 12 inches outside diameter, the thickness of the metal being 1 inch?

SOLUTION.—The formula for obtaining the square of the radius of gyration for a hollow cylindrical cast-iron column is, according to Table I,

$$R^2 = \frac{D^2 + A^2}{16}$$

Substituting the given values, we have

$$R^2 = \frac{12^2 + 10^2}{16} = \frac{244}{16} = 15.2. \quad \text{Ans.}$$

EXAMPLE 2.—Find the proper working load for a 10-inch square hollow cast-iron column, 20 feet long, the thickness of the metal being 1 inch.

SOLUTION.—The length L is $20 \times 12 = 240$ inches; and R^2 for a hollow square rectangular column is, according to Table I,

$$\frac{D^2 + A^2}{12} = \frac{10^2 + 8^2}{12} = 13.67$$

Therefore, $R = \sqrt{13.67} = 3.7$, about.

$$S_s = 11{,}300 - \frac{30 \times 240}{3.7} = 9{,}354 \text{ lb. per sq. in.}$$

The net area of the section of the column is

$$10^2 - 8^2 = 100 - 64 = 36 \text{ sq. in.}$$

The entire load that it will support with safety is, therefore,

$$36 \times 9{,}354 = 336{,}744 \text{ lb. Ans.}$$

DESIGN OF CAST-IRON COLUMNS

7. Fig. 3 shows a design of a circular cast-iron column. *B* shows the elevation for the cap and brackets supporting steel floorbeams. Attention should be paid to the design of the bracket *a* and the web made as shown at *c*, care being taken that it is brought to the edge of the plate *m*, on which

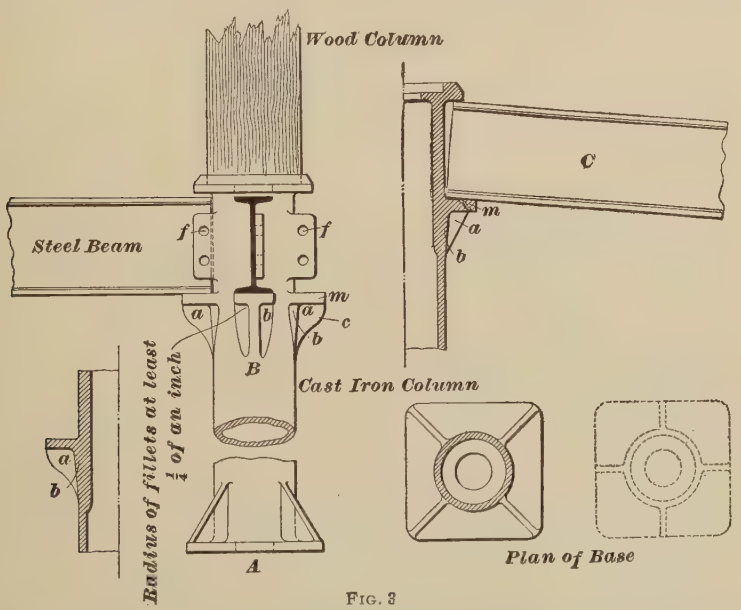


FIG. 3

the steel beams rest. Otherwise, if the beam takes a bearing on the edge of the plate *m*, as shown at *C*, the tendency will be to fracture the edge of the bracket. The web of the bracket should, if possible, form an angle of 60° with the horizontal plate *m*, as shown in Fig. 3.

The bolt holes *f* should be always drilled either in the casting or in the steel beams after these beams are in place, because if the holes were cored in the casting and the holes punched in the beams at the mill, it would more than likely be found in the course of erection at the building that the beams would not rest on the supporting brackets, but would be supported entirely by the bolts, thus inducing a shearing stress in these bolts. It is well to have at least $\frac{1}{4}$-inch fillets, as shown, but larger, if possible, in all the corners of the casting. It is good practice to thicken up the metal in the column, where the brackets are cast on it, as shown at *b*.

In designing the base for this column, it is well to place the ribs strengthening the web in such position that they may be most effective, which, in this case, is on the diagonals, for, if placed parallel to the sides, the corners will have a tendency to break off, and part of the bearing surface at the base of the column may prove ineffective.

Designs for the base and cap of cast-iron columns are as numerous as various conditions may require them. It would be almost impossible to give examples of the different connections demanded in actual building practice. The foregoing remarks will, if kept in mind, apply to every case, and, if followed, insure good design as well as secure construction.

8. Inspection.—In examining castings used in building construction, to ascertain their quality and soundness, several points are to be considered. The edges should be struck with a light hammer; if the blow makes a slight impression, the iron is probably of good quality, provided it is uniform throughout; if fragments fly off and no sensible indentation is made, the iron is hard and brittle. Air bubbles and blowholes are a common and dangerous source of weakness. They should be searched for by tapping all over the surface of the casting with a hammer. Bubbles, or flaws, filled in with sand from the mold, or purposely stopped with loam, cause a dullness in the sound, leading to their detection. The metal of a casting should be free from bubbles, core nails, or flaws of any kind. The exterior surface

should be smooth and clean, and the edges of the casting should be sharp and perfect. An uneven or wavy surface indicates unequal shrinkage, caused by want of uniformity in the texture of the iron.

The surface of a fracture, examined before becoming rusty, should present a fine-grained texture of uniform bluish-gray color and high metallic luster.

When inspecting cast-iron columns, care should be taken to see that they are straight and cored directly through the center, and that the metal is of the same thickness throughout. It is not unusual to find, in molding cast-iron columns, that the core has not been placed in the mold in a central position, or that, having been insecurely fastened, it has floated over to one side. Hence, the column that should have been, say, $\frac{3}{4}$ inch in thickness throughout, may be $1\frac{1}{4}$ inches on one side and $\frac{1}{4}$ inch on the other. The base and the cap of a cast-iron column should be turned accurately, being true and perpendicular to the center line of the column.

EXAMPLES FOR PRACTICE

1. The section of a hollow cast-iron column is 16 inches square, outside measurement; the thickness of the material in the column is 1 inch; what is the radius of gyration of this column? Ans. 6.13

2. What will be the working load of a cast-iron column 20 feet long, 12 inches in diameter outside, made of 1-inch metal?
Ans. 327,000 lb.

3. Find the strength of a 10-inch square, outside measurement, cast-iron column; the thickness of the metal is 1 inch, and the length of the column is 20 feet. Ans. 337,000 lb.

STEEL COLUMNS

INTRODUCTION

9. The calculations involved in the design of such *built-up* members of a building as steel columns and plate girders—members that are formed by the combination of several of the simple sections produced by the rolling mills—require a knowledge of certain mathematical properties of

the simpler sections, together with the methods by which these properties may be calculated. In many cases, the exact determination of the required properties is based on complicated mathematical principles; there are, however, numerous formulas and practical methods by means of which the values for all sections used in ordinary practice may be determined, either exactly or with a degree of approximation sufficiently close for all practical purposes.

THE NEUTRAL AXIS

10. Location of Neutral Axis.—In *Design of Beams*, it was stated that the neutral axis—the line separating the fibers in tension from those subjected to compression in a section of a beam—always passes through the center of gravity of the section. It is also evident from what was there stated that the neutral axis is perpendicular to the direction in which the load acts on the beam; therefore, to find the neutral axis of a section with reference to a set of

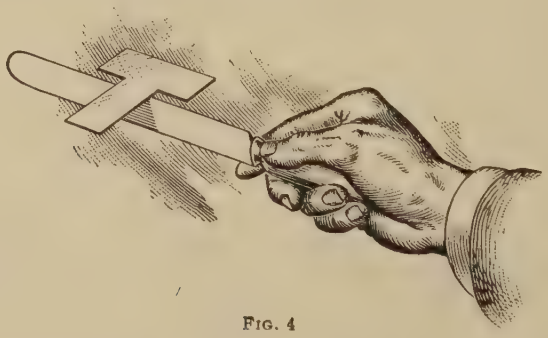


FIG. 4

loads applied in a given direction, it is only necessary to pass a line through the center of gravity perpendicular to the direction of the load.

A simple approximate method of locating the center of gravity and neutral axis of a section is shown in Fig. 4. Draw the outline of the section, either full size, or to some convenient scale, on a piece of heavy cardboard. Cut out the section and balance it carefully over a knife edge, as

shown in the figure; the line along which it rests on the edge of the knife is a line passing through the center of gravity, and by locating two such lines in different directions, the center of gravity will be found at their point of intersection.

11. Locating Neutral Axis by Means of Principle of Moments.—A convenient method of locating the neutral axis is based on the principle that the moment of any figure, with respect to a given line as an *axis* or *origin of moments*, is equal to the sum of the moments of its separate parts with respect to the same axis. Thus, in Fig. 5, take the

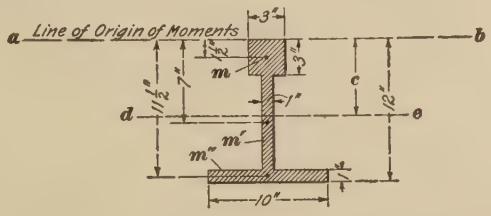


FIG. 5

line $a\,b$ as the line of origin of moments. Divide the figure into the three rectangles m, m', m''. In accordance with the principles stated in *Design of Beams*, the center of gravity of each of these rectangles is midway between its edges; the distances of the respective centers from the axis are, therefore, $1\frac{1}{2}$, 7, and $11\frac{1}{2}$ inches. The areas of the figures are respectively $3 \times 3 = 9$ square inches, $8 \times 1 = 8$ square inches, and $10 \times 1 = 10$ square inches. The moments of these areas about the axis $a\,b$ are:

Section m	$9 \times 1\frac{1}{2}$	= 13.5
Section m'	8×7	= 56.0
Section m''	$10 \times 11\frac{1}{2}$	= 115.0
Total		= 184.5

The area of the whole section is equal to $9 + 8 + 10 = 27$ square inches, the sum of the areas of the rectangles; the distance c of its neutral axis from the line of the origin of moments is, therefore, $184.5 \div 27 = 6.83$ inches, or nearly $6\frac{23}{32}$ inches.

It is not necessary that the line of origin of moments should coincide with an edge of the figure, as in Fig. 5, since any other line parallel with the direction of the required neutral axis gives the same results; in most cases, however, it will be found more convenient to take the axis about which the moments are calculated on one of the extreme edges of the section.

Since the section shown in Fig. 5 is symmetrical with respect to an axis perpendicular to the neutral axis, it is evident that its center of gravity is at their intersection. If, however, there were no axis of symmetry, the center of gravity could have been located by taking a second line perpendicular to $a\,b$ as an origin of moments and finding the neutral axis parallel to it. The intersection of this neutral axis with the one first found is the center of gravity of the section. In accordance with the principles just illustrated, we have the following rule:

Rule.—*To find the neutral axis of any section, first divide it into a number of simple parts, each of whose areas and centers of gravity can be readily found; then, find the sum of the moments of the areas of each of these parts with respect to an axis parallel to the required neutral axis. Finally, divide this sum by the sum of the areas of the parts of the section, and the result will be the perpendicular distance from the axis of the origin of moments to the required neutral axis.*

12. Application of Rule to a Built-Up Section. Fig. 6 shows a section of the rafter member of a large roof truss formed of a $\frac{3}{8}'' \times 16''$ web-plate and a $\frac{3}{8}'' \times 12''$ flange plate, the two joined by two $4'' \times 4'' \times \frac{1}{2}''$ angles. What is the distance of the neutral axis of the section from the top edge of the flange plate? By means of the principles previously given, the centers of gravity of the two rectangular plates are easily located as shown. The centers of gravity of the angles might also be located by applying the rule given in the preceding article; this, however, is unnecessary, since the center of gravity can be obtained directly by referring to the tables of the properties of rolled sections. By referring

to Table V, the center of gravity of a $4'' \times 4'' \times \frac{1}{2}''$ angle is found to be 1.18 inches from the back of flange, nearly $1\frac{3}{16}$ inches, thus giving the distance $1\frac{3}{16} + \frac{3}{8} = 1\frac{9}{16}$ inches from the top edge of the flange plate to the axis through the

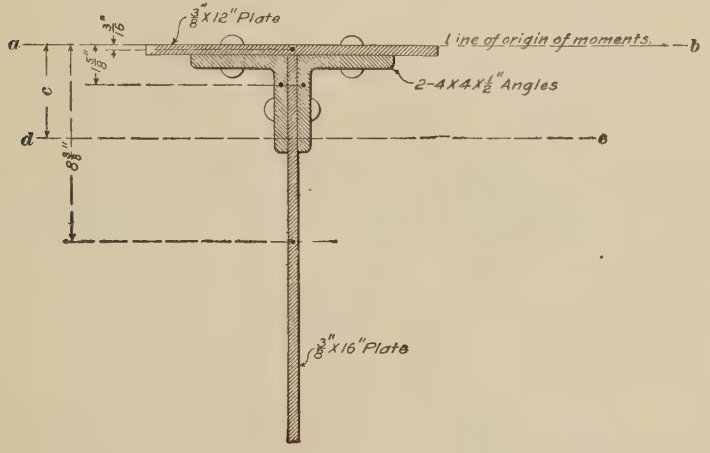


FIG. 6

centers of gravity of the angles. From this same table, it is also found that the area of the section of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 square inches.

The area of the section of the flange plate is $\frac{3}{8} \times 12 = 4.5$ square inches, and of the web-plate, $\frac{3}{8} \times 16 = 6$ square inches; the area of the whole section is therefore $2 \times 3.75 + 4.5 + 6 = 18$ square inches.

The moments of the areas of the separate sections, with respect to the line $a\,b$, are as follows:

Flange plate	$4.5 \times \frac{3}{16}$	$= \quad .8\,4$
Two angles	$2 \times 3.75 \times 1\frac{9}{16}$	$= 1\,1.7\,2$
Web-plate	$6 \times 8\frac{3}{8}$	$= 5\,0.2\,5$
Total		$= 6\,2.8\,1$

The distance c from the top edge of the flange plate to the neutral axis $d\,e$ of the section is therefore $63.28 \div 18 = 3.49$ inches.

THE MOMENT OF INERTIA

13. The term **moment of inertia** is a mathematical expression that depends on the distribution and the area of a surface with respect to a given axis. As applied to the area of a plane figure, the moment of inertia, with respect to an axis lying in the same plane, is numerically equal to the sum of the products obtained by multiplying each of the elementary areas of which the figure is composed by the square of its distance from the given axis.

By **elementary area** is meant an area smaller than any that it is customary to deal with in ordinary calculations; it is therefore impossible to find an exact expression for the

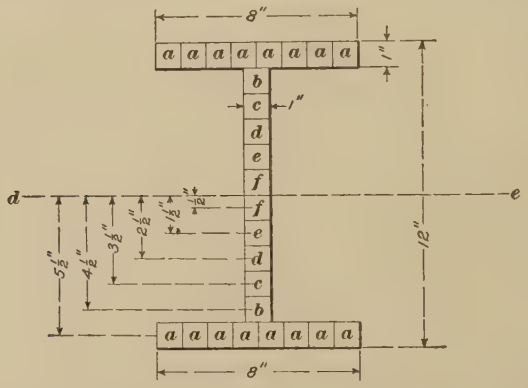


FIG. 7

moment of inertia of a figure by the methods of calculation in ordinary use. By means of the calculus, however, exact formulas have been deduced, by means of which the moments of inertia of many of the simple geometrical forms, with respect to axes through their centers of gravity, have been found. There are also a number of approximate methods by means of which the moment of inertia of an irregular section, to which these formulas do not apply, may be found. Further, the moment of inertia of any section that can be divided into parts, the moments of each of which, with respect to an axis through its center of gravity, can be found, is easily calculated by means of a principle to be given.

To illustrate the meaning of the term *moment of inertia*, together with a simple approximate method of computing the moment of inertia of a figure, consider the relation of the small I section shown in Fig. 7 to the axis *de* through its center of gravity. Divide the section into a number of little squares (in this case, each with an area of 1 square inch) and consider the distance of each square from the axis to be the distance from the axis to its center of gravity. Then the products of the area of each square, multiplied by the square of its distance from the axis, are as follows:

Squares *a*	$1 \times (5\frac{1}{2})^2 = \frac{121}{4}$
Squares *b*	$1 \times (4\frac{1}{2})^2 = \frac{81}{4}$
Squares *c*	$1 \times (3\frac{1}{2})^2 = \frac{49}{4}$
Squares *d*	$1 \times (2\frac{1}{2})^2 = \frac{25}{4}$
Squares *e*	$1 \times (1\frac{1}{2})^2 = \frac{9}{4}$
Squares *f*	$1 \times (\frac{1}{2})^2 = \frac{1}{4}$

Adding these products for all the squares, we have:

16 squares *a*	$\frac{121}{4} \times 16 = 484$
2 squares *b*	$\frac{81}{4} \times 2 = 40\frac{1}{2}$
2 squares *c*	$\frac{49}{4} \times 2 = 24\frac{1}{2}$
2 squares *d*	$\frac{25}{4} \times 2 = 12\frac{1}{2}$
2 squares *e*	$\frac{9}{4} \times 2 = 4\frac{1}{2}$
2 squares *f*	$\frac{1}{4} \times 2 = \frac{1}{2}$
Total	$= 566\frac{1}{2}$

which is the sum of the products of each of the small areas multiplied by the square of its distance from the axis. This result, however, is only a rough approximation to the moment of inertia, owing to the fact that the assumed areas are so large. The actual value of the moment of inertia of the section, found by higher mathematics, is $568\frac{2}{3}$.

14. Rules and Formulas for Moments of Inertia. Exact formulas for computing the moment of inertia of such regular figures as are most often met in the design of structures, and also approximate formulas for computing this factor for common rolled sections are given in *Design of Beams* or at the end of this Section. The tables of properties of rolled sections published by the rolling mills give

accurate values of the moment of inertia of all the principal sections used in the construction of buildings, so that it is not generally necessary to make the calculations for these sections; the approximate formulas just referred to are, however, sometimes useful in making calculations when the tables published by the rolling mills are not at hand.

Rule.—*To find the moment of inertia, with respect to any axis of any figure whose moment of inertia with respect to a parallel axis through its center of gravity is known, add its moment of inertia with respect to the axis through its center of gravity to the product of its area multiplied by the square of the distance from its center of gravity to the required axis.*

This rule may be expressed by the formula

$$I' = I + a r^2. \qquad (1)$$

in which I' = required moment of inertia;

I = moment of inertia of section, with respect to axis through its center of gravity and parallel to given axis;

a = area of figure;

r = distance from its center of gravity to required axis.

The moment of inertia of a section about an axis through its center of gravity is equal to the sum of the moments of inertia of its component parts about the same axis. Therefore, if the section can be divided into figures of such shape that the moment of inertia of each figure can be found about an axis through its own center of gravity, parallel to the desired axis of the entire section, the moment of inertia of the entire section about its own axis can be found by applying the foregoing principle and formula **1.**

Since the moment of inertia of any figure, with respect to any axis, is given by the formula $I' = I + a r^2$, if we denote the sum of the moments of the separate figures making up a section, with respect to an axis through the center of gravity of that section, by $\Sigma I'$ (in which the Greek letter Σ, read *sigma*, means *sum of*), we have

$$I_s = \Sigma I' = \Sigma (I + a r^2) \qquad (2)$$

which is a general formula often used to denote the moment of inertia I_s of any built-up section.

EXAMPLE 1.—What is the moment of inertia of the section of a $10'' \times 16''$ beam about an axis through its center of gravity parallel to its shorter side?

SOLUTION.—From *Design of Beams*, the formula for the moment of inertia of a solid rectangle is $I = \frac{b\,d^3}{12}$. Substituting the given dimensions, we have

$$I = \frac{10 \times 16^3}{12} = 3{,}413\tfrac{1}{3}. \quad \text{Ans.}$$

EXAMPLE 2.—Using the approximate formula given in *Design of Beams*, compute the moment of inertia of a section of a 10-inch I beam, the area of the section being 10.29 square inches.

SOLUTION.—Substituting in the formula, we have

$$I = \frac{A\,h^2}{6.66} = \frac{10.29 \times 10^2}{6.66} = 154.5. \quad \text{Ans.}$$

EXAMPLE 3.—Compute the moment of inertia, with respect to the axis $d\,e$ through its center of gravity, of the section shown in Fig. 7.

SOLUTION.—This section is made up of three rectangles, the moments of inertia of which, with respect to the given axis, can be found by means of formula **1**. The moment of inertia of one of the flanges, with respect to an axis through its center of gravity parallel to $d\,e$, is

$$I = \frac{8 \times 1^3}{12} = \tfrac{2}{3}$$

The area of this figure is $8 \times 1 = 8$ sq. in., and the distance of its center of gravity from $d\,e$ is $5\frac{1}{2}$ in.; therefore, its moment of inertia, with respect to $d\,e$, is

$$I' = \tfrac{2}{3} + 8 \times (5\tfrac{1}{2})^2 = 242\tfrac{2}{3}$$

The axis through the center of gravity of the web section coincides with the axis $d\,e$; hence, the moment of inertia of this section, with respect to $d\,e$, is

$$I' = \frac{1 \times 10^3}{12} = 83\tfrac{1}{3}$$

Then, the moment of inertia I_s of the whole section is

$$\Sigma I' = 242\tfrac{2}{3} + 242\tfrac{2}{3} + 83\tfrac{1}{3} = 568\tfrac{2}{3}. \quad \text{Ans.}$$

EXAMPLE 4.—What is the moment of inertia, with respect to the axis $d\,e$, of the column section shown in Fig. 8?

SOLUTION.—The moment of inertia of one of the flange plates, with respect to an axis through its center of gravity, parallel to the axis $d\,e$, is

$$\frac{12 \times \frac{3}{8}^3}{12} = .05$$

The area of the plate is $12 \times \frac{3}{8} = 4.5$ sq. in., and the distance of its center of gravity from the axis is $6\frac{3}{16}$ in. Therefore, the moment of inertia of the plate, with respect to the axis de, is

$$.05 + 4.5 \times (6\tfrac{3}{16})^2 = 172.33$$

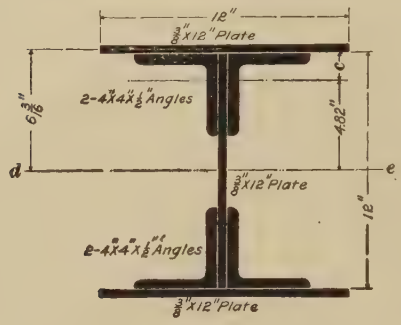


Fig. 8

From Table V, which gives the properties of standard angles having equal legs, the area of a $4'' \times 4'' \times \frac{1}{2}''$ angle is 3.75 sq. in., and the distance of its center of gravity from the back of a flange is approximately $1\frac{3}{16}$ in. The distance of the center of gravity of the angle from the axis de, in accordance with the dimensions given in the figure, is $6 - 1\frac{3}{16} = 4\frac{13}{16}$ in. In accordance with the approximate formula $\frac{A h^2}{10.2}$ given in *Design of Beams*, for finding the moment of inertia of an angle with equal legs, the moment of inertia of the $4'' \times 4'' \times \frac{1}{2}''$ angle, with respect to the axis through its center of gravity, is

$$\frac{3.75 \times 4^2}{10.2} = 5.9, \text{ nearly}$$

Or, Table V could be used if desired and a more accurate result obtained.

The moment of inertia of the angle, with respect to de, is, therefore,

$$5.9 + 3.75 \times (4\tfrac{13}{16})^2 = 92.75$$

The center of gravity of the web-plate lies on the axis de; therefore, the moment of inertia of the plate, with respect to de, is

$$\frac{\frac{3}{8} \times 12^3}{12} = 54$$

Thus the moment of inertia of the whole section, with respect to de, is

$$2 \times 172.33 + 4 \times 92.75 + 54$$
$$= 769.66. \quad \text{Ans.}$$

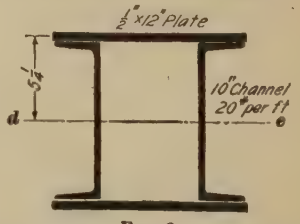


Fig. 9

Example 5.—What is the moment of inertia, with respect to the axis de, of the column section shown in Fig. 9?

Solution.—The moment of inertia of one of the cover-plates, shown in Fig. 9, with respect to an axis through its center of gravity, parallel to de, is

$$\frac{12 \times \frac{1}{2}^3}{12} = .125$$

The area of the plate is $12 \times \frac{1}{2} = 6$ sq. in., and the distance of its center of gravity from de is $5\frac{1}{4}$ in.; therefore, its moment of inertia, with respect to de, is

$$I' = .125 + 6 \times (5\tfrac{1}{4})^2 = 165.5$$

From Table IV, which gives the properties of standard channels, the moment of inertia of a standard 10-inch 20-pound channel is 78.7; therefore, the moment of inertia of the whole section, with respect to de, is

$$2 \times 165.5 + 2 \times 78.7 = 488.4. \quad \text{Ans.}$$

RADIUS OF GYRATION

15. In computing the strength of columns, frequent use is made of a property of a section that is numerically equal to the square root of the quotient of its moment of inertia, with respect to an axis through its center of gravity, divided by its area. This property is called the **radius of gyration** of the section, with respect to the given axis, and has been mentioned in *Design of Beams*. It is usually expressed by the letter R, and its value, with respect to a given axis, for any section whose area A and moment of inertia I, with respect to the same axis, are known, is given by the formula

$$R = \sqrt{\frac{I}{A}} \qquad (1)$$

The form in which the radius of gyration appears in most formulas for calculating the strength of columns is its square; hence, it is convenient to express the above relation by the formula

$$R^2 = \frac{I}{A} \qquad (2)$$

which gives directly the value to be substituted in the column formulas. Formulas for computing the least radius of gyration and its square for the sections most often used in the design of structures are given in *Design of Beams* or can be derived directly from the moment of inertia and the area.

TYPES OF COLUMNS

16. Columns of rolled-steel shapes riveted together are now largely used in the construction of buildings, and, especially in tall-building construction, have superseded cast-iron

columns. On account of their unreliability under sudden stress, cast-iron columns are no longer used in bridge work. In buildings, although the loads are generally quiescent, and the liability to sudden shock is more remote, the columns seldom receive their loads as favorably as in bridges, because in most cases they are subjected to considerable eccentricity in loading; that is, the loads on one side of a column are heavier than those on the other, thus tending to produce bending stress and materially decreasing its ultimate strength to direct compression.

FIG. 10

It may readily be seen that the placing of columns centrally, one over another, necessitates the application of loads to their sides; and, unless the loads are equal and on opposite sides, the effect is to increase the stress on the side where the greatest load occurs.

The method of securing the ends of columns, or struts, also exercises an important influence on their resistance

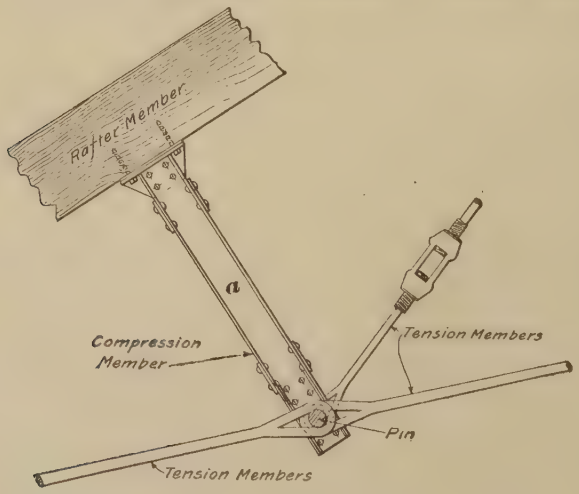


FIG. 11

to bending, and, consequently, on their ability to resist compression stresses. Columns may therefore be classified,

according to the arrangement of their ends, into the three following forms: (*a*) columns with *round*, or *hinged ends;* (*b*) columns with *one end round*, or *hinged*, and *one end fixed;* and (*c*) columns with *fixed ends.*

FIG. 12

Columns with **round,** or **hinged ends** are struts that have only central points or lines of contact, such as balls or pins resting on flat plates. The centers of the balls or pins should lie in a line through the center of gravity of a section of the strut, as shown in Fig. 10.

Columns with **one end hinged** and **one end fixed** occur frequently in roof-truss work, but are seldom met with elsewhere. Fig. 11 gives an example of such a column and Fig. 12 shows the same kind of column as it might occur in bridge construction.

FIG. 13

Columns with **fixed ends** have flat ends or ends rigidly secured to the structure. They are the most common form of column met with in framed structures. This type of column is illustrated in Fig. 13.

FORMULAS FOR STEEL COLUMNS

17. The formula for steel columns about to be given is a general formula that can be used equally well for columns of wood but not cast iron. The formulas already given, however, for these two materials will be found more convenient than the one that follows.

$$S = \frac{U}{1 + \frac{UL^2}{10\,m\,E\,R^2} + \frac{PC}{R^2}V}$$

in which S = ultimate breaking strength of column, in pounds per square inch of section;

U = ultimate compressive strength of material, in pounds per square inch;

L = length of columns, in inches;

R = radius of gyration of section in the direction of the eccentric load;

E = modulus of elasticity of material of which column is composed;

P = distance, in inches, between center line of column and line of action of load;

C = distance, in inches, between center line of column and surface of column, measured in same direction as P was measured;

m = a constant, which is 1 for round, or hinged, ends, $\frac{9}{4}$ for one end round, or hinged, and one end fixed, and 4 for both ends fixed;

V = ratio of eccentric part of load to entire load.

It should be noted that in this formula, as in the preceding column formulas, it is first necessary to assume the size of column and then figure to find if it has the required strength. Should the column assumed prove too weak, it will be necessary to choose larger sizes in succession until a column section of sufficient strength is found.

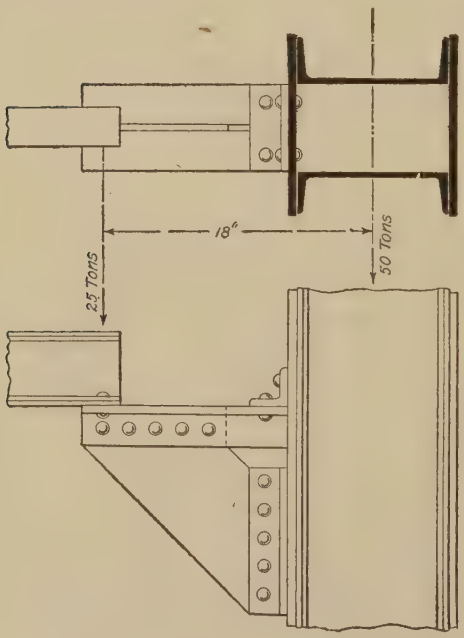


Fig. 14

Example.—A channel column 20 feet long, made of medium steel, having cover-plates with fixed ends, supports a direct load of 50 short tons and an eccentric load of 25 short tons, as shown in Fig. 14. Find the size of column required, if a factor of safety of 6 is to be used.

Solution.—Assume that 10-in. 30-lb. channels are used with $1'' \times 12''$ cover-plates. Since the cover-plates are made so wide, the column

will not have to be investigated for failing by bending at right angles to the eccentric load. R is therefore measured as stated, namely, in the direction of the eccentricity. The area of each plate is 12 sq. in.; the moment of inertia of each plate about an axis through its own center of gravity perpendicular to the line of eccentricity of the column is $\frac{12 \times 1^3}{12} = 1$; the distance of the plates from the center of gravity of the column is $5\frac{1}{2}$ in. Therefore, the moment of inertia of each plate about the neutral axis of the entire section is, from formula **1**, Art. **14**,

$$1 + 12 \times 5\tfrac{1}{2}^2 = 364$$

The moment of inertia of the channels can be obtained from Table IV. The moment of inertia of the entire section is

$$2 \times 103.2 + 2 \times 364 = 934.4$$

The area of the entire section is

$$2 \times 12 + 2 \times 8.82 = 41.64 \text{ sq. in.}$$

Therefore, $R^2 = 934.4 \div 41.64 = 21.96$

Assisted with the knowledge of the properties of structural steel, as given in *Design of Beams*, the terms in the formula can now be substituted.

$$S = \frac{64{,}000}{1 + \dfrac{64{,}000 \times 20^2 \times 12^2}{10 \times 4 \times 29{,}000{,}000 \times 21.96} + \dfrac{18 \times 6}{21.96} \times \dfrac{25}{50 + 25}}$$

$$= \frac{64{,}000}{1 + .15 + 1.64} = 22{,}940$$

Now, with a factor of safety of 6, the allowable working stress is $22{,}940 \div 6 = 3{,}823$ lb. The area of the column section is 41.64 sq. in. Therefore, the entire safe load is $3{,}823 \times 41.64 = 159{,}190$ lb., or 79.595 T. As it was desired that the column should only hold $50 + 25 = 75$ T., it will be seen that the load is safe. If any smaller section had been figured, the column would have been found too weak; and if any larger section had been figured, it would have been found that the column would have been able to carry more load than required, which means that more material than necessary would be used. The section of column selected is therefore the smallest section that is safe to use. Ans.

18. In modern buildings, nearly all columns are subjected to more or less eccentric loading. In fact, it would be a difficult matter to find a column loaded with a perfectly concentric load. It is, however, good practice to disregard the eccentricity generally encountered in ordinary building construction, since the preceding formula makes due allowance in the second term in the denominator, for any slight eccentricity caused by the lack of symmetry in the arrangement

of the brackets and the application of the loads with regard to the central axis of the column. For example, the beam connections may all be on one side of the column, and of the form shown in Fig. 15, or the beams attached to one side of the column may be more heavily loaded than those attached to the other side. Some of these conditions are usually unavoidable and tend to produce the undesirable effect of eccentric loading.

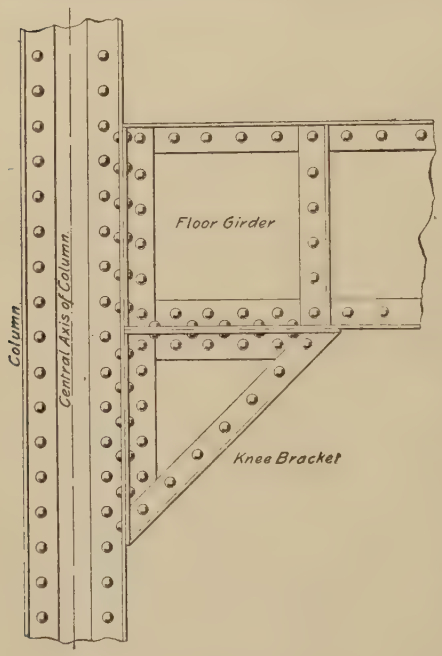


FIG. 15

Should the eccentricity of the load be considerable, and liable to produce dangerous transverse or bending stresses, it would materially decrease the ability of the column to withstand direct compressive stresses. The bending or transverse stresses should in such a case be calculated by the formula of Art. **17.** In general practice, however, in all cases of slight eccentricity of loading, as shown in Fig. 15, the following formula is used. This formula is the same as the formula of Art. **17,** except that the eccentricity P is made equal to zero. This makes the last term in the denominator vanish, and using the same letters as before, the formula is

$$S = \frac{U}{1 + \frac{UL^2}{10\,m\,E\,R^2}}$$

This formula is known as **Merriman's formula,** and is an adaptation of the celebrated Rankin's formula. There is one matter of importance to note in regard to the relation

between this formula and the formula of Art. **17.** In the formula of Art. **17** the column was assumed to tend to bend in the plane of the eccentricity of the load and with the bulge *away* from the eccentric load. R was therefore measured in the same plane as the eccentric load and not at right angles to it. In the formula just given, however, there is no eccentric load considered and the column will tend to bend about that axis where its moment of inertia is least. Therefore, in this formula, the *least* radius of gyration must be substituted for R, no matter in what direction it may be.

EXAMPLE 1.—Using a factor of safety of 4, compute the safe bearing strength of a strut of a pin-connected truss of medium steel, the length of the strut being 20 feet from center to center of pins, and its section made up of four $6'' \times 6'' \times \frac{3}{8}''$ angles connected as shown in Fig. 16.

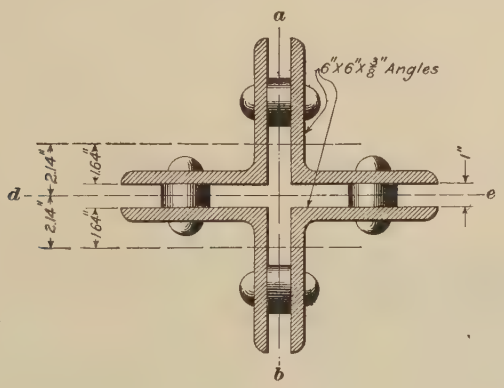


FIG. 16

SOLUTION.—Since the relation of the section to each of the axes $a\,b$ and $d\,e$ is the same, its moment of inertia and radius of gyration is the same about either axis, and it is therefore necessary to compute these factors for only one of these axes. From Table V, the area of the section of a $6'' \times 6'' \times \frac{3}{8}''$ angle is 4.36 sq. in., and the distance of its center of gravity from the back of either leg is 1.64 in., nearly. The moment of inertia of one of the angles, with respect to an axis through its center of gravity parallel with the given axis is 15.39.

From Fig. 16 the distance of the axis through the center of gravity of the angle from the axis $d\,e$ of the section is 2.14 in.; therefore, substituting the known values in formula **1**, Art. **14**, the moment of inertia of a single angle, with respect to the axis $d\,e$, is

$$I' = I + a r^2 = 15.39 + 4.36 \times 2.14^2 = 35.35$$

The moment of inertia of the whole section is, therefore,

$$I_s = \Sigma I' = 4 \times 35.35 = 141.40$$

The total area of the section is $4.36 \times 4 = 17.44$ sq. in.; therefore, the square of its radius of gyration, from formula **2** of Art. **15**, is

$$R^2 = \frac{I}{A} = \frac{141.4}{17.44} = 8.1, \text{ nearly}$$

In *Design of Beams*, the ultimate compressive strength U of structural steel is given as 64,000 lb. per sq. in., while E is 29,000,000. In accordance with the statement of the problem, the length of the strut is $L = 20$ ft. $= 240$ in. Since the strut is pin-connected, that is, hinged at the ends, $m = 1$. Substituting in the formula given in this article,

$$S = \frac{64{,}000}{1 + \frac{64{,}000 \times 240^2}{10 \times 29{,}000{,}000 \times 8.1}} = 24{,}900 \text{ lb.}$$

The safe bearing strength of the strut is, therefore,

$$\frac{24{,}900 \times 17.44}{4} = 109{,}000 \text{ lb., nearly.} \quad \text{Ans.}$$

EXAMPLE 2.—The moment of inertia, with respect to the axis YY of the steel **Z**-bar column shown in Fig. 17, is 287.92, and, with respect to the axis XX, 337.17. The total area of the section is 21.36 square inches. What is the safe bearing strength with one end round and one end fixed, if the length of the column is 25 feet and a factor of safety of 3 is used, the column to be made of medium steel?

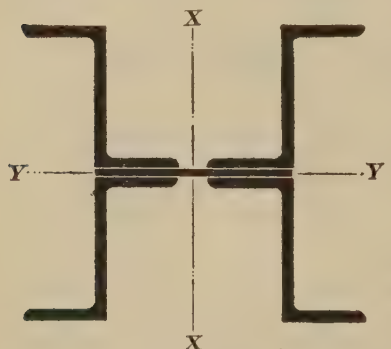


FIG. 17

SOLUTION.—Using the least moment of inertia, which is that with respect to the axis YY, the square of the least radius of gyration is found to be

$$R^2 = \frac{287.92}{21.36} = 13.48, \text{ nearly}$$

Therefore, the ultimate strength per square inch of section is

$$S = \frac{64{,}000}{1 + \frac{64{,}000 \times 25 \times 25 \times 12 \times 12}{10 \times \frac{9}{4} \times 29{,}000{,}000 \times 13.48}} = 38{,}680 \text{ lb.}$$

The safe bearing strength is, therefore,

$$\frac{38{,}680 \times 21.36}{3} = 275{,}300 \text{ lb.} \quad \text{Ans.}$$

19. Factors of Safety.—The ability of a column to resist the transverse or bending stresses due to eccentric

loading decreases as its length increases, and this ability is less for columns with round, or hinged, ends than for those with flat, or fixed, ends. For these reasons, it is good practice to assume a minimum factor of safety for the shortest columns or struts and increase the factor with the increase in length, making the rate of increase greater for round, or hinged, than for flat, or fixed, ends.

TABLE II
FACTORS OF SAFETY

$\frac{l}{R}$	Fixed and Flat Ends	Hinged and Round Ends	$\frac{l}{R}$	Fixed and Flat Ends	Hinged and Round Ends	$\frac{l}{R}$	Fixed and Flat Ends	Hinged and Round Ends
20	3.2	3.30	110	4.1	4.65	200	5.0	6.00
30	3.3	3.45	120	4.2	4.80	210	5.1	6.15
40	3.4	3.60	130	4.3	4.95	220	5.2	6.30
50	3.5	3.75	140	4.4	5.10	230	5.3	6.45
60	3.6	3.90	150	4.5	5.25	240	5.4	6.60
70	3.7	4.05	160	4.6	5.40	250	5.5	6.75
80	3.8	4.20	170	4.7	5.55	260	5.6	6.90
90	3.9	4.35	180	4.8	5.70	270	5.7	7.05
100	4.0	4.50	190	4.9	5.85	280	5.8	7.20

Assuming a minimum factor of safety of 3 for very short struts, the factors prescribed by good practice for longer struts with flat or fixed ends are given by the formula

$$F = 3 + .01\frac{l}{R} \qquad (1)$$

and for struts with round or hinged ends,

$$F = 3 + .015\frac{l}{R} \qquad (2)$$

in which F = required factor of safety;
l = length of strut, in inches;
R = least radius of gyration of section.

EXAMPLE.—What is the least factor of safety that should be used with (*a*) a column with flat ends, and (*b*) a column with hinged ends, the length of the column being 20 feet, and its least radius of gyration 2.5?

SOLUTION.—(*a*) Applying formula **1**, we have

$$F = 3 + .01 \times \frac{240}{2.5} = 3.96, \text{ say } 4. \quad \text{Ans.}$$

(*b*) Applying formula **2**,

$$F = 3 + .015 \times \frac{240}{2.5} = 4.44. \quad \text{Ans.}$$

Table II gives values of the factor of safety obtained by substituting different values of $\frac{l}{R}$ in formulas **1** and **2**.

The table gives values of the factor of safety for columns whose length is as great as 280 times their least radius of gyration; it is not, however, good practice to use a column in which the value of $\frac{l}{R}$ is greater than 150. Another condition to be observed is that the length of the column should not exceed 45 times its least dimension.

20. Column Formulas for Practical Use.—The formulas in Art. **19** are quite accurate, but they are not convenient to use, and for that reason the following formulas are given. These are for columns with fixed, or flat, ends, which is the almost universal condition of columns in buildings. These formulas will give the value of the *safe working stress* per square inch and not the *ultimate breaking stress*. They, therefore, may be said to embody their own factor of safety.

$$S_r(\text{for medium steel}) = \begin{cases} 12{,}000 \text{ for lengths up to 50 radii of gyration} \\ 15{,}000 - 57\frac{L}{R} \text{ for lengths from 50 to 150 radii of gyration} \end{cases} \quad (1), (2)$$

$$S_r(\text{for soft steel}) = \begin{cases} 12{,}000 \text{ for lengths up to 30 radii of gyration} \\ 13{,}500 - 50\frac{L}{R} \text{ for lengths from 30 to 150 radii of gyration} \end{cases} \quad (3), (4)$$

EXAMPLE.—A column in a building is 18 feet long. It is made of soft steel. The area of the section is 14.98 square inches and the least radius of gyration is 1.72. What safe load will it carry?

SOLUTION.— $\frac{L}{R} = \frac{18 \times 12}{1.72} = 126$, about. As this is more than 30 and less than 150, formula **4** is used.

$$S_t = 13{,}500 - 50 \times 126 = 7{,}200 \text{ lb. per sq. in.;}$$
$$7{,}200 \times 14.98 = 107{,}856 \text{ lb. Ans.}$$

EXAMPLES FOR PRACTICE

1. Design an I-beam column of medium steel, 15 feet long, to carry a safe load of 35 short tons. Ans. 12 in. 40 lb.

2. What safe load will a column composed of a single square rod of soft steel 2 inches on a side and 4 feet long sustain? Ans. 37,400 lb.

FORMS OF STEEL COLUMNS

21. Conditions That Affect the Choice of a Type of Column.—There are at present numerous forms in which rolled-steel shapes are combined to make up columns for structural purposes. The type of column to be used in a building is sometimes prescribed by the owner, or the design of the building is furnished by the architect, thus leaving the engineer little latitude in his choice of design. There are, however, a number of conditions demanded by considerations, partly practical and partly theoretical, that should be carefully studied and compared before selecting the type of column for a particular purpose. In many cases, it will be found that these considerations impose conditions that conflict with one another; and in order to make such a compromise as will meet this difficulty in the most satisfactory manner, there is demanded of the engineer a most careful exercise of judgment guided by practical experience. Some of the most important points to be considered in choosing the type of column to be used, in any case, are the following:

1. The cost and availability of the material.
2. The amount of labor required and the facility with which it may be performed in both shop and field.
3. The distribution of material in the column so as to give the maximum strength with the least weight.
4. The facility with which connections may be made between the column and the members that it supports.

5. The application of the connections in such a manner that they will transfer the compressive stresses directly to the axis of the column.

6. The facility with which it can be painted and inspected on all sides, to detect corrosion.

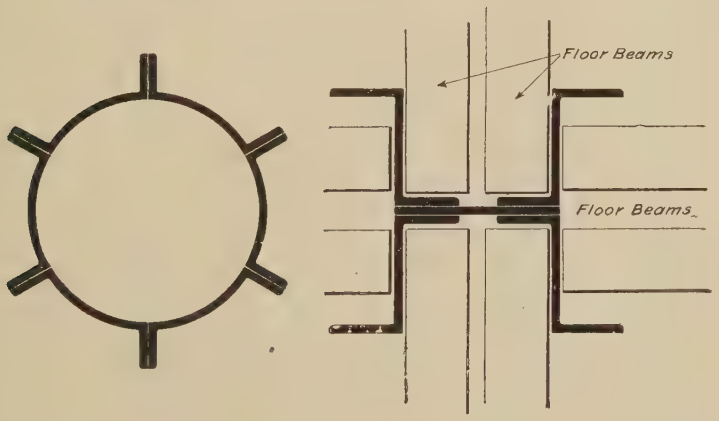
7. The facility with which the thickness of the metal in the different parts composing the column can be reduced in order to meet the reduced loads of the upper floors. It is not desirable to make the columns supporting the upper floors of the building very small, since the beams and girders supporting the upper floors are usually of the same dimensions as those for the lower floors, and consequently require connections as heavy and secure; it is almost impossible to make such connections with small columns, consequently, in order to reduce the weight of the column for the lighter load that it will carry, it is better to reduce the thickness of the material used and keep the section the same.

8. The facility with which fireproofing may be attached to the section. Columns of circular sections may be fireproofed more compactly than rectangular columns. Architects in some instances have utilized the space lost by the use of rectangular columns for the purpose of conduits in which to run pipe lines, etc. In one instance, for example, circular holes were cut in the bed and cap plates between the columns of several stories to allow for the passage of pipes inside of the column. Such practice cannot be condemned too severely. It is bad practice to run pipe lines or electric wires inside of, or alongside of, the fireproofing of columns; separate conduits, provided with suitable covers, which will permit inspection, should be provided in the walls.

In regard to the cost and availability of the material, shapes that are easily rolled and placed on the market at reasonable prices should be selected. I beams, channels, angles, and Z bars, together with plates, are the most common commercial sections; these are manufactured at nearly every structural-steel mill, and large orders may be obtained promptly at any locality where a skeleton-constructed building is likely to be erected. Patented sections do not fulfil

this condition, and should therefore be avoided in most cases. The consideration of prompt delivery is an important one, and greatly influences the cost and facility with which the modern building is erected.

The consideration of the facility with which the labor in both shop and field can be performed is one that should receive careful attention. In the shop, the complexity of the column section, and the number of pieces of which it is composed, greatly influence the element of labor. If there are numerous small pieces, such as brackets or splice plates, each of which requires cutting, bending, and fitting together,



(a) (b)

FIG. 18

with frequent handling, the cost of labor may be proportionately very great. The number of rivets also greatly influences the cost of a column; not only should there be as few rivets as is consistent with strength, but the construction should permit the rivets to be readily driven by machine, so as to avoid hand riveting, which is slow and expensive and now generally admitted to be inferior to machine work.

The same general remarks apply to the labor in the field; the connections should be as simple as possible, and with as few rivets as is consistent with the strength required; they should be easy of access, so that the work on them may be executed conveniently and rapidly.

In connection with the question of the distribution of the material in the column so as to develop the maximum amount of strength with the least weight of material, it is necessary to consider the facility with which economical connections may be made between the columns and floor-beams, and the directness with which the connections transfer the stresses to the central axis of the column. The relation between these conditions may be illustrated and analyzed by comparing the sections shown in Fig. 18, where (*a*) represents a section of a Phœnix column, while (*b*) is a section of a **Z**-bar column. The apparent advantage of the Phœnix column is that the material composing it is placed where it will be the most efficient, thus fulfilling the third condition of the foregoing list in a satisfactory manner; the radius of gyration of the column is also practically the same on any of its diameters, and the metal is placed at the greatest possible distance from its center.

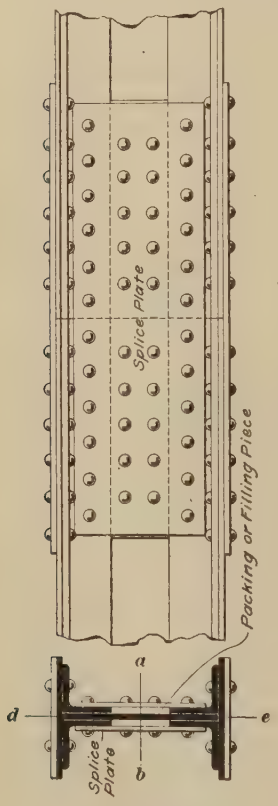


FIG. 19

On the other hand, by referring to the section of the **Z**-bar column, it is seen that a considerable portion of the material is concentrated on the axis, a condition that gives it a relatively small radius of gyration and demands the use of a greater weight of material for a given load. This column, however, offers greater advantages for the proper connection of the floorbeams than does the other, and, owing to its open construction, the beams transmit their loads almost directly to the central axis of the column, thus avoiding the disadvantages of eccentric loading. The **Z**-bar column is also more easily painted than the Phœnix column.

Thus, it is seen that the section having the best theoretical distribution of material is not always the best to use, on account of these several practical considerations; for example, the section shown in Fig. 19 is probably as much used for structural-steel columns as any other, although it is not an economical section, for the reason that its radius of gyration on the axis *de* is very small in proportion to the weight of material used, and, since the least radius of gyration is always used in calculating the strength of a structural column, all the material that does not go to form a greater radius of gyration adds nothing to the theoretical strength.

The great advantage of this section is that it is composed of the cheapest rolled sections, which are put together with a minimum of labor. It is also one of the best forms for attaching the beams and girders, and, as will be seen by a study of the details of the splice shown in the figure, for making connections between two adjacent columns.

22. Sections of Columns Frequently Used in Skeleton-Building Construction.—The following list shows the general forms of the principal types of steel-column sections now in use:

Larimer column, one row of rivets.

Z-bar column without cover-plates, two rows of rivets.

Z-bar column with one cover-plate, six rows of rivets.

Z-bar column with two cover-plates, six rows of rivets.

Z-bar column, additional section obtained by the use of angles and plates, eight rows of rivets.

Z-bar column, rectangular section, six rows of rivets.

Channel column with plates or lattice, four rows of rivets.

Box column of plates and angles, eight rows of rivets.

Latticed angle column, eight rows of rivets.

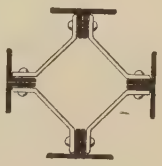
A column much used by the Pennsylvania Railroad, composed of angles and plates, ten rows of rivets.

Keystone octagonal column, four rows of rivets.

Four-section Phœnix column, four rows of rivets.

Eight-section Phœnix column, eight rows of rivets.

Grey column, four rows of rivets.



DETAILING OF STRUCTURAL COLUMNS AND CONNECTIONS

23. Rivets and Rivet Spacing.—Before discussing the design of structural-steel columns and their connections, it will be well to consider the best practice in proportioning and spacing the rivets that connect the several rolled sections of which the completed column is composed.

Where brackets are riveted to the column to support floor-beams, a sufficient number of rivets should be used to take the entire shear due to the reaction at the end of the beam; also where a plate girder, which forms the principal member of a floor system, is riveted directly to the column, sufficient

rivets should be provided so that their combined shearing strength is equal to the end reaction on the girder. If, as shown in Fig. 15, there is a bracket or knee brace in connection with the girder, the calculation may be based on the combined shearing strength of the rivets in both bracket and girder.

EXAMPLE.—If the reaction at the end of a plate girder is 60,000 pounds, and $\frac{7}{8}$-inch rivets, with an allowable shearing strength of 6,500 pounds per rivet, are used, how many rivets will be required to support the end of the girder?

SOLUTION.— 60,000 lb. ÷ 6,500 lb. = 9.2, say 10 rivets. Ans.

It is usual to space the rivets closer at the joints and foot of a column than in the body; for example, where $\frac{3}{4}$-inch rivets are used, it is customary to space them on 3-inch centers at the joints and bottom and from $4\frac{1}{2}$-inch to 6-inch centers throughout the remainder of the column, and where $\frac{7}{8}$-inch rivets are used, they should be spaced about 4 inches at the joints and about 6 inches throughout the length of the column.

In a compression member, the pitch of the rivets, in inches, should never exceed the thickness in sixteenths of an inch of the thinnest outside plate. For example, in Fig. 20, where the outside plate is $\frac{5}{16}$ inch thick, the greatest allowable pitch of the rivets would be 5 inches; under no conditions, however, should the pitch of the rivets in a compression member be greater than 6 inches, center to center, except where the rivets are staggered, in which case the pitch should not be more than 6 inches in a staggered line.

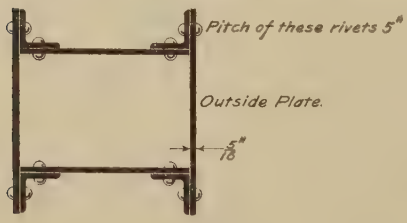


FIG. 20

The diameter of rivets to be used in built-up columns is determined by the thickness of the metal in the parts to be joined. Where the aggregate thickness of the metal between the heads of the rivet is not more than $1\frac{1}{4}$ inches, a $\frac{3}{4}$-inch rivet may be used; if the aggregate thickness of the several

plates is more than $1\frac{1}{4}$ inches and less than 3 inches, it would be advisable to use $\frac{7}{8}$-inch rivets; and if the aggregate thickness of the metal is 3 inches or more, 1-inch rivets should be used.

As far as practicable the rivets should be of the same size throughout the column, as this saves annoyance in both shop and field. The size of the rivets to be used in the column splices, girder and beam connections, knee braces, and brackets is determined by the size of the rivets in the column.

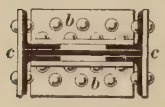
In conjunction with the foregoing statements, judgment in the matter of details should be exercised and the requirements of each condition to be fulfilled should be carefully studied.

24. Column Splices and Connections.—The excellence of the detail design of structural-steel columns is largely governed by the experience and judgment of the designer, and the care with which he studies the local conditions that will be met with in nearly every new piece of work.

The several points of excellence to be attained in the design of structural columns have already been discussed, and it now remains to consider a few first-class details of column splices and connections.

As the strength of a building depends largely on the strength of the connections to the columns, great care should be taken in their design. Rigid column connections for the floorbeams and girders at the several floors and efficient splice connections between the sections of the columns on the different floors are of the utmost importance, and should be given the most careful attention. The ideal system of column construction would be to make the various columns on the several floors of one continuous set of sections running from foundation to roof; it is evident that this is impossible in high modern building construction, therefore the next best thing to do is to make the spliced connections as rigid as possible.

Columns may be spliced as shown in Fig. 21. The bedplate *a* separates the two columns, and through it, by means of the angles *b*, *b*, the columns are riveted securely together; they are also additionally secured by means of the two splice plates *c*, *c*.



The abutting ends of structural columns should be milled, or planed, so as to secure a square and firm bearing and obviate the danger of their being thrown out of line when erected.

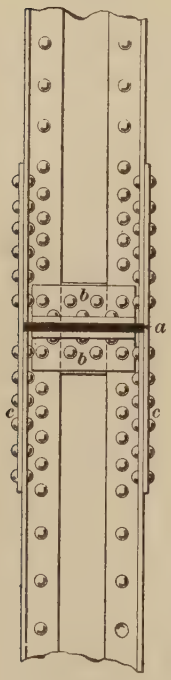


FIG. 21

Another very good method of connecting two columns is by the use of splice plates on all sides, thus doing away with the bedplates, or packing pieces, between the ends; this construction is shown in Fig. 22.

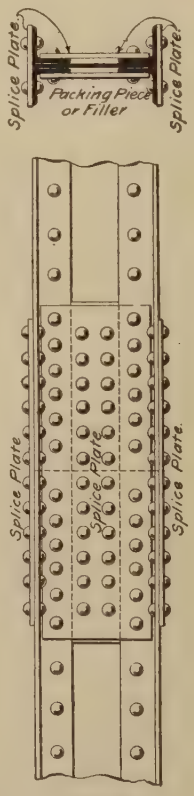


FIG. 22

The bedplate is sometimes extended beyond the column and forms a rest or support for the floorbeams or girders. If this is done, it is advisable to further support the bedplate by a bracket directly under the bearing of the beams, as shown in Fig. 23. If, in the same floor, the beams or girders are of different depths and are at the same level with regard to the top flange, the shallow beams may be supported by introducing cast-iron blocks beneath them, as shown at *a*, Fig. 23.

Where the floor girders are of the built-up plate-girder type, there is no difficulty in securing a rigid connection to the column. The connection to be recommended where the architectural features of

the interior arrangement of the building do not interfere is shown in Fig. 24. This connection is particularly efficient when the building is high and narrow, and in danger of being acted on by heavy wind pressure.

Other forms of connections, where plate girders are the

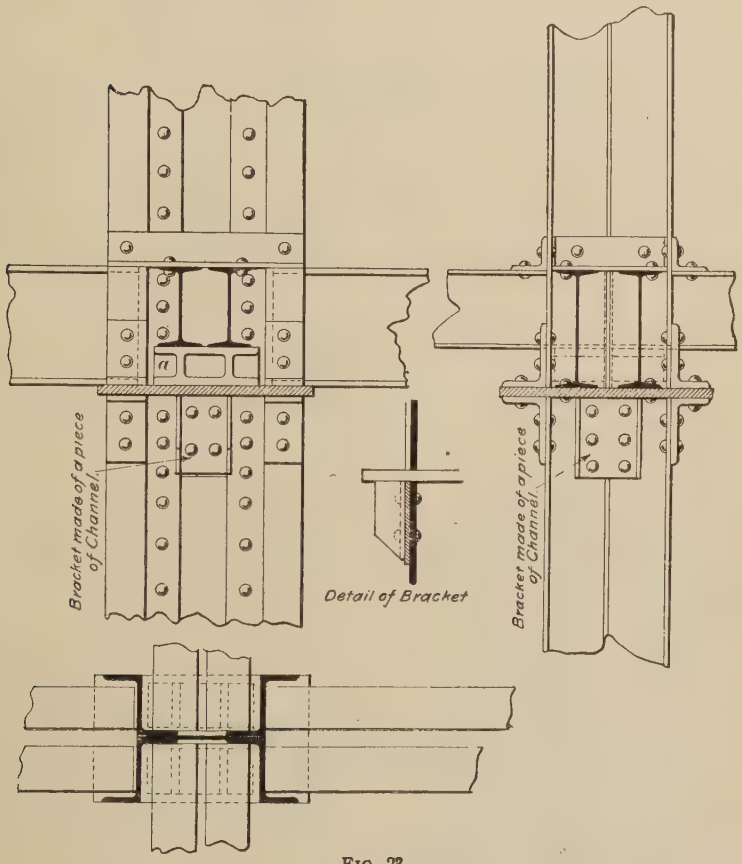


FIG. 23

principal supporting members of a floor system, are shown in Figs. 25 and 26. In Fig. 25, it will be seen that a filler is introduced at *c* because the upper column is smaller than the lower and it becomes necessary to pack between the splice plates. It is, however, preferable to pack equally on both

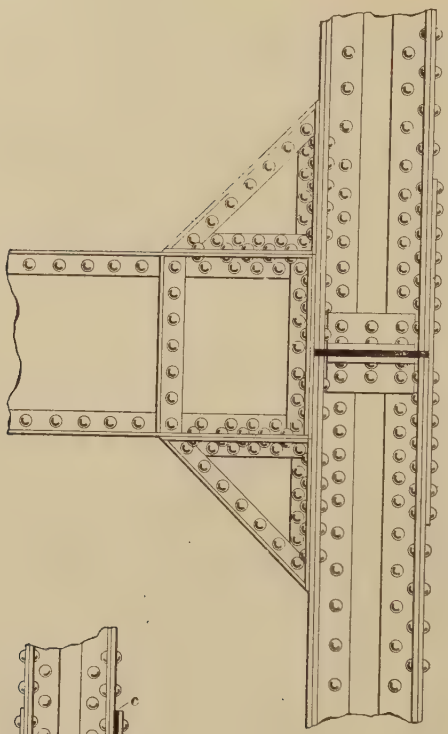


FIG. 24

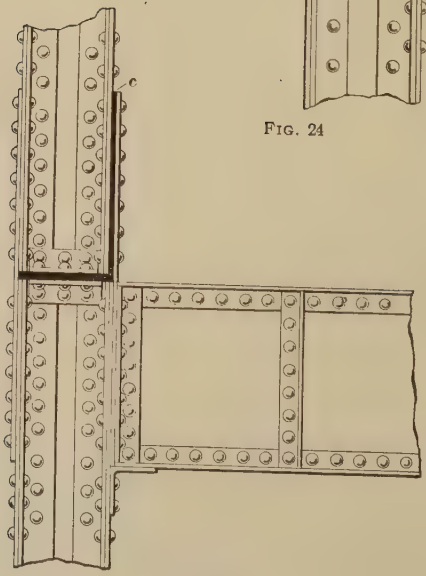


FIG. 25

sides of the upper column whenever practicable, as this insures the central axis of the one column being over the central axis of the other.

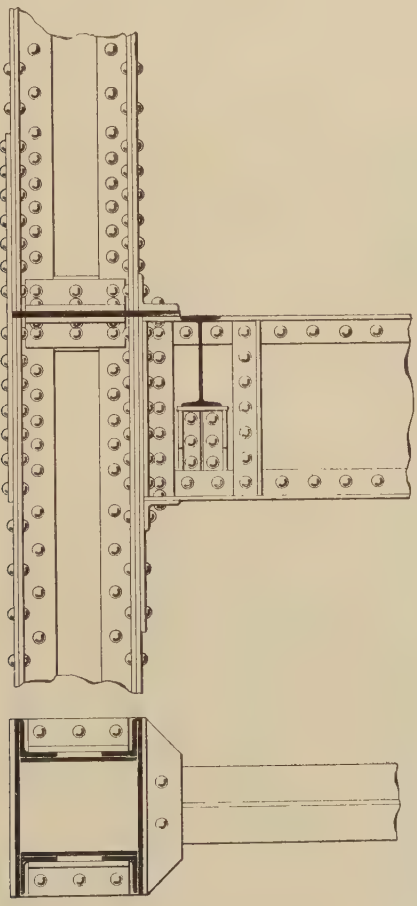


FIG. 26

25. Examples of Good Column Design.—In Figs. 27 and 28 are shown working drawings of the first-floor columns of a building designed according to the best modern practice.

Fig. 27 is a channel column. The floorbeams are rigidly secured to the column at a considerable distance below the

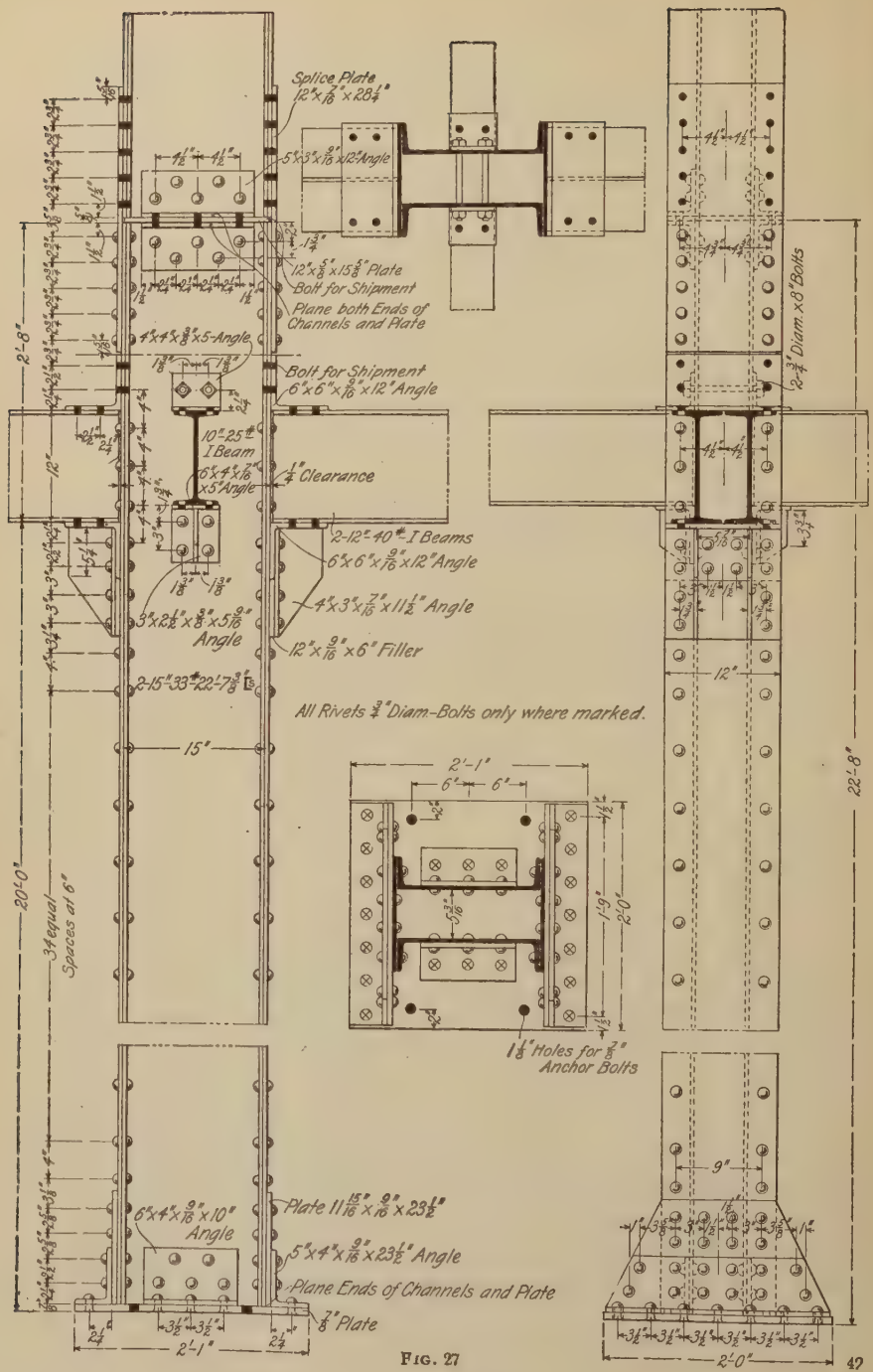

Splice Plate 12"x 7/16" x 28 1/4"
5"x3"x 9/16" x 12" Angle
12"x 5/8" x 15 5/8" Plate
Bolt for Shipment
Plane both Ends of Channels and Plate
4"x4"x 9/16" x 5" Angle
Bolt for Shipment
6"x6"x 9/16" x 12" Angle
10"-25# I Beam
6"x4"x 7/16" x 5" Angle
1/4" Clearance
12-12"-40# I Beams
6"x6"x 9/16" x 12" Angle
4"x3"x 7/16" x 11 1/2" Angle
12"x 9/16" x 6" Filler
3"x2 1/2"x 5/16" x 5 1/8" Angle
2-15"-33#-22'-7 3/8"
All Rivets 3/4" Diam-Bolts only where marked.
15"
20'-0"
3 Equal Spaces at 6"
2'-1"
6'-6"
1 1/8" Holes for 7/8" Anchor Bolts
6"x4"x 9/16" x 10" Angle
Plate 11 15/16" x 9/16" x 23 1/2"
5"x4"x 9/16" x 23 1/2" Angle
Plane Ends of Channels and Plate
7/8" Plate
2-3/4" Diam x 8" Bolts
22'-8"
12"
9"
2'-0"

FIG. 27

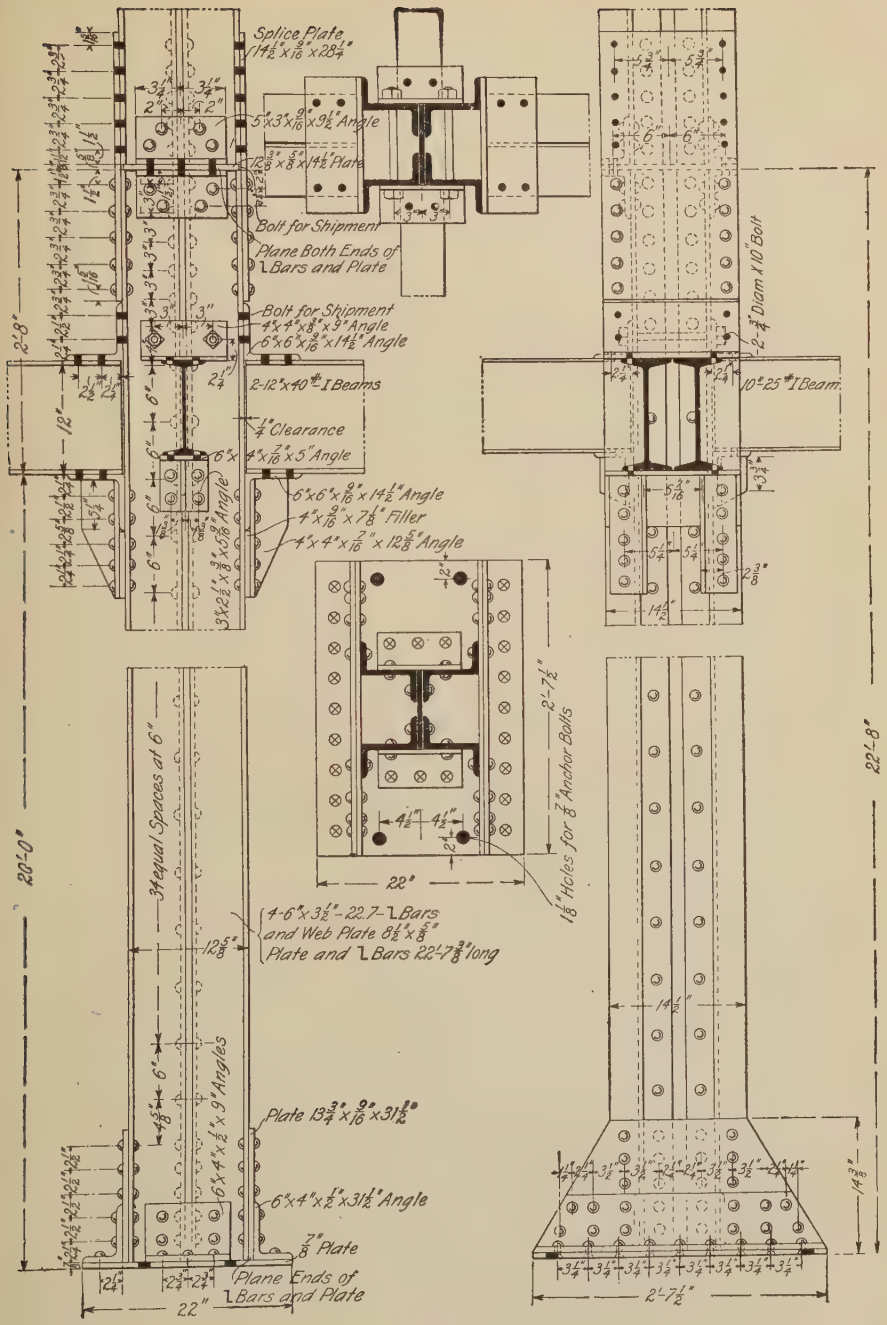

Splice Plate 14½" x 9/16" x 28¼"
5"x3"x9/16"x9½" Angle
12⅜" x 8" x 14½" Plate
Bolt for Shipment
Plane Both Ends of L Bars and Plate
Bolt for Shipment
4"x4"x9/16"x9" Angle
6"x6"x9/16"x14½" Angle
2-12"x40# I Beams
1" Clearance
6"x 4"x7/16"x5" Angle
6"x6"x9/16"x14½" Angle
4"x9/16"x7⅛" Filler
4"x4"x7/16"x12⅝" Angle
3"x2½"x3/8"x9/16" Angle
2-¾" Diam x 10" Bolt
10"-25# I Beam
2'-8"
12"
20'-0"
22'-8"
2'-7½"
22"
1⅛" Holes for ⅞" Anchor Bolts
34 equal Spaces at 6"
12⅝"
1-6"x3½"-22.7-L Bars and Web Plate 8½"x⅝" Plate and L Bars 22'-7⅞" long
Plate 13¾"x9/16"x31½"
6"x4"x7/16"x9" Angles
6"x4"x7/16"x31½" Angle
⅞" Plate
Plane Ends of L Bars and Plate
22"
14½"
14⅜"
2'-7½"

FIG. 28

splice, which has been designed with a bedplate between the two sections. The splice could have been made equally as well, and would possibly have been more efficient, by the use of side splice plates, as previously explained.

In Fig. 28 is shown a **Z**-bar column, the principal points in the detail of which are similar to those in the channel column.

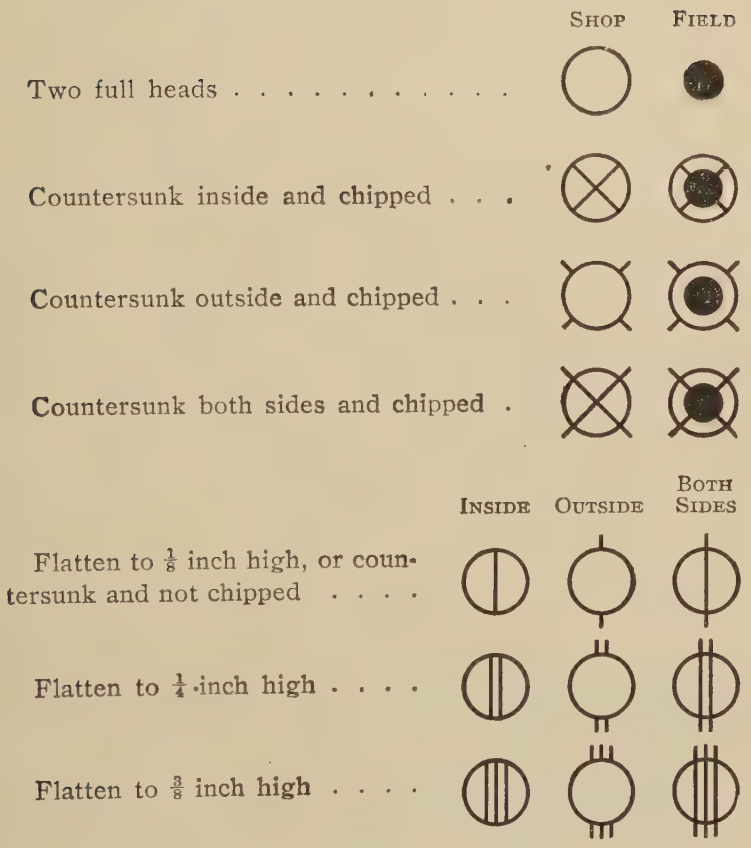
In each of the foregoing examples, the shear on the rivets supporting the floorbeam brackets has been calculated to safely sustain the load carried on the ends of the floorbeams.

All the connections and splices to a column should be riveted together with hot rivets; this insures more rigidity against wind pressure than can be obtained by bolted connections.

26. Rivet Signs.—In making drawings of structural-steel work, the form of rivet to be used should be designated by some system of conventional signs. The accompanying list shows Osborn's code of conventional signs for rivets, which has been adopted by many of the bridge companies and consulting engineers, and by many of the railway companies.

The basis of this system consists of the open circle to represent a shop rivet and the blackened circle to represent a field rivet, the diagonal cross to indicate a countersunk head and the vertical stroke to indicate a flattened head. The position of the diagonal lines with reference to the circle (inside, outside, or both) indicates whether the rivet head is countersunk into the inside, outside, or both sides of the material. Similarly, the number and position of the vertical strokes indicate the height and position of the flattened head. Any combination of shop or field rivets, with full, countersunk, or flattened heads, may be readily indicated by the proper combination of these signs. The diagonal cross indicates not only that the rivet head is to be countersunk, but that it is also to be chipped off even with the surrounding material; if the rivet is to be countersunk, but not chipped, the countersunk sign may be combined with the sign to flatten to $\frac{1}{8}$ inch.

CONVENTIONAL SIGNS FOR RIVETS


SHOP
FIELD
Two full heads . . . . . .
Countersunk inside and chipped . . .
Countersunk outside and chipped . . .
Countersunk both sides and chipped .
INSIDE
OUTSIDE
BOTH SIDES
Flatten to ⅛ inch high, or countersunk and not chipped . . . .
Flatten to ¼ inch high . . . .
Flatten to ⅜ inch high . . . .

In the case of simple flat joints the *front side*, or the side that is seen, is considered as the *outside*, while the *rear side*, or the side that is hidden, is considered as the *inside*.

27. Conventional Signs for Rolled Shapes.—When designating the rolled structural shapes, such as angles, tees, channels, and I beams, they should be represented by their conventional signs. For instance, a $6'' \times 6'' \times \frac{1}{2}''$ angle, weighing 20 pounds per foot, would be designated on the drawing as 1–$6'' \times 6'' \times \frac{1}{2}''$–20 # L, or, to be more explicit, it could be written 1–$6'' \times 6'' \times \frac{1}{2}''$ L–20 pounds per foot. If two

angles, or a pair of angles of this size, are used, they could be designated as 2–6″ × 6″ × ½″–20 # L's. It is not usual, however, to give the weight of an angle, when its thickness is given; consequently, a 6″ × 6″ × ½″ angle, weighing 20 pounds per foot, would generally be expressed on a drawing as 1–6″ × 6″ × ½″ L. A 12-inch channel, weighing 40 pounds per foot, would be marked on the drawing 1–12″–40 # C. Sometimes the length of the rolled section is marked on the drawing, together with its size and weight, as 2–6″ × 6″ × ½″ × 10′ 0″ L's.

The foregoing remarks, together with a careful study of the working drawings given in Figs. 27 and 28, are sufficient to give the student such information as will enable him to make a businesslike drawing of any column made up of rolled-steel sections. It is customary to put center lines through each rivet or rivet hole; these have, however, in most cases been omitted in Figs. 27 and 28 to make the drawings more clear.

REINFORCED-CONCRETE COLUMNS

28. The formulas here given will aid in designing reinforced-concrete columns.

Let A = area of concrete in column section, in square inches;

a = area of steel in column section, in square inches;

n = ratio of modulus of elasticity of steel to modulus of elasticity of concrete, which is about 8.25;

P = total load on column, in pounds;

S_c = unit stress, in pounds, produced in concrete by load P;

S_s = unit stress, in pounds, produced in steel by load P.

Then, $$P = S_c(A + na) \qquad (1)$$

Transposing, $$S_c = \frac{P}{A + na} \qquad (2)$$

This gives the unit compressive stress produced in the concrete by the load P.

Now, $$S_s = n S_c \qquad (3)$$

Therefore, $$S_s = \frac{n P}{A + n a} \qquad (4)$$

which gives the unit compressive stress in the steel in terms of the load P.

The average ultimate compressive unit stress of concrete may be taken from Table I in *Concrete Construction*. It can be assumed, however, to use even numbers, that the ultimate compressive unit stress of good concrete, 1-3-6 mixture, is about 2,000 pounds and the ultimate unit compressive stress of steel is 64,000 pounds. Now, suppose that a factor of safety of 5 is used, then the safe unit working stress of the concrete would be $2{,}000 \div 5 = 400$ pounds, and of the steel $64{,}000 \div 5 = 12{,}800$ pounds. Neither of these values may be exceeded. By formula **3**, if a stress of 400 pounds is produced in the concrete, the stress in the steel will be $400 \times 8.25 = 3{,}300$ pounds per square inch. It will therefore be seen that in reinforced-concrete columns, if the allowable unit stress in the concrete is not exceeded, the stress in the steel will not reach the allowable unit stress in that material. This is usually the case in practice. We are now in a position to design a column.

EXAMPLE 1.—Design a column to safely withstand a load of 100,000 pounds. Assume that the steel is 3 per cent. of the area of the concrete.

SOLUTION.—From formula **1**, $100{,}000 = 400 \times (A + 8.25\ a)$; therefore, $A + 8.25\ a = 250$. Since it is assumed that $a = \frac{3}{100} A$, it is

$$A + 8.25 \times \tfrac{3}{100} A = 250$$

$$1.2475 A = 250. \qquad A = 200 \text{ sq. in., nearly.}$$

$$a = \tfrac{3}{100} \times 200 = 6 \text{ sq. in.} \quad \text{Ans.}$$

EXAMPLE 2.—A column, whose area of cross-section consists of 250 square inches of concrete and 10 square inches of steel, supports a load of 120,000 pounds; what is the stress in the steel and in the concrete?

SOLUTION.—Applying formula **2**,

$$S_c = \frac{120{,}000}{250 + 8.25 \times 10} = 361 \text{ lb. per sq. in.}$$

$$S_s = 361 \times 8.25 = 2{,}978 \text{ lb. per sq. in.} \quad \text{Ans.}$$

In designing reinforced concrete, great care must be taken to conform to the building laws of the city in which the

structure is to be erected. Many cities allow a value for S_c of 350 pounds per square inch, but n is made equal to 12 or 15, or even 20. Some cities will allow a pressure of 500 pounds per square inch on the concrete but the strength of the steel, though present, must not enter into the calculation. In all reinforced-concrete-column formulas, it is assumed that the column fails by direct crushing and not by bending. Some building laws, therefore, specify that the height of the column may not exceed twelve to eighteen times the diameter of the column. Still other building laws consider that the function of the concrete is simply to keep the steel from bending sidewise and will not allow stress to be considered in the concrete, but will allow 16,000 pounds unit working stress for the steel. To be brief, there is no uniform method of calculating the strength of reinforced-concrete columns and every municipal building inspector has his own ideas on the subject.

29. In designing hooped concrete columns, there is no formula that is constructed by rigid analysis of the actual stresses produced, as these columns seem to fail by bending. Formula **1**, Art. **28**, may be used, however, by making a represent 2.4 times the area of the metal in the spiral as cut by a section right angles to the axis of the column. This formula, while not absolutely accurate, is close enough for practical purposes.

EXAMPLES FOR PRACTICE

1. A column has 100 square inches of concrete in section and 4 square inches of steel; what safe load will it carry with an allowable strength in the concrete of 400 pounds per sq. in.?

Ans. 53,200 lb.

2. A column 11 in. × 12 in. is reinforced with four angles, each containing 3 square inches in section with a load of 85,000 pounds; what stresses will be produced in the concrete and in the steel?

Ans. { In concrete, 388 lb.
In steel, 3,201 lb.

3. Design a round column, one-tenth steel and nine-tenths concrete, to hold a safe load of 4,000,000 pounds.

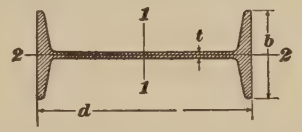
Ans. { Column $85\frac{7}{8}$ in. in diameter containing 580 sq. in. of steel

TABLES OF STEEL SHAPES

30. The following tables give the various weights, section moduli, moments of inertia, and radii of gyration of the more common steel shapes. These shapes are of standard section to which most of the steel mills conform. Almost every mill, however, rolls shapes that differ somewhat. The shapes referred to in the tables are rolled by the Cambria Steel Company, and conform more or less closely to the standards of other manufacturers. To obtain uniform results, they are used in all the examples in this Section.

TABLE III

PROPERTIES OF STANDARD I BEAMS



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Depth of Beam Inches	Weight per Foot Pounds	Area of Section Square Inches	Thickness of Web Inches	Width of Flange Inches	Moment of Inertia, Axis 1-1	Section Modulus, Axis 1-1	Radius of Gyration, Axis 1-1	Moment of Inertia, Axis 2-2	Radius of Gyration, Axis 2-2	Increase of Thickness of Web for Each Pound Increase in Weight. Inches	Coefficient of Strength		Coefficient of Deflection	
											For Fiber Stress of 16,000 Pounds per Square Inch for Buildings	For Fiber Stress of 12,500 Pounds per Square Inch for Bridges	Uniform Load	Center Load
d		*A*	*t*	*b*	*I*	*s*	*r*	*I'*	*r'*	*f'*	*F*	*F'*	*N*	*N'*
3	5.50	1.63	.17	2.33	2.5	1.7	1.23	.46	.53	.098	17,650	13,790	.00031253	.00050006
3	6.50	1.91	.26	2.42	2.7	1.8	1.19	.53	.52		19,140	14,950	.00028827	.00046124
3	7.50	2.21	.36	2.52	2.9	1.9	1.15	.60	.52		20,710	16,180	.00026644	.00042630
4	7.50	2.21	.19	2.66	6.0	3.0	1.64	.77	.59	.074	31,810	24,850	.00013009	.00020815
4	8.50	2.50	.26	2.73	6.4	3.2	1.59	.85	.58		33,890	26,480	.00012209	.00019535
4	9.50	2.79	.34	2.81	6.7	3.4	1.54	.93	.58		35,980	28,110	.00011500	.00018400
4	10.50	3.09	.41	2.88	7.1	3.6	1.52	1.01	.57		38,070	29,750	.00010868	.00017389
5	9.75	2.87	.21	3.00	12.1	4.8	2.05	1.23	.65	.059	51,590	40,300	.00006417	.00010267
5	12.25	3.60	.36	3.15	13.6	5.4	1.94	1.45	.63		58,100	45,390	.00005698	.00009117
5	14.75	4.34	.50	3.29	15.1	6.1	1.87	1.70	.63		64,630	50,490	.00005122	.00008195
6	12.25	3.61	.23	3.33	21.8	7.3	2.46	1.85	.72	.049	77,460	60,520	.00003561	.00005698
6	14.75	4.34	.35	3.45	24.0	8.0	2.35	2.09	.69		85,270	66,610	.00003235	.00005177
6	17.25	5.07	.47	3.57	26.2	8.7	2.27	2.36	.68		93,110	72,740	.00002963	.00004741

7	15.00	4.42	.25	3.66	36.2	10.4	2.86	2.67	.78	.042	110,410	86,260	.00002142	.00003427
7	17.50	5.15	.35	3.76	39.2	11.2	2.76	2.94	.76		119,400	93,290	.00001980	.00003168
7	20.00	5.88	.46	3.87	42.2	12.1	2.68	3.24	.74		128,560	100,430	.00001839	.00002942
8	18.00	5.33	.27	4.00	56.9	14.2	3.27	3.78	.84	.037	151,660	118,490	.00001364	.00002183
8	20.25	5.96	.35	4.08	60.2	15.0	3.18	4.04	.82		160,510	125,400	.00001289	.00002062
8	22.75	6.69	.44	4.17	64.1	16.0	3.10	4.36	.81		170,970	133,570	.00001210	.00001936
8	25.25	7.43	.53	4.26	68.0	17.0	3.03	4.71	.80		181,430	141,740	.00001140	.00001825
9	21.00	6.31	.29	4.33	84.9	18.9	3.67	5.16	.90	.033	201,300	157,260	.00000914	.00001462
9	25.00	7.35	.41	4.45	91.9	20.4	3.54	5.65	.88		217,930	170,260	.00000844	.00001350
9	30.00	8.82	.57	4.61	101.9	22.6	3.40	6.42	.85		241,460	188,640	.00000762	.00001219
9	35.00	10.29	.73	4.77	111.8	24.8	3.30	7.31	.84		264,990	207,020	.00000694	.00001110
10	25.00	7.37	.31	4.66	122.1	24.4	4.07	6.89	.97	.029	260,470	203,500	.00000635	.00001017
10	30.00	8.82	.45	4.80	134.2	26.8	3.90	7.65	.93		286,250	223,630	.00000578	.00000925
10	35.00	10.29	.60	4.95	146.4	29.3	3.77	8.52	.91		312,390	244,050	.00000530	.00000848
10	40.00	11.76	.75	5.10	158.7	31.7	3.67	9.50	.90		338,530	264,480	.00000489	.00000782
12	31.50	9.26	.35	5.00	215.8	36.0	4.83	9.50	1.01	.025	383,670	299,740	.00000360	.00000575
12	35.00	10.29	.44	5.09	228.3	38.0	4.71	10.07	.99		405,800	317,030	.00000340	.00000544
12	40.00	11.76	.56	5.21	245.9	41.0	4.57	10.95	.96		437,170	341,540	.00000316	.00000505
15	42.00	12.48	.41	5.50	441.8	58.9	5.95	14.62	1.08	.020	628,270	490,840	.00000176	.00000281
15	45.00	13.24	.46	5.55	455.8	60.8	5.87	15.09	1.07		648,310	506,490	.00000170	.00000272
15	50.00	14.71	.56	5.65	483.4	64.5	5.73	16.04	1.04		687,530	537,130	.00000161	.00000257
15	55.00	16.18	.66	5.75	511.0	68.1	5.62	17.06	1.03		726,740	567,770	.00000152	.00000243
15	60.00	17.65	.76	5.84	538.6	71.8	5.52	18.17	1.01		765,960	598,410	.00000144	.00000231
18	55.00	15.93	.46	6.00	795.6	88.4	7.07	21.19	1.15	.016	942,880	736,620	.00000098	.00000156
18	60.00	17.65	.56	6.10	841.8	93.5	6.91	22.38	1.13		997,680	779,440	.00000092	.00000148
18	65.00	19.12	.64	6.18	881.5	97.9	6.79	23.47	1.11		1,044,740	816,200	.00000088	.00000141
18	70.00	20.59	.72	6.26	921.2	102.4	6.69	24.62	1.09		1,091,800	852,970	.00000084	.00000135
20	65.00	19.08	.50	6.25	1,169.5	117.0	7.83	27.86	1.21	.015	1,247,490	974,600	.00000066	.00000106
20	70.00	20.59	.58	6.33	1,219.8	122.0	7.70	29.04	1.19		1,301,110	1,016,490	.00000064	.00000102
20	75.00	22.06	.65	6.40	1,268.8	126.9	7.58	30.25	1.17		1,353,400	1,057,340	.00000061	.00000098
24	80.00	23.32	.50	7.00	2,087.2	173.9	9.46	42.86	1.36	.0123	1,855,310	1,449,460	.00000037	.00000060
24	85.00	25.00	.57	7.07	2,167.8	180.7	9.31	44.35	1.33		1,926,950	1,505,430	.00000036	.00000057
24	90.00	26.47	.63	7.13	2,238.4	186.5	9.20	45.70	1.31		1,989,700	1,554,450	.00000035	.00000056
24	95.00	27.94	.69	7.19	2,309.0	192.4	9.09	47.10	1.30		2,052,440	1,603,470	.00000034	.00000054
24	100.00	29.41	.75	7.25	2,379.6	198.3	8.99	48.55	1.28		2,115,190	1,652,490	.00000033	.00000052

TABLE IV

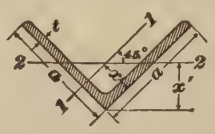
PROPERTIES OF STANDARD CHANNELS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Depth of Channel Inches	Weight per Foot Pounds	Area of Section Square Inches	Thickness of Web Inches	Width of Flange Inches	Moment of Inertia, Axis 1-1	Section Modulus, Axis 1-1	Radius of Gyration, Axis 1-1	Moment of Inertia, Axis 2-2	Section Modulus, Axis 2-2	Radius of Gyration, Axis 2-2	Distance of Center of Gravity From Outside of Web Inches	Increase of Thickness of Web for Each Pound Increase in Weight, Inches	Coefficient of Strength		Coefficient of Deflection	
													For Fiber Stress of 16,000 Pounds per Square Inch for Buildings	For Fiber Stress of 12,500 Pounds per Square Inch for Bridges	Uniform Load	Center Load
d		A	t	b	I	S	r	I'	S'	r'	x	f	F	F'	N	N'
3	4.00	1.19	.17	1.41	1.6	1.1	1.17	.20	.21	.41	.44	.098	11,630	9,090	.0004743	.0007589
3	5.00	1.47	.26	1.50	1.8	1.2	1.12	.25	.24	.41	.44		13,140	10,270	.0004199	.0006718
3	6.00	1.76	.36	1.60	2.1	1.4	1.08	.31	.27	.42	.46		14,710	11,490	.0003751	.0006001
4	5.25	1.55	.18	1.58	3.8	1.9	1.56	.32	.29	.45	.46	.074	20,230	15,800	.0002046	.0003273
4	6.25	1.84	.25	1.65	4.2	2.1	1.51	.38	.32	.45	.46		22,270	17,400	.0001858	.0002973
4	7.25	2.13	.33	1.73	4.6	2.3	1.46	.44	.35	.46	.46		24,360	19,030	.0001698	.0002717
5	6.50	1.95	.19	1.75	7.4	3.0	1.95	.48	.38	.50	.49	.059	31,640	24,720	.0001046	.0001674
5	9.00	2.65	.33	1.89	8.9	3.5	1.83	.64	.45	.49	.48		37,860	29,570	.0000875	.0001399
5	11.50	3.38	.48	2.04	10.4	4.2	1.75	.82	.54	.49	.51		44,390	34,680	.0000746	.0001193

6	8.00	2.38	.20	1.92	13.0	4.3	2.34	.70	.50	.54	.52	.049	46,210	36,100	.0000597	.0000855
6	10.50	3.09	.32	2.04	15.1	5.0	2.21	.88	.57	.53	.50		53,750	42,000	.0000513	.0000821
6	13.00	3.82	.44	2.16	17.3	5.8	2.13	1.07	.65	.53	.52		61,600	48,120	.0000448	.0000717
6	15.50	4.56	.56	2.28	19.5	6.5	2.07	1.28	.74	.53	.55		69,440	54,250	.0000397	.0000636
7	9.75	2.85	.21	2.09	21.1	6.0	2.72	.98	.63	.59	.55	.042	64,270	50,210	.0000368	.0000588
7	12.25	3.60	.32	2.20	24.2	6.9	2.59	1.19	.71	.57	.53		73,650	57,540	.0000321	.0000514
7	14.75	4.34	.42	2.30	27.2	7.8	2.50	1.40	.79	.57	.53		82,740	64,690	.0000286	.0000457
7	17.25	5.07	.53	2.41	30.2	8.6	2.44	1.62	.87	.56	.55		91,950	71,840	.0000257	.0000411
7	19.75	5.81	.63	2.51	33.2	9.5	2.39	1.85	.96	.56	.58		101,100	78,990	.0000234	.0000374
8	11.25	3.35	.22	2.26	32.3	8.1	3.10	1.33	.79	.63	.58	.037	86,140	67,300	.0000240	.0000384
8	13.75	4.04	.31	2.35	36.0	9.0	2.98	1.55	.87	.62	.56		95,990	75,000	.0000216	.0000345
8	16.25	4.78	.40	2.44	39.9	10.0	2.89	1.78	.95	.61	.56		106,450	83,170	.0000194	.0000311
8	18.75	5.51	.49	2.53	43.8	11.0	2.82	2.01	1.02	.60	.57		116,910	91,340	.0000177	.0000283
8	21.25	6.25	.58	2.62	47.8	11.9	2.76	2.25	1.11	.60	.59		127,370	99,510	.0000162	.0000260
9	13.25	3.89	.23	2.43	47.3	10.5	3.49	1.77	.97	.67	.61	.033	112,170	87,630	.0000164	.0000262
9	15.00	4.41	.29	2.49	50.9	11.3	3.40	1.95	1.03	.66	.59		120,540	94,170	.0000153	.0000244
9	20.00	5.88	.45	2.65	60.8	13.5	3.21	2.45	1.19	.65	.58		144,070	112,550	.0000128	.0000204
9	25.00	7.35	.61	2.81	70.7	15.7	3.10	2.98	1.36	.64	.62		167,590	130,930	.0000110	.0000176
10	15.00	4.46	.24	2.60	66.9	13.4	3.87	2.30	1.17	.72	.64	.029	142,680	111,470	.0000116	.0000186
10	20.00	5.88	.38	2.74	78.7	15.7	3.66	2.85	1.34	.70	.61		167,940	131,210	.0000099	.0000158
10	25.00	7.35	.53	2.89	91.0	18.2	3.52	3.40	1.50	.68	.62		194,090	151,630	.0000085	.0000136
10	30.00	8.82	.68	3.04	103.2	20.6	3.42	3.99	1.67	.67	.65		220,230	172,660	.0000075	.0000120
10	35.00	10.29	.82	3.18	115.5	23.1	3.35	4.66	1.87	.67	.69		246,380	192,480	.0000067	.0000107
12	20.50	6.03	.28	2.94	128.1	21.4	4.61	3.91	1.75	.81	.70	.025	227,750	177,930	.0000061	.0000097
12	25.00	7.35	.39	3.05	144.0	24.0	4.43	4.53	1.91	.78	.68		256,000	200,000	.0000054	.0000086
12	30.00	8.82	.51	3.17	161.6	26.9	4.28	5.21	2.09	.77	.68		287,370	224,510	.0000048	.0000077
12	35.00	10.29	.64	3.30	179.3	29.9	4.17	5.90	2.27	.76	.69		318,750	249,020	.0000043	.0000069
12	40.00	11.76	.76	3.42	196.9	32.8	4.09	6.63	2.46	.75	.72		350,120	273,530	.0000039	.0000063
15	33.00	9.90	.40	3.40	312.6	41.7	5.62	8.23	3.16	.91	.79	.020	444,520	347,280	.0000025	.0000040
15	35.00	10.29	.43	3.43	319.9	42.7	5.57	8.48	3.22	.91	.79		455,030	355,500	.0000024	.0000039
15	40.00	11.76	.52	3.52	347.5	46.3	5.44	9.39	3.43	.89	.78		494,250	386,130	.0000022	.0000036
15	45.00	13.24	.62	3.62	375.1	50.0	5.32	10.29	3.63	.88	.79		533,470	416,770	.0000021	.0000033
15	50.00	14.71	.72	3.72	402.7	53.7	5.23	11.22	3.85	.87	.80		572,680	447,410	.0000019	.0000031
15	55.00	16.18	.82	3.82	430.2	57.4	5.16	12.19	4.07	.87	.82		611,900	478,050	.0000018	.0000029

TABLE V

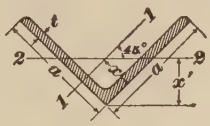
PROPERTIES OF STANDARD ANGLES HAVING EQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Flange. Inches	Moment of Inertia, Axis *1–1*	Section Modulus, Axis *1–1*	Radius of Gyration, Axis *1–1*	Distance of Center of Gravity From External Apex. Inches	Least Moment of Inertia, Axis *2–2*	Section Modulus, Axis *2–2*	Least Radius of Gyration, Axis *2–2*
$a \times a$	t		A	x	I	S	r	x'	I'	S'	r'
$\frac{3}{4} \times \frac{3}{4}$	$\frac{1}{8}$	.58	.17	.23	.009	.017	.22	.33	.004	.011	.14
$\frac{3}{4} \times \frac{3}{4}$	$\frac{3}{16}$	.84	.25	.25	.012	.024	.22	.36	.005	.014	.14
1×1	$\frac{1}{8}$	.80	.23	.30	.022	.031	.30	.42	.009	.021	.19
1×1	$\frac{3}{16}$	1.16	.34	.32	.030	.044	.30	.45	.013	.028	.19
1×1	$\frac{1}{4}$	1.49	.44	.34	.037	.056	.29	.48	.016	.034	.19
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{1}{8}$	1.02	.30	.36	.044	.049	.38	.51	.018	.035	.24
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{3}{16}$	1.47	.43	.38	.061	.071	.38	.54	.025	.047	.24
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{1}{4}$	1.91	.56	.40	.077	.091	.37	.57	.033	.057	.24
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{5}{16}$	2.32	.68	.42	.090	.109	.36	.60	.040	.066	.24
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{1}{8}$	1.23	.36	.42	.08	.072	.47	.60	.031	.053	.30
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{3}{16}$	1.79	.53	.44	.11	.104	.46	.63	.045	.072	.29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{1}{4}$	2.34	.69	.47	.14	.134	.45	.66	.058	.088	.29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{5}{16}$	2.86	.84	.49	.16	.162	.44	.69	.070	.101	.29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{3}{8}$	3.35	.98	.51	.19	.188	.44	.72	.082	.114	.29
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{7}{16}$	3.81	1.12	.53	.21	.214	.43	.75	.094	.126	.29
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{7}{16}$	2.11	.62	.51	.18	.14	.54	.72	.073	.10	.34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{1}{4}$	2.76	.81	.53	.23	.19	.53	.75	.094	.13	.34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{5}{16}$	3.39	1.00	.55	.27	.23	.52	.78	.113	.15	.34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{3}{8}$	3.98	1.17	.57	.31	.26	.51	.81	.133	.16	.34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{7}{16}$	4.56	1.34	.59	.35	.30	.51	.84	.152	.18	.34
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{1}{2}$	5.10	1.50	.61	.38	.33	.50	.87	.171	.20	.34
2×2	$\frac{7}{16}$	2.43	.71	.57	.27	.19	.62	.80	.11	.14	.39
2×2	$\frac{1}{4}$	3.19	.94	.59	.35	.25	.61	.84	.14	.17	.39
2×2	$\frac{5}{16}$	3.92	1.15	.61	.42	.30	.60	.87	.17	.20	.39
2×2	$\frac{3}{8}$	4.62	1.36	.64	.48	.35	.59	.90	.20	.22	.39
2×2	$\frac{7}{16}$	5.30	1.56	.66	.54	.40	.59	.93	.23	.25	.38
2×2	$\frac{1}{2}$	5.95	1.75	.68	.59	.45	.58	.96	.26	.27	.38
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{7}{16}$	3.1	.90	.69	.55	.30	.78	.98	.22	.22	.49
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{4}$	4.0	1.19	.72	.70	.39	.77	1.01	.29	.28	.49
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{5}{16}$	5.0	1.46	.74	.80	.48	.76	1.05	.35	.33	.49
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{3}{8}$	5.9	1.73	.76	.98	.57	.75	1.08	.41	.38	.48
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{7}{16}$	6.8	2.00	.78	1.11	.65	.75	1.11	.46	.42	.48
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{2}$	7.7	2.25	.81	1.23	.72	.74	1.14	.52	.46	.48
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{9}{16}$	8.5	2.50	.83	1.34	.80	.73	1.17	.58	.49	.48

TABLE V—(Continued)

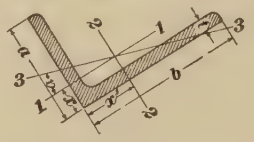
PROPERTIES OF STANDARD ANGLES HAVING EQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Flange. Inches	Moment of Inertia, Axis 1-1	Section Modulus, Axis 1-1	Radius of Gyration, Axis 1-1	Distance of Center of Gravity From External Apex. Inches	Least Moment of Inertia, Axis 2-2	Section Modulus, Axis 2-2	Least Radius of Gyration, Axis 2-2
$a \times a$	t		A	x	I	S	r	x'	I'	S'	r'
3 × 3	1/4	4.9	1.44	.84	1.24	.58	.93	1.19	.50	.42	.59
3 × 3	5/16	6.0	1.78	.87	1.51	.71	.92	1.22	.61	.50	.59
3 × 3	3/8	7.2	2.11	.89	1.76	.83	.91	1.26	.72	.57	.58
3 × 3	7/16	8.3	2.43	.91	1.99	.95	.91	1.29	.82	.64	.58
3 × 3	1/2	9.4	2.75	.93	2.22	1.07	.90	1.32	.92	.70	.58
3 × 3	9/16	10.4	3.06	.95	2.43	1.19	.89	1.35	1.02	.76	.58
3 × 3	5/8	11.4	3.36	.98	2.62	1.30	.88	1.38	1.12	.81	.58
3 × 3	11/16	12.4	3.65	1.00	2.81	1.40	.88	1.41	1.22	.86	.58
3½ × 3½	5/16	7.1	2.09	.99	2.45	.98	1.08	1.40	.99	.71	.69
3½ × 3½	3/8	8.4	2.48	1.01	2.87	1.15	1.07	1.43	1.16	.81	.68
3½ × 3½	7/16	9.8	2.87	1.04	3.26	1.32	1.07	1.46	1.33	.91	.68
3½ × 3½	1/2	11.1	3.25	1.06	3.64	1.49	1.06	1.50	1.50	1.00	.68
3½ × 3½	9/16	12.3	3.62	1.08	3.99	1.65	1.05	1.53	1.66	1.09	.68
3½ × 3½	5/8	13.5	3.98	1.10	4.33	1.81	1.04	1.56	1.82	1.17	.68
3½ × 3½	11/16	14.8	4.34	1.12	4.65	1.96	1.04	1.59	1 97	1.24	.67
3½ × 3½	3/4	15.9	4.69	1.15	4.96	2.11	1.03	1.62	2.13	1.31	.67
3½ × 3½	13/16	17.1	5.03	1.17	5.25	2.25	1.02	1.65	2.28	1.38	.67
3½ × 3½	7/8	18.3	5.36	1.19	5.53	2.39	1.02	1.68	2.43	1.45	.67
4 × 4	5/16	8.2	2.40	1.12	3.71	1.29	1.24	1.58	1.50	.95	.79
4 × 4	3/8	9.7	2.86	1.14	4.36	1.52	1.23	1.61	1.77	1.10	.79
4 × 4	7/16	11.3	3.31	1.16	4.97	1.75	1.23	1.64	2.02	1.23	.78
4 × 4	1/2	12.8	3.75	1.18	5.56	1.97	1.22	1.67	2.28	1.36	.78
4 × 4	9/16	14.2	4.18	1.21	6.12	2.19	1.21	1.71	2.52	1.48	.78
4 × 4	5/8	15.7	4.61	1.23	6.66	2.40	1.20	1.74	2.76	1.59	.77
4 × 4	11/16	17.1	5.03	1.25	7.17	2.61	1.19	1.77	3.00	1.70	.77
4 × 4	3/4	18.5	5.44	1.27	7.66	2.81	1.19	1.80	3.23	1.80	.77
4 × 4	13/16	19.9	5.84	1.29	8.14	3.01	1.18	1.83	3.46	1.89	.77
4 × 4	7/8	21.2	6.23	1.31	8.59	3.20	1.17	1.86	3.69	1.99	.77
6 × 6	3/8	14.8	4.36	1.64	15.39	3.53	1.88	2.32	6.19	2.67	1.19
6 × 6	7/16	17.2	5.06	1.66	17.68	4.07	1.87	2.34	7.13	3.04	1.19
6 × 6	1/2	19.6	5.75	1.68	19.91	4.61	1.86	2.38	8.04	3.37	1.18
6 × 6	9/16	21.9	6.43	1.71	22.07	5.14	1.85	2.41	8.94	3.70	1.18
6 × 6	5/8	24.2	7.11	1.73	24.16	5.66	1.84	2.45	9.81	4.01	1.17
6 × 6	11/16	26.4	7.78	1.75	26.19	6.17	1.83	2.48	10.67	4.31	1.17
6 × 6	3/4	28.7	8.44	1.78	28.15	6.66	1.83	2.51	11.52	4.59	1.17
6 × 6	13/16	30.9	9.09	1.80	30.06	7.15	1.82	2.54	12.35	4.86	1.17
6 × 6	7/8	33.1	9.73	1.82	31.92	7.63	1.81	2.57	13.17	5.12	1.16
6 × 6	15/16	35.3	10.37	1.84	33.72	8.11	1.80	2.60	13.98	5.37	1.16
6 × 6	1	37.4	11.00	1.86	35.46	8.57	1.80	2.64	14.78	5.61	1.16

TABLE VI

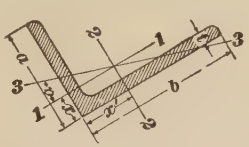
PROPERTIES OF STANDARD ANGLES HAVING UNEQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Longer Flange Inches	Moment of Inertia, Axis 1-1	Section Modulus, Axis 1-1	Radius of Gyration, Axis 1-1	Distance of Center of Gravity From Back of Shorter Flange, Inches	Moment of Inertia, Axis 2-2	Section Modulus, Axis 2-2	Radius of Gyration, Axis 2-2	Tangent of Angle	Least Radius of Gyration, Axis 3-3
$b \times a$	t		A	x	I	S	r	x'	I'	S'	r'	a	r''
2½ × 2	1/16	2.7	.81	.51	.29	.20	.60	.76	.51	.29	.79	.632	.43
2½ × 2	⅛	3.6	1.06	.54	.37	.25	.59	.79	.65	.38	.78	.626	.42
2½ × 2	3/16	4.5	1.31	.56	.45	.31	.58	.81	.79	.47	.78	.620	.42
2½ × 2	¼	5.3	1.55	.58	.51	.36	.58	.83	.91	.55	.77	.614	.42
2½ × 2	5/16	6.1	1.78	.60	.58	.41	.57	.85	1.03	.62	.76	.607	.42
2½ × 2	⅜	6.8	2.00	.63	.64	.46	.56	.88	1.14	.70	.75	.600	.42
2½ × 2	7/16	7.6	2.21	.65	.69	.51	.56	.90	1.24	.77	.75	.592	.42
3 × 2½	⅛	4.5	1.31	.66	.74	.40	.75	.91	1.17	.56	.95	.684	.53
3 × 2½	3/16	5.5	1.62	.68	.90	.49	.74	.93	1.42	.69	.94	.680	.53
3 × 2½	¼	6.5	1.92	.71	1.04	.58	.74	.96	1.66	.81	.93	.676	.52
3 × 2½	5/16	7.5	2.21	.73	1.18	.66	.73	.98	1.88	.93	.92	.672	.52
3 × 2½	⅜	8.5	2.50	.75	1.30	.74	.72	1.00	2.08	1.04	.91	.666	.52
3 × 2½	7/16	9.4	2.78	.77	1.42	.82	.72	1.02	2.28	1.15	.91	.661	.52
3 × 2½	½	10.4	3.05	.79	1.53	.90	.71	1.04	2.46	1.26	.90	.655	.52
3½ × 2½	⅛	4.9	1.44	.61	.78	.41	.74	1.11	1.80	.75	1.12	.506	.54
3½ × 2½	3/16	6.0	1.78	.64	.94	.50	.73	1.14	2.19	.93	1.11	.501	.54
3½ × 2½	¼	7.2	2.11	.66	1.09	.59	.72	1.16	2.56	1.09	1.10	.496	.54
3½ × 2½	5/16	8.3	2.43	.68	1.23	.68	.71	1.18	2.91	1.26	1.09	.491	.54
3½ × 2½	⅜	9.4	2.75	.70	1.36	.76	.70	1.20	3.24	1.41	1.09	.486	.53
3½ × 2½	7/16	10.4	3.06	.73	1.49	.84	.70	1.23	3.55	1.56	1.08	.480	.53
3½ × 2½	½	11.4	3.36	.75	1.61	.92	.69	1.25	3.85	1.71	1.07	.472	.53
3½ × 2½	9/16	12.4	3.65	.77	1.72	.99	.69	1.27	4.13	1.85	1.06	.468	.53
3½ × 2½	⅝	13.4	3.94	.79	1.83	1.07	.68	1.29	4.40	1.99	1.06	.461	.54
3½ × 3	5/16	6.6	1.93	.81	1.58	.72	.90	1.06	2.33	.95	1.10	.724	.63
3½ × 3	⅜	7.8	2.30	.83	1.85	.85	.90	1.08	2.72	1.13	1.09	.721	.62
3½ × 3	7/16	9.0	2.65	.85	2.09	.98	.89	1.10	3.10	1.29	1.08	.718	.62
3½ × 3	½	10.2	3.00	.88	2.33	1.10	.88	1.13	3.45	1.45	1.07	.714	.62
3½ × 3	9/16	11.4	3.34	.90	2.55	1.21	.87	1.15	3.79	1.61	1.07	.711	.62
3½ × 3	⅝	12.5	3.67	.92	2.76	1.33	.87	1.17	4.11	1.76	1.06	.707	.62
3½ × 3	11/16	13.6	4.00	.94	2.96	1.44	.86	1.19	4.41	1.91	1.05	.703	.62
3½ × 3	¾	14.7	4.31	.96	3.15	1.54	.85	1.21	4.70	2.05	1.04	.698	.62
3½ × 3	13/16	15.7	4.62	.98	3.33	1.65	.85	1.23	4.98	2.20	1.04	.694	.62
3½ × 3	⅞	16.8	4.92	1.00	3.50	1.75	.84	1.25	5.24	2.33	1.03	.689	.63
4 × 3	5/16	7.1	2.09	.76	1.65	.73	.89	1.26	3.38	1.23	1.27	.554	.65
4 × 3	⅜	8.4	2.48	.78	1.92	.87	.88	1.28	3.96	1.46	1.26	.551	.64
4 × 3	7/16	9.8	2.87	.80	2.18	.99	.87	1.30	4.52	1.68	1.25	.547	.64
4 × 3	½	11.1	3.25	.83	2.42	1.12	.86	1.33	5.05	1.89	1.25	.543	.64
4 × 3	9/16	12.3	3.62	.85	2.66	1.23	.86	1.35	5.55	2.09	1.24	.538	.64
4 × 3	⅝	13.5	3.98	.87	2.87	1.35	.85	1.37	6.03	2.30	1.23	.534	.64
4 × 3	11/16	14.8	4.34	.89	3.08	1.46	.84	1.39	6.49	2.49	1.22	.529	.64
4 × 3	¾	15.9	4.69	.92	3.28	1.57	.84	1.42	6.93	2.68	1.22	.524	.64
4 × 3	13/16	17.1	5.03	.94	3.47	1.68	.83	1.44	7.35	2.87	1.21	.518	.64
4 × 3	⅞	18.3	5.36	.96	3.66	1.79	.83	1.46	7.75	3.05	1.20	.512	.64

TABLE VI—(Continued)

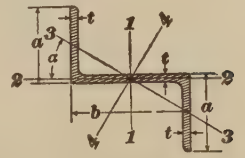
PROPERTIES OF STANDARD ANGLES HAVING UNEQUAL LEGS



1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dimensions Inches	Thickness Inches	Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Back of Longer Flange Inches	Moment of Inertia, Axis 1-1	Section Modulus, Axis 1-1	Radius of Gyration Axis 1-1	Distance of Center of Gravity From Back of Shorter Flange. Inches.	Moment of Inertia, Axis 2-2	Section Modulus, Axis 2-2	Radius of Gyration, Axis 2-2	Tangent of Angle	Least Radius of Gyration, Axis 3-3
$b \times a$	t		A	x	I	S	r	x'	I'	S'	r'	α	r''
5 × 3	5/16	8.2	2.40	.68	1.75	.75	.85	1.68	6.26	1.89	1.61	.368	.66
5 × 3	3/8	9.7	2.86	.70	2.04	.89	.84	1.70	7.37	2.24	1.61	.364	.65
5 × 3	7/16	11.3	3.31	.73	2.32	1.02	.84	1.73	8.43	2.58	1.60	.361	.65
5 × 3	1/2	12.8	3.75	.75	2.58	1.15	.83	1.75	9.45	2.91	1.59	.357	.65
5 × 3	9/16	14.2	4.18	.77	2.83	1.27	.82	1.77	10.43	3.23	1.58	.353	.65
5 × 3	5/8	15.7	4.61	.80	3.06	1.39	.82	1.80	11.37	3.55	1.57	.349	.64
5 × 3	11/16	17.1	5.03	.82	3.29	1.51	.81	1.82	12.28	3.86	1.56	.345	.64
5 × 3	3/4	18.5	5.44	.84	3.51	1.62	.80	1.84	13.15	4.16	1.55	.340	.64
5 × 3	13/16	19.9	5.84	.86	3.71	1.74	.80	1.86	13 98	4.46	1.55	.336	.64
5 × 3	7/8	21.2	6.23	.88	3.91	1.85	.79	1.88	14.78	4.75	1.54	.331	.64
5 × 3½	5/16	8.7	2.56	.84	2.72	1.02	1.03	1.59	6.60	1.94	1.61	.489	.77
5 × 3½	3/8	10.4	3.05	.86	3.18	1.21	1.02	1.61	7.78	2.29	1.60	.485	.76
5 × 3½	7/16	12.0	3.53	.88	3.63	1.39	1.01	1.63	8.90	2.64	1.59	.482	.76
5 × 3½	1/2	13.6	4.00	.91	4.05	1.56	1.01	1.66	9.99	2.99	1.58	.479	.75
5 × 3½	9/16	15.2	4.46	.93	4.45	1.73	1.00	1.68	11.03	3.32	1.57	.476	.75
5 × 3½	5/8	16.7	4.92	.95	4.83	1.90	.99	1.70	12.03	3.65	1.56	.472	.75
5 × 3½	11/16	18.3	5.37	.97	5.20	2.06	.98	1.72	12.99	3.97	1.56	.468	.75
5 × 3½	3/4	19.8	5.81	1.00	5.55	2.22	.98	1.75	13.92	4.28	1.55	.464	.75
5 × 3½	13/16	21.2	6.25	1.02	5.89	2.37	.97	1.77	14.81	4.58	1.54	.460	.75
5 × 3½	7/8	22.7	6.67	1.04	6.21	2.52	.96	1.79	15.67	4.88	1.53	.455	.75
5 × 3½	15/16	24.1	7.09	1.06	6.52	2.67	.96	1.81	16.49	5.17	1.53	.451	.75
6 × 3½	3/8	11.6	3.42	.79	3.34	1.23	.99	2.04	12.86	3.24	1.94	.350	.77
6 × 3½	7/16	13.5	3.96	.81	3.81	1.41	.93	2.06	14.76	3.75	1.93	.347	.76
6 × 3½	1/2	15.3	4.50	.83	4.25	1.59	.97	2.08	16.59	4.24	1.92	.344	.76
6 × 3½	9/16	17.1	5.03	.86	4.67	1.77	.96	2.11	18.37	4.72	1.91	.341	.75
6 × 3½	5/8	18.9	5.55	.88	5.08	1.94	.96	2.13	20.08	5.19	1.90	.338	.75
6 × 3½	11/16	20.6	6.06	.90	5.47	2.11	.95	2.15	21.74	5.65	1.89	.334	.75
6 × 3½	3/4	22.3	6.56	.93	5.84	2.27	.94	2.18	23.34	6.10	1.89	.331	.75
6 × 3½	13/16	24.0	7.06	.95	6.20	2.43	.94	2.20	24.89	6.55	1.88	.327	.75
6 × 3½	7/8	25.7	7.55	.97	6.55	2.59	.93	2.22	26.39	6.98	1.87	.323	.75
6 × 3½	15/16	27.3	8.03	.99	6.88	2.74	.93	2.24	27.84	7.41	1.86	.320	.75
6 × 3½	1	28.9	8.50	1.01	7.21	2.90	.92	2.26	29.15	7.80	1.85	.317	.75
6 × 4	3/8	12.3	3.61	.94	4.90	1.60	1.17	1.94	13.47	3.32	1.93	.446	.88
6 × 4	7/16	14.2	4.18	.96	5.60	1.85	1.16	1.96	15.46	3.83	1.92	.443	.87
6 × 4	1/2	16.2	4.75	.99	6.27	2.08	1.15	1.99	17.40	4.33	1.91	.440	.87
6 × 4	9/16	18.1	5.31	1.01	6.91	2.31	1.14	2.01	19.26	4.83	1.90	.438	.87
6 × 4	5/8	19.9	5.86	1.03	7.52	2.54	1.13	2.03	21.07	5.31	1.90	.434	.86
6 × 4	11/16	21.8	6.40	1.06	8.11	2.76	1.13	2.06	22.82	5.78	1.89	.431	.86
6 × 4	3/4	23.6	6.94	1.08	8.68	2.97	1.12	2.08	24.51	6.25	1.88	.428	.86
6 × 4	13/16	25.4	7.46	1.10	9.23	3.18	1.11	2.10	26.15	6.70	1.87	.425	.86
6 × 4	7/8	27.2	7.98	1.12	9.75	3.39	1.11	2.12	27.73	7.15	1.86	.421	.86
6 × 4	15/16	28.9	8.50	1.14	10.26	3.59	1.10	2.14	29.26	7.59	1.86	.418	.86
6 × 4	1	30.6	9.00	1.17	10.75	3.79	1.09	2.17	30.75	8.02	1.85	.414	.86

TABLE VII

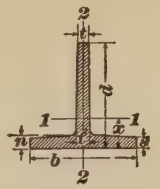
PROPERTIES OF Z BARS



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Depth of Bar Inches	Length of Legs Inches	Thickness of Web and Legs Inches	Weight per Foot Pounds	Area of Section Square Inches	Moment of Inertia, Axis 1–1	Section Modulus, Axis 1–1	Radius of Gyration, Axis 1–1	Moment of Inertia, Axis 2–2	Section Modulus, Axis 2–2	Radius of Gyration, Axis 2–2	Tangent of Angle	Least Radius of Gyration, Axis 3–3	Coefficient of Strength		Coefficient of Deflection	
													For Fiber Stress of 16,000 Pounds per Square Inch	For Fiber Stress of 12,500 Pounds per Square Inch	Uniform Load	Center Load
b	a	t		A	I	S	r	I'	S'	r'		r''	F	F'	N	N'
3	$2\frac{1}{16}$	$\frac{1}{4}$	6.7	1.97	2.87	1.92	1.21	2.81	1.10	1.19	.986	.55	20,400	16,000	.000270	.000432
3	$2\frac{3}{16}$	$\frac{5}{16}$	8.4	2.48	3.64	2.38	1.21	3.64	1.40	1.21	1.000	.55	25,400	19,800	.000213	.000341
3	$2\frac{1}{16}$	$\frac{3}{8}$	9.7	2.86	3.85	2.57	1.16	3.92	1.57	1.17	.990	.54	27,400	21,400	.000201	.000322
3	$2\frac{1}{16}$	$\frac{5}{16}$	11.4	3.36	4.57	2.98	1.17	4.75	1.88	1.19	.975	.55	31,800	24,800	.000170	.000272
3	$2\frac{1}{16}$	$\frac{1}{2}$	12.5	3.69	4.59	3.06	1.12	4.85	1.99	1.15	.965	.53	32,600	25,500	.000169	.000271
3	$2\frac{1}{4}$	$\frac{5}{16}$	14.2	4.18	5.26	3.43	1.12	5.68	2.30	1.17	.951	.54	36,600	28,600	.000148	.000236

4	3 1/16	1/4	8.2	2.41	6.28	3.14	1.62	4.23	1.44	1.33	.778	.67	33,500	26,200	.000123	.000198
4 1/16	3 1/8	1/8	10.3	3.03	7.94	3.91	1.62	5.46	1.84	1.34	.788	.68	41,700	32,600	.000098	.000156
4 1/8	3 3/16	3/8	12.4	3.66	9.63	4.67	1.62	6.77	2.26	1.36	.798	.69	49,800	38,900	.000081	.000129
4	3 1/8	1/16	13.8	4.05	9.66	4.83	1.54	6.73	2.37	1.29	.794	.66	51,500	40,200	.000080	.000129
4 1/16	3 1/8	1/4	15.8	4.66	11.18	5.50	1.55	7.96	2.77	1.31	.804	.67	58,700	45,900	.000069	.000111
4 1/8	3 3/16	1/16	17.9	5.27	12.74	6.18	1.55	9.26	3.19	1.32	.814	.68	65,900	51,500	.000061	.000097
4	3 1/16	5/8	18.9	5.55	12.11	6.05	1.48	8.73	3.18	1.25	.808	.65	64,600	50,500	.000064	.000103
4 1/16	3 1/8	1/16	20.9	6.14	13.52	6.65	1.48	9.95	3.58	1.27	.818	.67	71,000	55,500	.000057	.000092
4 1/8	3 3/16	3/4	23.0	6.75	14.97	7.26	1.49	11.24	4.00	1.29	.828	.68	77,400	60,500	.000052	.000083
5	3 1/4	5/16	11.6	3.40	13.36	5.34	1.98	6.18	2.00	1.35	.611	.75	57,000	44,500	.000058	.000093
5 1/16	3 5/16	3/8	13.9	4.10	16.18	6.39	1.99	7.65	2.45	1.37	.619	.76	68,200	53,300	.000048	.000077
5 1/8	3 3/8	7/16	16.4	4.81	19.07	7.44	1.99	9.20	2.92	1.38	.628	.76	79,400	62,000	.000041	.000065
5	3 1/4	1/2	17.9	5.25	19.19	7.68	1.91	9.05	3.02	1.31	.616	.74	81,900	64,000	.000040	.000065
5 1/16	3 5/16	9/16	20.2	5.94	21.83	8.62	1.92	10.51	3.47	1.33	.623	.75	92,000	71,900	.000036	.000057
5 1/8	3 3/8	5/8	22.6	6.64	24.53	9.57	1.92	12.06	3.94	1.35	.631	.76	102,100	79,800	.000032	.000051
5	3 1/4	11/16	23.7	6.96	23.68	9.47	1.84	11.37	3.91	1.28	.619	.73	101,000	78,900	.000033	.000052
5 1/16	3 5/16	3/4	26.0	7.64	26.16	10.34	1.85	12.83	4.37	1.30	.626	.74	110,200	86,100	.000030	.000047
5 1/8	3 3/8	13/16	28.3	8.33	28.70	11.20	1.86	14.37	4.84	1.31	.633	.76	119,500	93,300	.000027	.000043
6	3 1/2	3/8	15.6	4.59	25.32	8.44	2.35	9.11	2.75	1.41	.519	.83	90,000	70,300	.000031	.000049
6 1/16	3 5/16	7/16	18.3	5.39	29.80	9.83	2.35	10.94	3.27	1.43	.526	.83	104,900	81,900	.000026	.000042
6 1/8	3 3/8	1/2	21.0	6.19	34.36	11.22	2.36	12.87	3.81	1.44	.532	.84	119,700	93,500	.000023	.000036
6	3 1/2	9/16	22.7	6.68	34.64	11.55	2.28	12.59	3.91	1.37	.520	.81	123,200	96,200	.000022	.000036
6 1/16	3 5/16	5/8	25.4	7.46	38.87	12.82	2.28	14.41	4.44	1.39	.526	.82	136,800	106,800	.000020	.000032
6 1/8	3 3/8	11/16	28.1	8.25	43.18	14.10	2.29	16.34	4.98	1.41	.532	.84	150,400	117,500	.000018	.000029
6	3 1/2	3/4	29.3	8.63	42.12	14.04	2.21	15.44	4.94	1.34	.519	.81	149,800	117,000	.000018	.000029
6 1/16	3 5/16	13/16	31.9	9.39	46.13	15.22	2.22	17.27	5.47	1.36	.525	.82	162,300	126,800	.000017	.000027
6 1/8	3 3/8	7/8	34.6	10.17	50.22	16.40	2.22	19.18	6.02	1.37	.530	.83	174,900	136,700	.000015	.000025
7 1/2	3	3/8	16.3	4.78	38.19	10.18	2.83	5.59	1.99	1.08	.29	.72	108,600	84,800	.000020	.000033
8	3	3/8	16.9	4.97	44.64	11.16	3.00	5.60	1.99	1.06	.27	.72	119,000	93,000	.000017	.000028
10	3	3/8	19.4	5.72	76.87	15.37	3.67	5.60	1.99	.99	.19	.71	163,900	128,100	.000010	.000016

TABLE VIII—PROPERTIES OF T BARS



Equal Legs

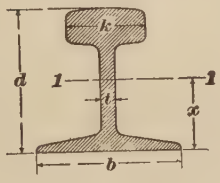
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Dimensions				Weight per Foot Pounds	Area of Section Square Inches	Distance of Center of Gravity From Outside of Flange Inches	Moment of Inertia, Axis 1–1	Section Modulus, Axis 1–1	Radius of Gyration, Axis 1–1	Moment of Inertia, Axis 2–2	Section Modulus, Axis 2–2	Radius of Gyration, Axis 2–2	Coefficient of Strength	
Width of Flange Inches	Depth of Bar Inches	Thickness of Flange Inches	Thickness of Stem Inches										For Fiber Stress of 16,000 Pounds per Square Inch	For Fiber Stress of 12,500 Pounds per Square Inch
b	d_1	s to n'	t to t'		A	x	I	S	r	I'	S'	r'	F	F'
1	1	1/8 to 3/32	1/8 to 3/32	.89	.26	.29	.02	.03	.30	.01	.02	.21	350	270
1 1/8	1 1/8	3/16 to 7/32	3/32 to 7/32	1.39	.41	.33	.04	.05	.32	.02	.04	.25	560	440
1 3/16	1 3/16	3/16 to 1/4	3/32 to 7/32	1.53	.45	.34	.05	.06	.33	.03	.05	.26	630	490
1 1/4	1 1/4	3/16 to 1/4	3/32 to 1/4	1.61	.47	.36	.06	.07	.35	.03	.05	.27	700	550
1 3/8	1 3/8	3/16 to 1/4	3/32 to 1/4	1.85	.54	.39	.08	.08	.39	.05	.07	.29	890	690
2	2	1/4 to 5/16	1/4 to 5/16	3.7	1.05	.59	.37	.26	.59	.18	.18	.42	2,770	2,160
2	2	5/16 to 3/8	5/16 to 3/8	4.3	1.26	.61	.43	.31	.59	.23	.23	.42	3,330	2,600
2 1/4	2 1/4	1/4 to 5/16	1/4 to 5/16	4.1	1.19	.68	.51	.32	.65	.24	.21	.45	3,440	2,690
2 1/4	2 1/4	5/16 to 3/8	5/16 to 3/8	4.9	1.45	.67	.64	.40	.66	.32	.29	.47	4,290	3,360
2 1/2	2 1/2	5/16 to 3/8	5/16 to 3/8	5.5	1.60	.73	.87	.49	.74	.44	.35	.52	5,250	4,100
3	3	5/16 to 3/8	5/16 to 3/8	6.6	1.95	.86	1.58	.74	.90	.75	.50	.62	7,860	6,140
3	3	3/8 to 7/16	3/8 to 7/16	7.8	2.27	.88	1.82	.86	.90	.92	.61	.64	9,180	7,180
3 1/2	3 1/2	3/8 to 7/16	3/8 to 7/16	9.3	2.74	.99	3.10	1.23	1.08	1.42	.81	.73	13,140	10,260

Unequal Legs

1 1/4	1 1/16	3/16 to 1/4	5/32 to 7/32	1.49	.44	.29	.04	.05	.29	.03	.01	.28	500	390
2 1/2	1 1/2	3/16 to 3/8	3/16 to 7/32	2.9	.84	.30	.08	.09	.31	.28	.22	.58	930	730
3	2 1/2	3/8 to 7/16	3/8 to 7/16	7.2	2.07	.71	1.08	.60	.64	.90	.60	.66	6,400	5,000
3 1/2	4	3/8 to 7/16	3/8 to 7/16	9.9	2.91	1.20	4.30	1.54	1.23	1.42	.81	.70	16,470	12,860
5	3	1/2 to 9/16	11/32 to 3/8	13.6	3.93	.72	2.42	1.06	.78	5.42	2.17	1.17	11,340	8,860

TABLE IX

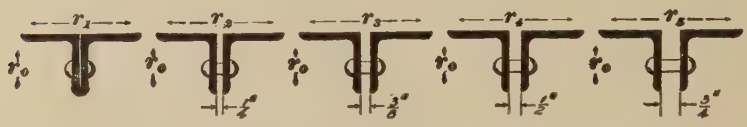
PROPERTIES AND PRINCIPAL DIMENSIONS OF STANDARD T RAILS



1	2	3	4	5	6	7	8	9
							Axis 1–1	
Weight per Yard Pounds	Area Square Inches	b Inches	d Inches	k Inches	t Inches	x Inches	Moment of Inertia	Section Modulus
							I	S
8	.78	$1\frac{1}{2}$	$1\frac{1}{2}$	$\frac{13}{16}$	$\frac{5}{32}$	.75	.23	.31
12	1.18	$1\frac{7}{8}$	$1\frac{7}{8}$	$1\frac{1}{16}$	$\frac{3}{16}$	.92	.55	.58
16	1.57	$2\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{3}{16}$	$\frac{7}{32}$	1.10	1.13	.99
20	2.00	$2\frac{3}{8}$	$2\frac{3}{8}$	$1\frac{3}{8}$	$\frac{1}{4}$	1.2	1.5	1.2
25	2.5	$2\frac{3}{8}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$\frac{19}{64}$	1.4	2.4	1.7
30	2.9	$3\frac{5}{16}$	3	$1\frac{11}{16}$	$\frac{11}{32}$	1.5	3.7	2.4
35	3.4	$3\frac{3}{8}$	$3\frac{1}{16}$	$1\frac{27}{32}$	$\frac{13}{32}$	1.5	4.3	2.8
40	3.9	$3\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{27}{32}$	$\frac{23}{64}$	1.7	6.0	3.4
45	4.4	$3\frac{3}{4}$	$3\frac{3}{4}$	$1\frac{59}{64}$	$\frac{3}{8}$	1.8	7.6	3.9
50	4.9	$3\frac{7}{8}$	$3\frac{7}{8}$	$2\frac{1}{8}$	$\frac{7}{16}$	1.9	10.1	5.1
55	5.4	$4\frac{1}{16}$	$4\frac{1}{16}$	$2\frac{1}{4}$	$\frac{15}{32}$	2.0	12.2	5.9
60	5.9	$4\frac{1}{4}$	$4\frac{1}{4}$	$2\frac{3}{8}$	$\frac{31}{64}$	2.1	14.7	6.7
65	6.4	$4\frac{7}{16}$	$4\frac{7}{16}$	$2\frac{13}{32}$	$\frac{1}{2}$	2.1	17.0	7.4
70	6.9	$4\frac{5}{8}$	$4\frac{5}{8}$	$2\frac{7}{16}$	$\frac{33}{64}$	2.2	20.0	8.4
75	7.4	$4\frac{13}{16}$	$4\frac{13}{16}$	$2\frac{15}{32}$	$\frac{17}{32}$	2.3	23.0	9.1
80	7.8	5	5	$2\frac{1}{2}$	$\frac{35}{64}$	2.4	26.7	10.1
85	8.3	$5\frac{3}{16}$	$5\frac{3}{16}$	$2\frac{9}{16}$	$\frac{9}{16}$	2.5	30.5	11.2
90	8.8	$5\frac{3}{8}$	$5\frac{3}{8}$	$2\frac{5}{8}$	$\frac{9}{16}$	2.6	35.2	12.6
100	9.8	$5\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{3}{4}$	$\frac{9}{16}$	2.8	44.4	15.0
150	14.7	6	6	$4\frac{1}{4}$	1	3.0	69.3	22.9

TABLE X

RADII OF GYRATION FOR TWO ANGLES, HAVING EQUAL LEGS, PLACED BACK TO BACK



Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
$\frac{3}{4} \times \frac{3}{4}$	$\frac{1}{8}$	.34	.22	.32	.42	.48	.53	.65
$\frac{3}{4} \times \frac{3}{4}$	$\frac{3}{16}$	.49	.22	.33	.44	.49	.55	.67
1×1	$\frac{1}{8}$	.47	.30	.42	.52	.57	.62	.74
1×1	$\frac{1}{4}$	.88	.29	.45	.55	.60	.66	.77
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{1}{8}$	.60	.38	.52	.62	.67	.72	.83
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{5}{16}$	1.37	.36	.56	.66	.71	.77	.88
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{3}{16}$	1.05	.46	.64	.73	.78	.83	.94
$1\frac{1}{2} \times 1\frac{1}{2}$	$\frac{3}{8}$	1.97	.44	.67	.77	.82	.88	.99
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{3}{16}$	1.24	.54	.74	.83	.88	.93	1.03
$1\frac{3}{4} \times 1\frac{3}{4}$	$\frac{7}{16}$	2.68	.51	.78	.88	.93	.98	1.09
2×2	$\frac{3}{16}$	1.43	.62	.84	.93	.98	1.03	1.13
2×2	$\frac{5}{16}$	2.30	.60	.86	.95	1.00	1.05	1.16
2×2	$\frac{7}{16}$	3.12	.59	.88	.98	1.03	1.08	1.19
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{3}{16}$	1.62	.70	.94	1.03	1.08	1.12	1.22
$2\frac{1}{4} \times 2\frac{1}{4}$	$\frac{3}{8}$	3.09	.67	.97	1.06	1.11	1.16	1.27
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{4}$	2.38	.77	1.05	1.14	1.19	1.24	1.34
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{3}{8}$	3.47	.75	1.07	1.16	1.21	1.26	1.36
$2\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{2}$	4.50	.74	1.09	1.19	1.24	1.29	1.39
$2\frac{3}{4} \times 2\frac{3}{4}$	$\frac{1}{16}$	1.99	.86	1.14	1.23	1.28	1.32	1.42
$2\frac{3}{4} \times 2\frac{3}{4}$	$\frac{5}{16}$	3.24	.84	1.16	1.25	1.30	1.35	1.45
$2\frac{3}{4} \times 2\frac{3}{4}$	$\frac{7}{16}$	4.43	.83	1.18	1.28	1.32	1.37	1.47
3×3	$\frac{1}{4}$	2.88	.93	1.26	1.34	1.39	1.43	1.53
3×3	$\frac{7}{16}$	4.87	.91	1.28	1.37	1.42	1.47	1.57
3×3	$\frac{5}{8}$	6.72	.88	1.32	1.41	1.46	1.51	1.61
$3\frac{1}{2} \times 3\frac{1}{2}$	$\frac{3}{8}$	4.97	1.07	1.48	1.56	1.61	1.66	1.75
$3\frac{1}{2} \times 3\frac{1}{2}$	$\frac{5}{8}$	7.97	1.04	1.52	1.61	1.66	1.71	1.81
$3\frac{1}{2} \times 3\frac{1}{2}$	$\frac{13}{16}$	10.05	1.02	1.55	1.65	1.70	1.75	1.85
4×4	$\frac{5}{16}$	4.80	1.24	1.67	1.76	1.80	1.85	1.94
4×4	$\frac{9}{16}$	8.37	1.21	1.71	1.80	1.85	1.89	1.99
4×4	$\frac{13}{16}$	11.68	1.18	1.75	1.85	1.89	1.94	2.04
$4\frac{1}{2} \times 4\frac{1}{2}$	$\frac{5}{16}$	5.43	1.40	1.87	1.96	2.00	2.05	2.14
$4\frac{1}{2} \times 4\frac{1}{2}$	$\frac{1}{2}$	8.50	1.38	1.90	1.99	2.04	2.08	2.18
$4\frac{1}{2} \times 4\frac{1}{2}$	$\frac{13}{16}$	10.47	1.36	1.92	2.01	2.06	2.10	2.20
5×5	$\frac{3}{8}$	7.22	1.56	2.09	2.17	2.22	2.26	2.35
5×5	$\frac{1}{2}$	9.50	1.54	2.10	2.19	2.24	2.28	2.38
5×5	$\frac{13}{16}$	11.72	1.52	2.12	2.21	2.26	2.30	2.40
6×6	$\frac{7}{16}$	10.12	1.87	2.50	2.58	2.63	2.67	2.76
6×6	$\frac{5}{8}$	14.22	1.84	2.53	2.62	2.66	2.71	2.80
6×6	$\frac{7}{8}$	19.47	1.81	2.57	2.66	2.70	2.75	2.85

TABLE XI

RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK

Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
2 × 1⅝	3/16	1.20	.63	.54	.62	.67	.72	.83
2 × 1⅝	3/8	2.25	.61	.56	.66	.71	.76	.88
2 × 1½	3/16	1.24	.63	.59	.68	.73	.78	.88
2 × 1½	3/8	2.34	.61	.62	.72	.77	.82	.93
2½ × 1¼	3/16	1.34	.80	.44	.52	.58	.63	.74
2½ × 1¼	3/8	2.53	.78	.47	.57	.62	.68	.79
2½ × 1½	3/16	1.43	.80	.55	.64	.69	.74	.84
2½ × 1½	3/8	2.72	.78	.58	.68	.73	.78	.89
2½ × 1¾	3/16	1.52	.80	.67	.75	.80	.85	.95
2½ × 1¾	¼	2.00	.79	.68	.77	.81	.86	.97
2½ × 2	3/16	1.62	.79	.79	.88	.92	.97	1.07
2½ × 2	3/8	3.09	.77	.82	.91	.96	1.01	1.12
2½ × 2	½	4.00	.75	.84	.94	.99	1.04	1.15
2¾ × 1½	3/16	1.52	.89	.53	.62	.67	.72	.82
2¾ × 1½	5/16	2.46	.87	.55	.65	.70	.75	.86
2¾ × 1½	7/16	3.34	.85	.58	.68	.73	.78	.89
3 × 2	3/16	1.80	.97	.75	.83	.88	.93	1.03
3 × 2	5/16	2.93	.95	.76	.85	.90	.95	1.05
3 × 2	7/16	3.99	.93	.79	.88	.93	.98	1.09
3 × 2½	¼	2.63	.95	1.00	1.09	1.13	1.18	1.28
3 × 2½	3/8	3.84	.93	1.02	1.11	1.16	1.21	1.31
3 × 2½	9/16	5.55	.91	1.05	1.15	1.20	1.25	1.35
3½ × 2½	¼	2.88	1.12	.96	1.04	1.09	1.13	1.23
3½ × 2½	½	5.50	1.09	1.00	1.09	1.14	1.19	1.29
3½ × 2½	11/16	7.30	1.06	1.03	1.13	1.18	1.23	1.33
3½ × 3	5/16	3.87	1.10	1.21	1.30	1.35	1.39	1.49
3½ × 3	9/16	6.68	1.07	1.25	1.34	1.39	1.44	1.54
3½ × 3	13/16	9.24	1.04	1.30	1.40	1.45	1.50	1.60

TABLE XI—(Continued)

RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK

Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
4 × 3	5/16	4.18	1.27	1.17	1.25	1.30	1.34	1.44
4 × 3	9/16	7.24	1.24	1.21	1.30	1.34	1.39	1.49
4 × 3	13/16	10.05	1.21	1.25	1.35	1.40	1.45	1.55
4 × 3½	5/16	4.49	1.26	1.42	1.50	1.55	1.59	1.69
4 × 3½	½	7.00	1.23	1.44	1.53	1.58	1.63	1.72
4 × 3½	⅝	8.59	1.22	1.46	1.55	1.60	1.65	1.75
4½ × 3	⅜	5.34	1.44	1.14	1.22	1.27	1.31	1.41
4½ × 3	½	7.00	1.42	1.15	1.24	1.29	1.34	1.44
4½ × 3	⅝	8.59	1.40	1.18	1.27	1.31	1.36	1.46
5 × 3	5/16	4.80	1.61	1.09	1.17	1.22	1.26	1.36
5 × 3	9/16	8.37	1.58	1.13	1.22	1.26	1.31	1.41
5 × 3	13/16	11.68	1.55	1.17	1.27	1.32	1.37	1.47
5 × 3½	⅜	6.09	1.60	1.34	1.42	1.46	1.51	1.60
5 × 3½	⅝	9.84	1.56	1.37	1.46	1.51	1.56	1.66
5 × 3½	⅞	13.34	1.53	1.42	1.51	1.56	1.61	1.71
5 × 4	⅜	6.47	1.59	1.58	1.66	1.71	1.75	1.85
5 × 4	½	8.50	1.57	1.60	1.68	1.73	1.78	1.87
5 × 4	⅝	10.47	1.55	1.62	1.71	1.75	1.80	1.90
6 × 3½	⅜	6.84	1.94	1.26	1.34	1.39	1.43	1.53
6 × 3½	⅝	11.09	1.90	1.30	1.39	1.43	1.48	1.58
6 × 3½	⅞	15.09	1.87	1.34	1.44	1.49	1.53	1.64
6 × 4	⅜	7.22	1.93	1.50	1.58	1.62	1.67	1.76
6 × 4	⅝	11.72	1.90	1.53	1.62	1.67	1.71	1.81
6 × 4	⅞	15.97	1.86	1.58	1.67	1.71	1.76	1.86
7 × 3½	7/16	8.80	2.26	1.16	1.29	1.33	1.38	1.47
7 × 3½	½	10.00	2.25	1.22	1.30	1.35	1.39	1.48
7 × 3½	⅝	12.34	2.24	1.24	1.32	1.37	1.42	1.51
7 × 3½	13/16	15.74	2.21	1.27	1.36	1.41	1.46	1.56
7 × 3½	1	19.00	2.19	1.31	1.40	1.45	1.50	1.60

TABLE XII

RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK

Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
2 × 1⅜	3/16	1.20	.41	.92	1.01	1.06	1.11	1.22
2 × 1⅜	⅜	2.25	.38	.95	1.05	1.10	1.15	1.26
2 × 1½	3/16	1.24	.44	.90	.99	1.05	1.09	1.20
2 × 1½	⅜	2.34	.42	.93	1.09	1.14	1.19	1.29
2½ × 1¼	3/16	1.34	.33	1.21	1.31	1.36	1.41	1.51
2½ × 1¼	⅜	2.53	.32	1.25	1.35	1.40	1.45	1.56
2½ × 1½	3/16	1.43	.42	1.17	1.26	1.31	1.36	1.47
2½ × 1½	⅜	2.72	.40	1.20	1.30	1.35	1.40	1.51
2½ × 1¾	3/16	1.52	.51	1.13	1.23	1.27	1.32	1.43
2½ × 1¾	¼	2.00	.50	1.14	1.24	1.29	1.34	1.44
2½ × 2	3/16	1.62	.60	1.10	1.19	1.24	1.29	1.39
2½ × 2	⅜	3.09	.58	1.13	1.23	1.28	1.33	1.43
2½ × 2	½	4.00	.56	1.15	1.25	1.30	1.35	1.46
2¾ × 1½	3/16	1.52	.41	1.31	1.40	1.45	1.50	1.60
2¾ × 1½	5/16	2.46	.40	1.33	1.43	1.48	1.53	1.63
2¾ × 1½	7/16	3.34	.39	1.36	1.45	1.51	1.56	1.66
3 × 2	3/16	1.80	.58	1.37	1.46	1.51	1.56	1.66
3 × 2	5/16	2.93	.57	1.39	1.48	1.53	1.58	1.68
3 × 2	7/16	3.99	.55	1.41	1.51	1.56	1.61	1.71
3 × 2½	¼	2.63	.75	1.31	1.40	1.45	1.50	1.60
3 × 2½	⅜	3.84	.74	1.33	1.42	1.47	1.52	1.63
3 × 2½	9/16	5.55	.72	1.37	1.46	1.51	1.56	1.66
3½ × 2½	¼	2.88	.74	1.58	1.67	1.72	1.76	1.86
3½ × 2½	½	5.50	.70	1.62	1.72	1.77	1.81	1.92
3½ × 2½	11/16	7.30	.69	1.66	1.75	1.80	1.86	1.96
3½ × 3	5/16	3.87	.90	1.52	1.61	1.66	1.71	1.80
3½ × 3	9/16	6.68	.87	1.57	1.66	1.71	1.76	1.86
3½ × 3	13/16	9.24	.85	1.61	1.71	1.76	1.81	1.91

TABLE XII—(Continued)

RADII OF GYRATION FOR TWO ANGLES, HAVING UNEQUAL LEGS, PLACED BACK TO BACK

Dimensions Inches	Thickness Inches	Area of Two Angles Square Inches	Radii of Gyration					
			r_0	r_1	r_2	r_3	r_4	r_5
4 × 3	5/16	4.18	.89	1.79	1.88	1.93	1.97	2.07
4 × 3	9/16	7.24	.86	1.83	1.93	1.97	2.02	2.12
4 × 3	13/16	10.05	.83	1.88	1.97	2.02	2.08	2.18
4 × 3½	5/16	4.49	1.07	1.73	1.81	1.86	1.91	2.00
4 × 3½	½	7.00	1.04	1.76	1.85	1.89	1.94	2.04
4 × 3½	⅝	8.59	1.02	1.78	1.87	1.92	1.97	2.07
4½ × 3	⅜	5.34	.86	2.07	2.16	2.21	2.26	2.35
4½ × 3	½	7.00	.85	2.09	2.18	2.23	2.28	2.38
4½ × 3	⅝	8.59	.83	2.11	2.21	2.26	2.31	2.41
5 × 3	5/16	4.80	.85	2.33	2.42	2.47	2.52	2.61
5 × 3	9/16	8.37	.82	2.37	2.47	2.52	2.57	2.67
5 × 3	13/16	11.68	.80	2.42	2.52	2.57	2.62	2.72
5 × 3½	⅜	6.09	1.02	2.27	2.36	2.41	2.45	2.55
5 × 3½	⅝	9.84	.99	2.31	2.40	2.45	2.50	2.60
5 × 3½	⅞	13.34	.96	2.36	2.45	2.50	2.55	2.65
5 × 4	⅜	6.47	1.20	2.20	2.29	2.34	2.38	2.48
5 × 4	½	8.50	1.18	2.22	2.31	2.36	2.41	2.50
5 × 4	⅝	10.47	1.17	2.24	2.33	2.38	2.43	2.53
6 × 3½	⅜	6.84	.99	2.81	2.90	2.95	3.00	3.09
6 × 3½	⅝	11.09	.96	2.86	2.95	3.00	3.05	3.15
6 × 3½	⅞	15.09	.93	2.90	3.00	3.05	3.10	3.20
6 × 4	⅜	7.22	1.17	2.74	2.83	2.87	2.92	3.02
6 × 4	⅝	11.72	1.13	2.78	2.87	2.92	2.97	3.06
6 × 4	⅞	15.97	1.11	2.82	2.92	2.97	3.02	3.12
7 × 3½	7/16	8.80	.95	3.37	3.47	3.52	3.56	3.66
7 × 3½	½	10.00	.94	3.39	3.48	3.53	3.58	3.67
7 × 3½	⅝	12.34	.93	3.40	3.50	3.55	3.60	3.70
7 × 3½	13/16	15.64	.91	3.45	3.54	3.59	3.64	3.74
7 × 3½	1	19.00	.89	3.48	3.58	3.63	3.68	3.78

DESIGN OF ROOF TRUSSES

LOADS ON ROOF TRUSSES

LIVE AND DEAD LOADS

1. Live Loads.—The loads on a roof truss may be divided into four classes, namely, live, dead, snow, and wind. The **live loads** on roof trusses differ greatly in character. The load may be due to a hoisting crane or to a line of shafting attached to the chords; in some instances, also, roofs are used for summer gardens, gymnasiums, and the like. As a rule, however, a roof has no live load and the designer may omit it entirely in his calculations.

2. Dead Loads.—In obtaining the **dead load** on roof trusses, it is necessary, after having found the weight of the sheathing and roofing, to add a certain weight per square foot, to represent the weight of the truss or members supporting the roof. Not knowing, as yet, the size and weight of the different members in the roof truss, approximate weights must be assumed.

Table I gives the approximate weights of the trusses, or principals, as they are called, for roofs of different spans. These weights are, of course, only assumed, and may not be within 25 per cent. of the actual weight of the principals. They are, however, generally on the side of safety.

It is required, in the application of Table I, to obtain the weight in pounds per square foot of *roof surface*. As the

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weights given in the table are in pounds per square foot of *area covered*, and as the area of the roof is considerably greater than this, owing to the pitch of the roof, it is necessary to divide the area covered by the area of the roof and multiply the result by the quantities given in the table. For example, the area of a building covered by a roof with a

TABLE I

POUNDS TO BE ADDED FOR WEIGHT OF PRINCIPALS, OR ROOF TRUSSES

Length of Span Feet	Weight per Square Foot of Area Covered Pounds
Up to 40	4
40 to 60	5
60 to 80	6
80 to 100	7

span of 50 feet is 10,000 square feet, and the area of the roof is 15,000 square feet; $10{,}000 \div 15{,}000 = \frac{2}{3}$, or .67, and, since the span of the roof is 50 feet, according to Table I, the weight of the truss is 5 pounds for each square foot of area covered. Therefore, $5 \times .67 = 3.35$ pounds must be added to the weight of each square foot of roof surface.

EXAMPLE.—The span of a roof truss is 40 feet, and its rise is 10 feet; what weight per square foot of roof surface should be assumed so as to allow for the weight of the principal or roof truss?

SOLUTION.—The area covered is 40 ft. per ft. in length of the building, while the area of the roof for the same length by the principles in geometry is $2\sqrt{10^2 + 20^2}$ or $2\sqrt{500}$. From Table I, the weight per square foot of area covered is 4 lb. for a span of 40 ft. Therefore, the weight per square foot of roof area is

$$4 \times \frac{40}{2\sqrt{500}} = 3.58 \text{ lb.} \quad \text{Ans.}$$

SNOW AND WIND LOADS

3. Snow Loads.—In calculating the weight on roofs, there are two other loads always to be considered when obtaining the stresses on the various members of the truss; these are snow and wind loads. Where the roof is comparatively flat, the **snow load** is estimated at 20 pounds per square foot of roof area; for a roof that has a steep slope, or a rise of more than 12 inches per foot of horizontal distance, it is good practice to assume the snow load to be 12 pounds per square foot of roof area. In the usual style of truss the snow load is conveniently included with the dead load.

4. Wind Pressure.—**Wind pressure** on roofs is always assumed as acting normal (that is, perpendicular) to the slope. In Fig. 1 the outline *a b c* of a roof is shown; the force *d* is normal to the slope *a b*, and represents the assumed pressure of the wind on the roof. The wind generally acts in a horizontal direction, as shown by the full arrow *e*. The maximum horizontal pressure of the wind is always considered to be 40 pounds per square foot; this pressure represents a wind velocity of from 80 to 100 miles per hour, which is a violent hurricane in intensity, and as this velocity is seldom realized and never exceeded except in cyclonic storms, the assumption may be considered reasonably safe. The wind, blowing with a horizontal pressure of 40 pounds, strikes the roof at an angle; consequently, the pressure *d*, normal to the slope, is considerably less than 40 pounds.

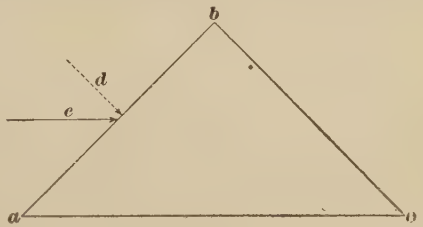


FIG. 1

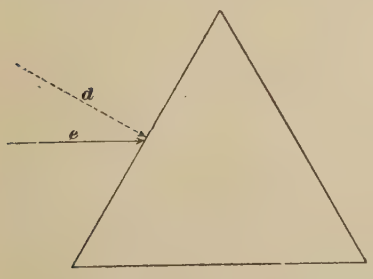


FIG. 2

unless the slope of the roof is very steep. Referring to Figs. 2 and 3 it is clear that the horizontal force *e* of the wind on the slope of the roof, shown in Fig. 2, is almost as intense as on a vertical surface; on the extremely flat roof in Fig. 3, however, the wind exerts hardly any force at all normal to the

FIG. 3

slope, because it strikes the slope at such an acute angle that it has a tendency to slide along and off. The more acute the angle between the lines *e* and *d*, the greater is the pressure normal to the slope; whereas, the greater the distance these lines are apart or the greater the angle, the less is the

TABLE II

WIND PRESSURE NORMAL TO THE SLOPE OF ROOF

Rise Inches per Foot Horizontal	Angle of Slope With Horizontal		Pitch, Proportion of Rise to Span	Wind Pressure Normal to Slope Pounds per Square Foot
	Degrees	Minutes		
4	18	25	$\frac{1}{6}$	16.8
6	26	33	$\frac{1}{4}$	23.7
8	33	42	$\frac{1}{3}$	29.1
12	45	0	$\frac{1}{2}$	36.1
16	53	7	$\frac{2}{3}$	38.7
18	56	20	$\frac{3}{4}$	39.3
24	63	27	1	40.0

pressure normal to the slope, until they form a right angle with each other, where the pressure on the roof may be disregarded. On the basis of a horizontal wind pressure of 40 pounds, the pressure normal to the slope has been reckoned by a formula known as *Hutton's formula*. This formula,

being complicated, is not given here, but results derived from it are given in Table II.

5. In order to more fully explain Table II, reference is made to Fig. 4. The rise in the slope ab is 6 inches for every 12 inches on the horizontal line ac; for instance, at 4 feet from a on the horizontal line ac, the rise is four times 6 inches, or 2 feet, the angle included between the line of slope ab and the horizontal base line ac is 26° 33′, and the

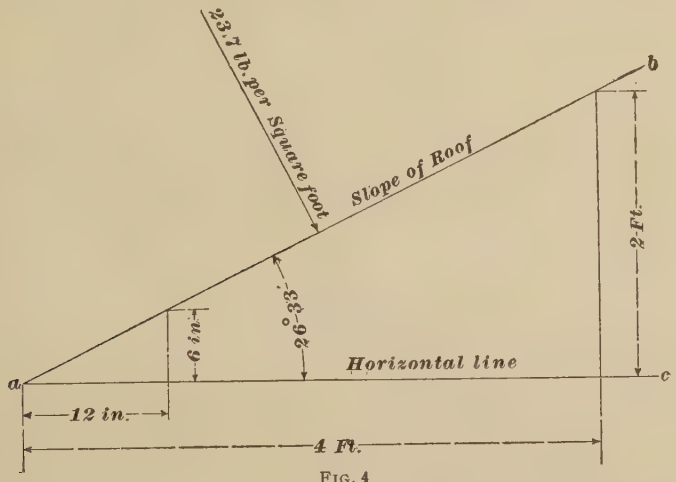


FIG. 4

pressure normal to the slope, according to Table II, is assumed at 23.7 pounds per square foot. Since the rise at the center is equal to one-half the length of one-half the span, the total rise is one-quarter of the span. Under these conditions, the pitch of the roof, that is, the ratio of the rise to the span, is $\frac{1}{4}$, and the roof is said to be $\frac{1}{4}$ *pitch*.

EXAMPLE.—(*a*) What will be the dead load, per square foot of roof surface, on a roof with a 12-inch rise, the span of the trusses being 50 feet, the roof covering 1 inch white-pine sheathing, two layers of Neponset roofing felt, and $\frac{1}{8}$-inch slate 3-inch lap? (*b*) What will be the wind pressure per square foot normal to the slope? (*c*) If the roof trusses are placed 12 feet apart, what will be the entire dead load on one truss? Fig. 5 shows a plan with elevation and detail section of the roof.

SOLUTION.—(*a*) By referring to Table I, it is seen that the approximate weight of a roof truss with a span of 50 ft. is 5 lb. for every square

foot of area covered. It is first necessary to obtain the length of the line of slope $a\,b$; this is done by calculating the hypotenuse of the triangle, or by laying the figure out to scale and measuring. In this case, it is found that $a\,b$ measures about 35 ft. 4 in., equal to 35.33 ft.

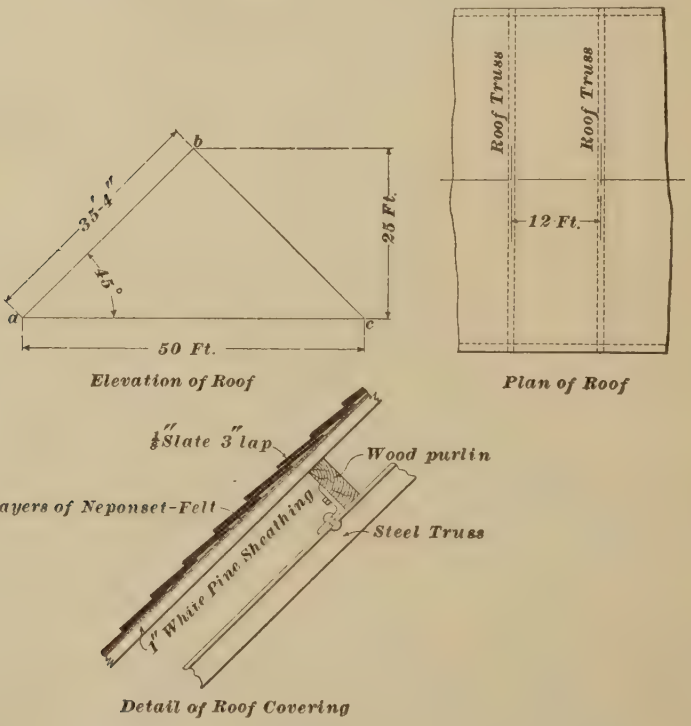


Fig. 5

The area covered by the roof supported on one truss is $12 \times 50 = 600$ sq. ft. The area of the roof supported by one truss is

$$2 \times 35.33 \times 12 = 847.92 \text{ sq. ft.}$$

Then, $600 \div 847.92 = .7$, which means that the weight of the truss per square foot of roof surface is .7 times 5 lb., or $5 \times .7 = 3.5$ lb. With the assistance of a table given in *Design of Beams*, the dead load per square foot of roof surface is, then, as follows:

Weight of supporting truss	3.5 lb. per sq. ft.
Weight of white-pine sheathing 1 in. thick . .	2.5 lb. per sq. ft.
Weight of two layers of Neponset roofing paper	.5 lb. per sq. ft.
Weight of slate ($\frac{1}{8}$ in. thick)	4.5 lb. per sq. ft.
Total	11.0 lb. per sq. ft.

Ans.

The weight of the purlins supporting the sheathing has not been estimated, it being safe in this case to assume that the weight used for the principals, or trusses, is sufficient to cover this item. A snow and accidental load of 12 lb. per sq. ft. of roof surface should also be added to the dead load to get the entire vertical load on the roof.

(*b*) The wind pressure normal to the slope of this roof, according to Table II, for a $\frac{1}{2}$-pitch roof is 36.1 lb., say 36 lb. per sq. ft. Ans.

(*c*) The area of the roof supported by one truss is, as previously found, 847.92 sq. ft., and the dead load is 11 lb. per sq. ft. Then, $847.92 \times 11 = 9{,}327.12$ lb. to be supported by one truss, not including the snow load. Ans.

6. Engineering is not, it must be remembered, an exact science, for the results obtained depend more or less on the judgment and experience of the designer. When, for instance, the wind is blowing a hurricane, snow never lodges on a roof; in such a case, the slates, shingles, and sheathing themselves are exposed and in danger of sudden removal. If, therefore, the full wind pressure is assumed, the snow load may, in most cases, be omitted, especially if it is desired to build an economical roof. It is, however, not well for the beginner to make such assumptions until his experience and judgment is sufficiently developed to enable him to make true deductions.

7. Careful designers sometimes make allowances for the **accidental load** caused by a mass of snow dropping from one roof to another. But this load may usually be ignored, because it is taken care of in the factor of safety, within the limit of which every member in a structure is designed.

EXAMPLES FOR PRACTICE

1. The area of one slope of a $\frac{1}{2}$-pitch roof is 800 square feet; what is the entire pressure on the slope of the roof, provided that the maximum horizontal wind pressure is taken at 40 pounds per square foot? Ans. 28,800 lb.

2. In a $\frac{1}{4}$-pitch roof, the trusses are 20 feet apart, and the length of the roof slope is 40 feet; what wind load is there on each roof truss, if the horizontal pressure is 40 pounds per square foot? Ans. 18,960 lb.

3. The purlins supporting a $\frac{3}{4}$-pitch roof are placed 6 feet apart, and the trusses are 12 feet from center to center; what is the maximum load due to the wind on each purlin, provided that the greatest horizontal pressure is 40 pounds per square foot? Ans. 2,830 lb.

GRAPHIC STATICS

8. In the use of graphic statics to determine the stresses on the various members entering into the construction of a frame, not only may the magnitude of the stresses on the members, but the direction in which they act, be determined.

The representation to the eye of the forces existing in the several parts of a frame structure possesses many advantages over their determination by calculation. Graphic analysis being founded on correct principles, the diagrams give results depending for accuracy on the exactness with which the lines have been drawn and on the scale by which they are measured. With ordinary care, the different forces may be obtained much more accurately than the several parts of the frame can be proportioned.

9. Frame and Stress Diagrams.—In Fig. 6, the weight W of 1,000 pounds is supported at c by the two branching cords $c\,a$ and $c\,b$, fastened to the pins a and b. The figure, which is drawn to scale and accurately represents the outline of the structure, is called a **frame diagram.** Now, to obtain the stress on the cords $c\,a$ and $c\,b$, draw the vertical line *1–2*, Fig. 7; this line must represent the direct pull on the rope, or cord, $c\,W$, so let each inch represent 400 pounds; then, to represent a force of 1,000 pounds, the length of the line *1–2* shall be $1{,}000 \div 400 = 2\frac{1}{2}$ inches. From *1* draw the line *1–3* parallel to $a\,c$, and from *2* draw the line *2–3* parallel to the line $c\,b$ of Fig. 6; they will intersect at the point *3* and form a triangle. If the

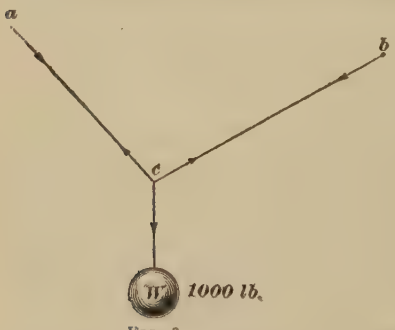


FIG. 6

lines *1–3* and *2–3* are measured with the 1-inch scale, the stresses *c a* and *c b* can be determined.

The diagram, Fig. 7, is called the **stress diagram.** In working out the stresses for a roof truss, it is first necessary to make the frame diagram, drawing it accurately to any scale and representing the outline of the truss and the members of which it is composed. It is then necessary to draw a stress diagram for the dead load on the roof, which sometimes includes the snow load, and another diagram to represent the stresses produced by the action of the wind on the roof.

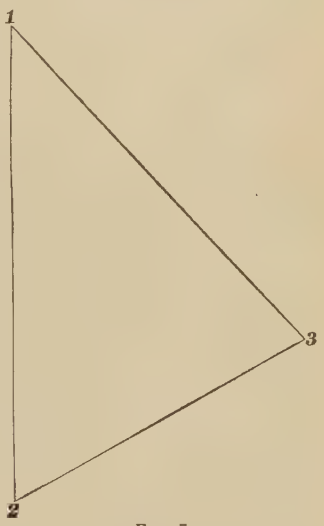


FIG. 7

10. Lettering the Diagrams.—In laying out the frame and stress diagram, it is useful to adopt a system of lettering, so that the relative position of the different members in the frame and stress diagram may be seen at a glance, as in Figs. 8 and 9. In Fig. 8, the reactions at the walls are R_1 and R_2.

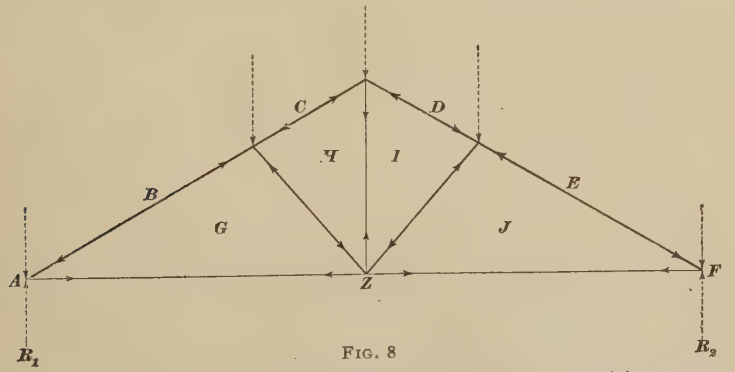


FIG. 8

It must be remembered that a roof truss is nothing more nor less than a beam, and, consequently, the sum of the reactions must be equal to the sum of the loads.

The loads, or forces, acting on a roof truss are always considered as concentrated at the **panel points**, that is, where several members in the structure join one another. In Fig. 8, the loads and reactions on the truss are shown by dotted lines and arrowheads.

The spaces outside of the truss, between the loads, together with each triangle inside of the truss, should be lettered with capitals, as in Fig. 8. It is well to begin at the left-hand reaction of the truss with the letter A, working around the outside of the truss in alphabetical order, until the right-hand reaction is reached. Then start with the first triangular space at the left-hand end of the truss, following with the next letter in the alphabet, continuing in alphabetical order, it being well to mark the space between the two reactions, at the center of the truss, Z.

It is not absolutely necessary to use this system of lettering, for any letters or numbers may be employed, so long as no two spaces in the truss are marked alike. It is, however, well to have some definite system in engineering work, the foregoing being as convenient as any that can be suggested.

Having marked the truss with letters, as in Fig. 8, the various members and forces may be designated by those letters between which they are located. The left-hand reaction, for instance, will be ZA, the first load on the truss AB, the second load BC, the load at the apex of the truss CD, next in succession the loads DE and EF—always following the truss in the same direction. The last external force on the truss is the right-hand reaction FZ.

The lower portion of the left-hand rafter is BG, the upper portion CH, the left-hand portion of the tie at the base of the truss is GZ, the left-hand compression member in the truss is HG, and the central tie IH, and so on. In designating the various members of the truss, the letters are given in the order in which they occur, always following the joints in the same direction, which in this case is that in which the hands of a watch travel. This being important, it is fully explained further on.

11. The stress diagram for the truss shown in Fig. 8 is represented by Fig. 9. The stress diagram is always drawn to some scale in which the unit measurement represents a certain number of pounds. Thus, suppose 1 inch represents 1,000 pounds, and suppose that in the stress diagram the line ch is 4 inches long; then the stress in the member CH in Fig. 8 is equal to 4,000 pounds. It will be noticed that small letters are used in the stress diagram and that the letter at the end of a line designates that line. Thus, the length of the line di in the stress diagram, Fig. 9, measures the stress in the corresponding member DI of the frame diagram, Fig. 8. The distance from z to a measures the magnitude of the reaction AZ; the length of the line ih in the stress diagram measures the stress in the corresponding member IH of the frame diagram.

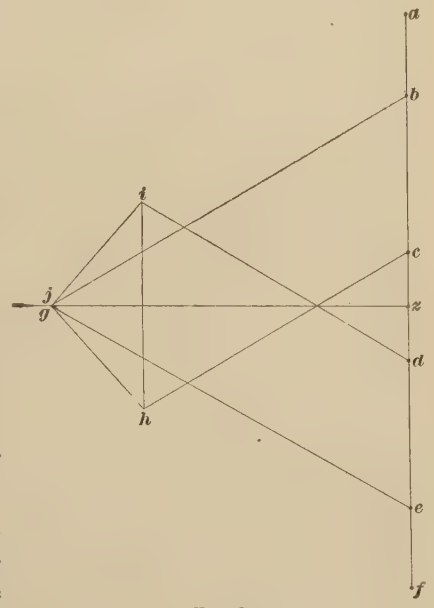


FIG. 9

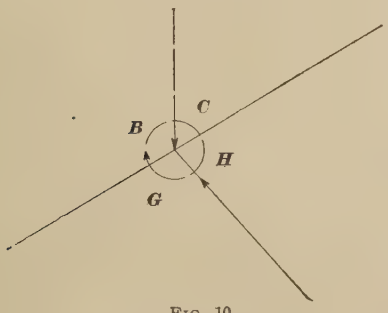


FIG. 10

12. Order of Letters. As previously mentioned, it is very important always to read the letters in the same direction, and in proper order. If, for instance, the joint at the middle of the left-hand rafter (Fig. 10) is examined, the members and stresses must be read off in their proper order, as BC,

CH, HG, and back again to GB. Leaps should not be made from BC to GB, etc., as this would lead to errors, and prevent the drawing of the stress diagram. Still more important is it to read around the joints in one direction, as in Fig. 10, that is, in the direction of the arrow. By doing this, the following method is devised to find the kind of stress in the members of the frame. Read forces on the stress diagram in the same order as in the frame diagram; thus, ch (not hc). Then the force acts from c to h (not from h to c), which is downwards and to the left. Returning to the frame diagram, it is seen that if the member CH exerts a force downwards and to the left on the joint in question it is in compression. Taking the diagonal member at the same joint, since we read HG (not GH) in the frame diagram, the force in the stress diagram acts from h to g (not from g to h), which is upwards and to the left. Returning to the frame diagram, notice that as HG pushes, on the joint shown, upwards and to the left it must be in compression. The direction of the other forces acting on the joint under consideration may be found in the same manner. Consider, for instance, the joint at Z in the same frame diagram. Here in the diagonal, if the members are read in the same direction as before, it is GH (not HG). Looking at the stress diagram, it will be observed that GH is acting downwards and to the right at the joint Z, which shows that the member is in compression. This agrees with the result obtained when examining the other end of this member, for, of course, if there is compression in one part of a member, there must be compression throughout its entire length, since it is only held at the ends.

13. Diagram for Simple Frames.—Fig. 11 shows a force, or load, of 1,000 pounds pulling on the cord EA, and held in position by the two cords AC and CE. Find by graphic statics the stress in these two cords, also the magnitudes of the forces (AB, BC) (CD, DE) required to act at the ends of the cords.

Draw the frame diagram, Fig. 11, accurately, say to a scale of $\frac{3}{8}$ inch to 1 foot. Then start to draw the stress

diagram, Fig. 12. The force EA in the frame diagram being already known, take some scale, say 400 pounds to 1 inch,

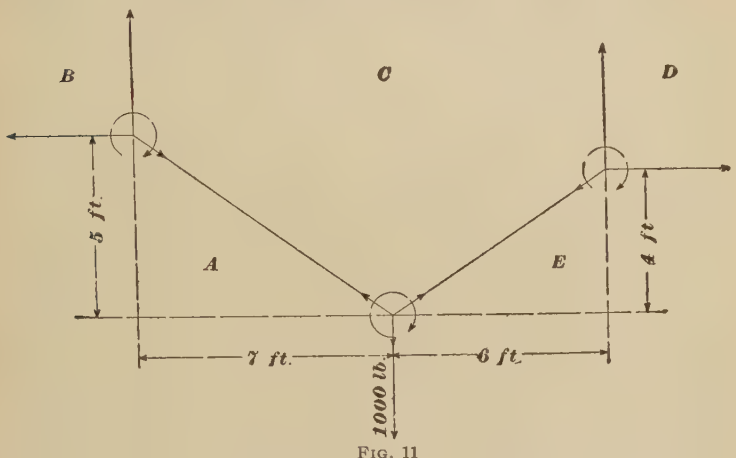


Fig. 11

and draw ea in the stress diagram. It must be drawn parallel to the line along which the force or load EA acts. This force EA being 1,000 pounds, and the scale to which the stress diagram is drawn being 400 pounds to each 1 inch, the line ea must be $2\frac{1}{2}$ inches long. Having drawn ea, work around the joint in the direction of the arrow. From a in the stress diagram draw a line parallel to AC, and from e in the stress diagram draw a line parallel to CE until it intersects the line ac at the point c.

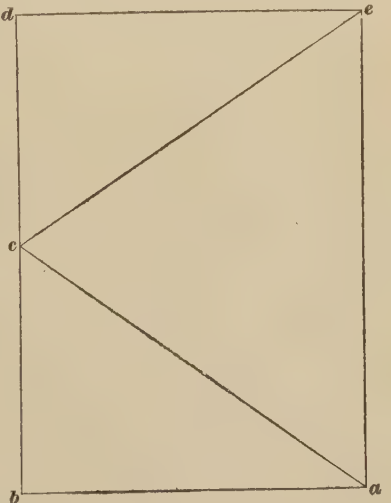


Fig. 12

In going around the stress diagram, the same direction is to be followed. Thus, in going around the joint AE, it is read in the stress diagram from e to a, from a to c, from

c to *e*, always arriving at the same point from which the start was made. As the stresses are read, their direction should be marked with arrows on the frame diagram, Fig. 11. This shows the direction of the stress and designates whether it is compression or tension. Forces acting away from a joint are always tensile stresses; those acting toward a joint are always compressive stresses.

If there is tension at one end of a member, it is evident there must be an equal amount of tension at the other end; if there is compression at one end of a member, there is an equal amount at the other.

To continue the solution of the problem in Fig. 11: Having gone around the joint *E A C*, proceed to go around the joint *A B C* in the same direction. Having the point *a*, draw from it a line parallel to *A B* in the frame diagram, then draw a line from *c* parallel to the line *B C* in the frame diagram; the point where the two lines intersect is *b*. The polygon of forces may be read from *c* to *a*, from *a* to *b*, from *b* to *c*, always moving in the same direction and arriving at the starting point. The next joint to work around is the joint *C D E*. Starting at the point *c* already determined, draw a line parallel to the direction of *C D*, and from *e* draw a line parallel to *D E*; the polygon of forces is from *a* to *c*, *c* to *d*, and *d* to *e*, the last point in the diagram, and the point from which the whole stress diagram was started. Then by measuring the lines (with the 1-inch scale, wherein every inch represents 400 pounds) in the stress diagram, the stresses in the different members of the frame diagram may be found. If, for instance, it is desired to obtain the stress in the cords *A C* and *C E*, measure the lines *a c* and *c e* in the stress diagram; likewise, to obtain the forces *A B*, *B C*, *C D*, and *D E*, measure the length of the lines *a b*, *b c*, *c d*, and *d e* in the stress diagram.

14. In Fig. 13 is shown a case similar to that in the preceding article. The compression member *B D* and the tension members *A D* and *D C* form a triangular frame, which supports the downward pull of 1,000 pounds. The

triangular frame is supported, in turn, by the reactions AB and BC. Draw the stress diagram to determine the stress in the various members.

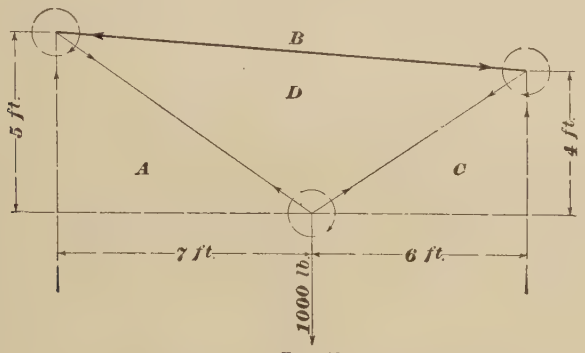


FIG. 13

Take a scale, in this case 100 pounds to $\frac{1}{4}$ inch, and draw the vertical line ca, Fig. 14, equal to 1,000 pounds. This line represents the force CA in the frame diagram, Fig. 13. Start to work around the joint ADC, in the direction of the arrow. The first member encountered is AD. Hence, from a in the stress diagram draw a line parallel to AD in the frame diagram. Then, DC being the next member met with, from c in the stress diagram draw a line parallel to DC. The point of intersection of the two lines just drawn is d. This done, go around the joint again, to see that none of the members have been omitted, and also to get the direction in which the stresses act. Starting at c in the stress diagram, and going around the joint CAD, the polygon of forces is as follows: from c to a, from a to d, and from d back

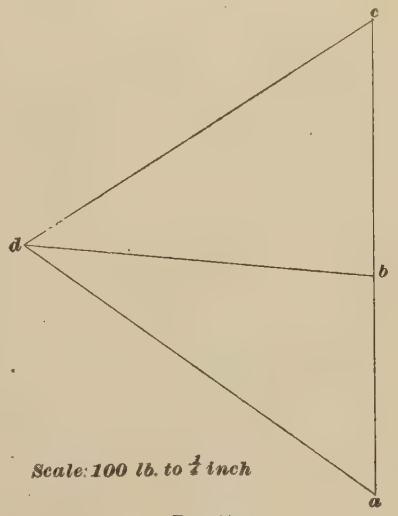


FIG. 14

again to *c*, thus arriving at the point from which the start was made. The next joint in the frame diagram is *A B D*. The point *b* on the line *c a* is not known, but may be determined by calculating the reactions *A B* and *B C* in the same manner as for a beam. Thus, the load of 1,000 pounds is placed on the assumed beam, 6 feet from the reaction *B C*. The moment about *C B* is $1{,}000 \times 6 = 6{,}000$ foot-pounds, while the reaction at *A B* equals $6{,}000 \div 13 = 461$ pounds. Knowing that the force *A B* is 461 pounds and that it acts upwards, the point *b* can easily be located by measuring from *a* on the line *a c* in the stress diagram; then the line *b d* may be drawn

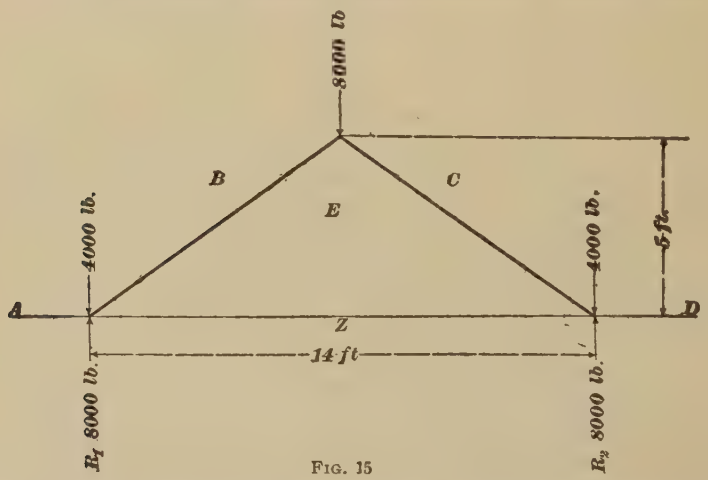


FIG. 15

and if found parallel to the member *B D* in the frame diagram, the stress diagram is correct. In this case, however, it is not necessary to calculate the reactions *A B* and *B C*, the point *d* having been already determined, and since we know that the line *d b* must be parallel to *D B*, all that is needed to locate the point *b* is to draw a line from *d* parallel to *D B*, and the point where it cuts the line *c a* is *b*. Having found the point *b* and drawn the line *d b*, go around the joints *A B D* and *C D B*, marking the direction of the stress by the arrowheads, as shown in the frame diagram, Fig. 13. Around the joint *A B D* the polygon of forces is from *a* to *b*, from *b* to *d*, and from *d* back again to *a*. Working around

the joint CDB, the polygon of forces is from c to d, from d to b, and from b back again to c. This completes the stress diagram; the magnitude of the stresses in the several members of the frame diagram is found by measuring the corresponding lines in the stress diagram.

15. Diagram for a Small Roof Truss.—Fig. 15 is the frame diagram for a small roof truss. Each of the rafter members EB and CE is connected at its foot by the tension member EZ. The loads and their reactions are as shown in the frame diagram. Determine the stresses in the several members composing the truss.

Draw the vertical line ad, shown in the stress diagram, Fig. 16. Lay off to any scale, say, in this case 2,000 pounds to $\frac{1}{2}$ inch, the load ab; then, to the same scale, the loads bc and cd. From the point d, the reaction dz, 8,000 pounds, acts upwards, which determines the point z. Now go around the joint $ABEZ$. The reaction ZA acts upwards and AB downwards. Then, from the point b in the stress diagram draw the line be parallel to BE in the frame diagram, and from z draw the line ez parallel to the member EZ in the frame diagram. The point of intersection will be the point e. Having gone thus far, again go around the joint, to get the direction of the stress in the members and to see whether the polygon of forces is correctly drawn. Go, for instance, from z to a upwards; a to b downwards; then from b to e, and from e back again to z, the starting point. The next

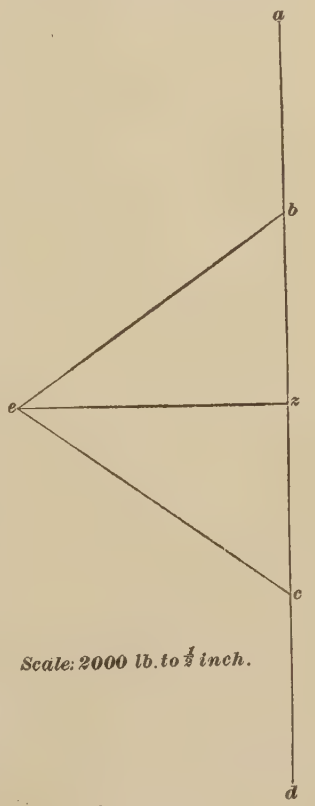


FIG. 16

joint in the frame diagram is BCE. The force bc being already determined, from point c, draw the line ce, parallel to the member CE in the frame diagram; this line passes through the point e if the diagram has been drawn correctly. The polygon of forces at EBC is from b to c downwards, then from c to e, and back again from e to b, the starting point. The next joint in the frame diagram is $ECDZ$, and the polygon of forces in the stress diagram is from e to c already drawn, from c to d, d to z, and then from z back to e.

The stress diagram completed, all that remains is to measure the various lines in the stress diagram that represent the corresponding members in the frame diagram. Thus, eb measures $1\frac{3}{4}$ inches, the scale being 2,000 pounds to $\frac{1}{2}$ inch; hence, the stress in this member is 7,000 pounds; the line ez measures about $1\frac{3}{8}$ inches, and the stress in the member ez is 5,500 pounds. In this manner, the stress in any member may be determined.

16. Diagram for a Jib Crane.—A jib crane proportioned as in Fig. 17 has a load of 30,000 pounds suspended

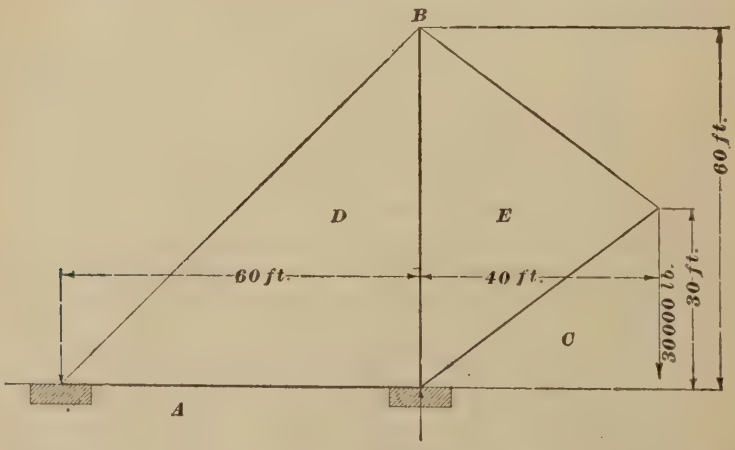


FIG. 17

at the end of the jib; what are the stresses in the guy ropes and in the different members of the crane, and what are the reactions CA and DA?

In the stress diagram, Fig. 18, draw the vertical line bc equal to 30,000 pounds, and from the point c draw the line ce parallel to CE in the frame diagram, Fig. 17. Then from b draw the line eb parallel to EB in the frame diagram. Again going around the joint to check the polygon of forces, they are found to be from c to e, from e to b, and from b back again to c. The next joint encountered is EDB. Hence, from e draw ed upwards parallel to ED, and from b draw db parallel to DB, the point where these two lines intersect being d. The polygon of forces about the joint EDB is from b to e, and from e to d, and from d back again to b, the starting point. Next, go around the joint $CADE$, and draw ca upwards; then from d draw ad parallel to AD in the frame diagram; where the lines just drawn intersect will be the point a; de and ec have already been drawn. The remaining joint to work around is ABD. On looking at the stress diagram, it may be seen that the forces around this joint have already been determined, completing the stress diagram.

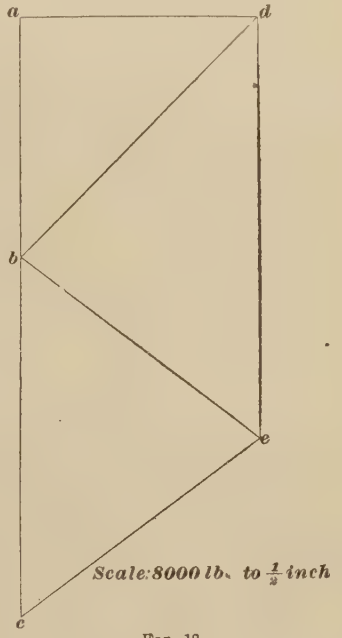


FIG. 18

The stresses in the members may be determined, as already stated, by measuring the lines corresponding to them in the stress diagram, with the scale to which the diagram has been drawn.

17. Roof Truss With a 40-Foot Span.—Fig. 19 shows the frame diagram for a 40-foot span roof truss. The loads are as shown, the compression members being indicated by heavy lines, and the tension members by

light lines. Required, to draw the stress diagram for this truss.

First, draw the vertical line $a\,f$ as shown in the stress diagram, Fig. 20; mark the point a and lay off on this vertical line, to any scale, using, for instance, 4,000 pounds to $\frac{1}{2}$ inch, the loads $a\,b$, $b\,c$, $c\,d$, $d\,e$, and $e\,f$ corresponding to the loads $A\,B$, $B\,C$, $C\,D$, $D\,E$, and $E\,F$ in the frame diagram. The truss being symmetrically loaded, the loads are the same in amount on the two sides of the center line. The reactions R_1 and R_2 are therefore each equal to one-half of the load, in this case, 16,500 pounds. Hence, $z\,a$ may

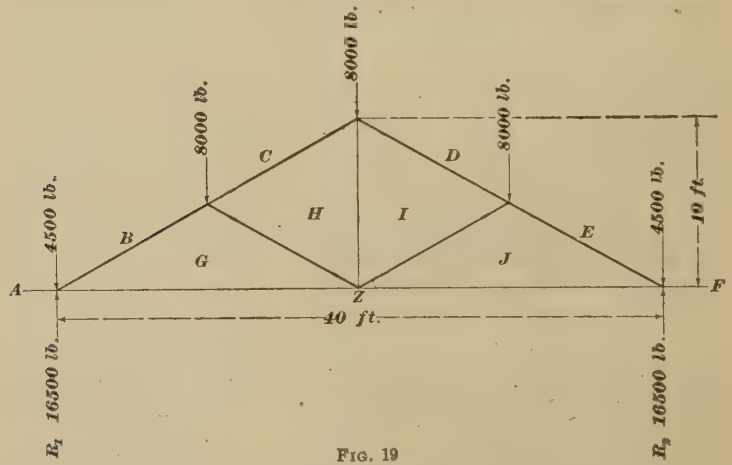


FIG. 19

be laid off on the vertical line, and as R_1 equals R_2, $z\,a$ must equal $f\,z$; consequently, z is located centrally between a and f, or between c and d. The point z having been determined, proceed with the diagram by going around the joint $A\,B\,G\,Z$. Draw $b\,g$ in the stress diagram parallel to $B\,G$ in the frame diagram; then from z draw $g\,z$ parallel to $G\,Z$, the point where the two lines intersect being g. The next joint is $B\,C\,H\,G$. As $b\,c$ in the stress diagram is already known, draw $c\,h$ parallel to $C\,H$ and $h\,g$ parallel to $H\,G$. Then the polygon of forces around this joint will be from b to c, from c to h, from h to g, and from g back again to b. It is now expedient to analyze the joint $C\,D\,I\,H$. In the stress diagram, $h\,c$

and $c\,d$ have already been obtained; then from d, draw $d\,i$ parallel to $D\,I$, and from the point h, already known, draw $i\,h$ parallel to $I\,H$. The polygon of forces around this joint will be from c to d, from d to i, from i to h, and h to c, the starting point, the direction marking the direction in

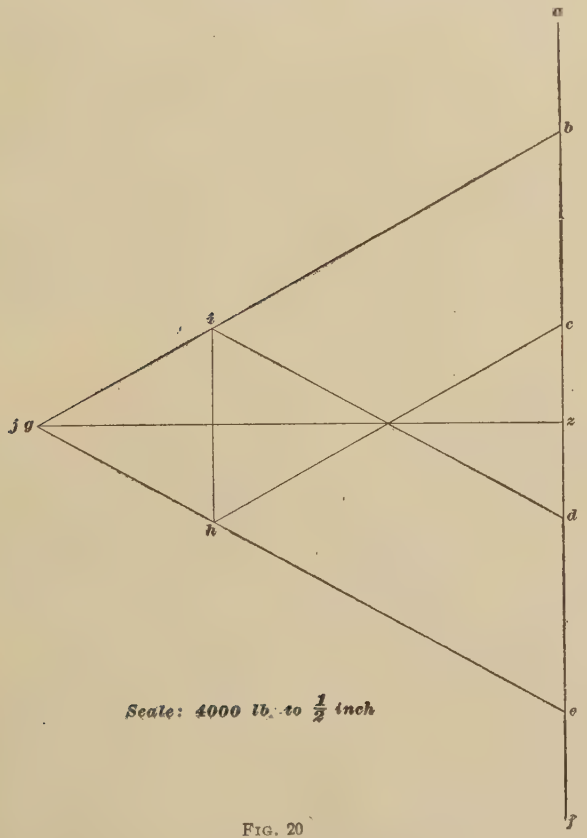


FIG. 20

which the stresses act in the stress diagram, on the members in the frame diagram. Now analyze the forces around the joint $G\,H\,I\,J\,Z$ in which the forces $z\,g$, $g\,h$, and $h\,i$ have been obtained. From the point i, $i\,j$ is drawn parallel to $I\,J$; and from z, $j\,z$ is drawn parallel to $J\,Z$; the point j is found to fall on the point g, and the polygon of forces around this joint

is from z to g, from g to h, from h to i, from i to j, and from j back again to z, the starting point. The stress in the members around the joint $IDEJ$ should next be determined, ji, id, and de being already known. From e draw ej parallel to EJ. The polygon of forces around this joint will then be from i to d, from d to e, from e to j, and from j back again to i.

The only remaining joint to go around is $JEFZ$. By referring to the stress diagram, it is seen that the stresses in these members have been determined, while the polygon of forces around this joint is from j to e, from e to f, from f to z, and back again from z to j.

The stress diagram completed, the magnitudes of the stresses may be determined by measuring the various lines with the scale to which the diagram has been drawn, in this case 4,000 pounds to $\frac{1}{2}$ inch.

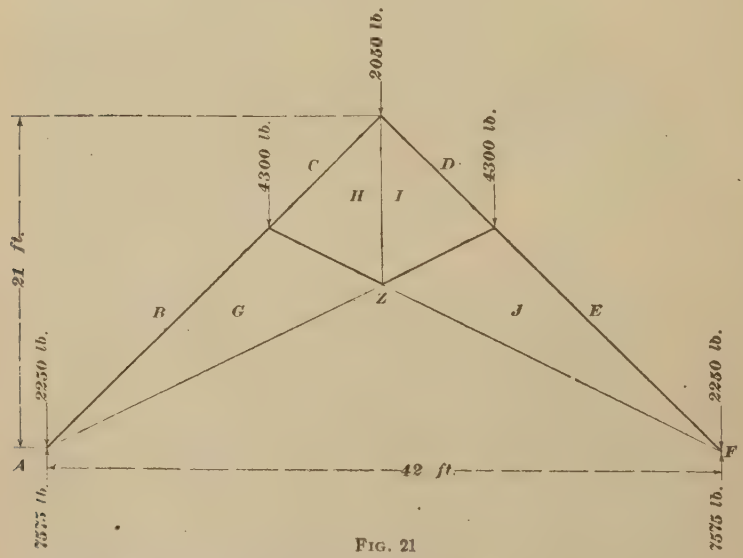


FIG. 21

18. Truss for a Church Roof.—Fig. 21 is the frame diagram of a form of truss sometimes used to support a church roof. Determine the stress diagram for the dead load and also the stress diagram for the wind pressure on this roof.

First, draw the stress diagram (Fig. 22) for the dead load. As the dead loads on the truss are symmetrical both in amount and location with regard to the center line of the truss, the reactions are the same at either end of the truss, and each one is equal in amount to one-half of the load, in this case 7,575 pounds.

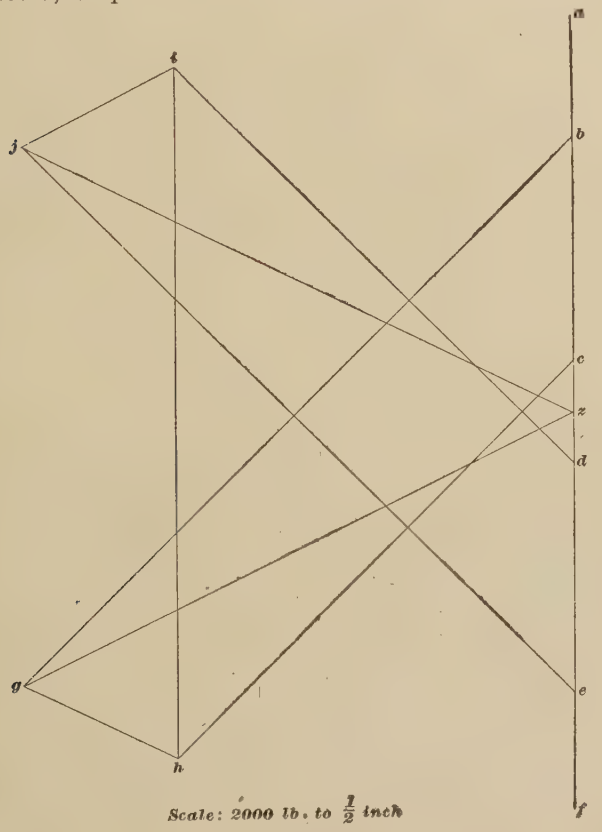


FIG. 22

Draw the vertical line af in the stress diagram; then, starting at the point a, lay off the scale of, say, 2,000 pounds to every $\frac{1}{2}$ inch, the force ab equal to AB in the frame diagram; then lay off bc equal to BC, cd equal to CD, de equal to DE, and ef equal to EF. Then, as the truss is

symmetrically loaded, the point z is located midway between the points a and f. If the truss were not symmetrically loaded, the reactions would have to be calculated in the same manner as in a beam.

Having located the loads and their reactions on the vertical line af, obtain the stresses in the members around the joint $ABGZ$ from the point b, by drawing a line bg parallel to BG in the stress diagram; then from z draw the line gz parallel to GZ, and the intersection of these two lines will be the point g. The polygon of forces around this joint is from b to g, from g to z, from z to a, and then from a back to b, the starting point. Bear in mind that the forces in the stress diagram representing the reactions must have the same direction as the reactions in the frame diagram. The lines determining the stresses around the joint $BCHG$ should next be drawn; bc having been determined, from c draw a line ch parallel to CH, and from g draw a line gh parallel to HG, the intersection of these two lines determining the point h; the polygon of forces is from b to c, from c to h, from h to g, and back again from g to b.

Now work around the joint $CDIH$; cd being already known, from the point d draw the line di parallel to DI, and from the point h draw the line hi parallel to IH, the intersection of the two lines being the point i. In going around the joint $GHIJZ$, the stresses in the members zg, gh, and hi have already been determined and drawn in the stress diagram. Then from the point i draw the line ij parallel to IJ, and from z draw the line zj parallel to JZ, and the intersection of these two lines will be the point j. The polygon of forces around this joint is from z to g, from g to h, from h to i, from i to j, and from j back to z, the starting point.

The next joint to analyze, in going around the truss, is $DEJI$; ji, id, and de being known, the only remaining force to determine is the stress in the member EJ. The point e being fixed, draw the line ej parallel to EJ in the frame diagram, and if this line, which completes the diagram, passes through the point j, the diagram is correct and

accurately drawn. The stresses around the right-hand heel of the truss are all known, the line ej just drawn having been the only unknown member at this joint.

The polygon of forces around the joint $DEJI$ is from d to e, from e to j, from j to i, and from i to d, the starting point. The polygon of forces around the joint $EFZJ$ is from e to f, from f to z, from z to j, and from j to back again to e, completing the stress diagram for the dead, or vertical, load on the roof truss.

19. The wind diagram, however, remains to be drawn. Redraw the frame diagram as shown in Fig. 23. The wind is always considered as acting normally, or at right angles, to the roof, the amount of its pressure at the different joints

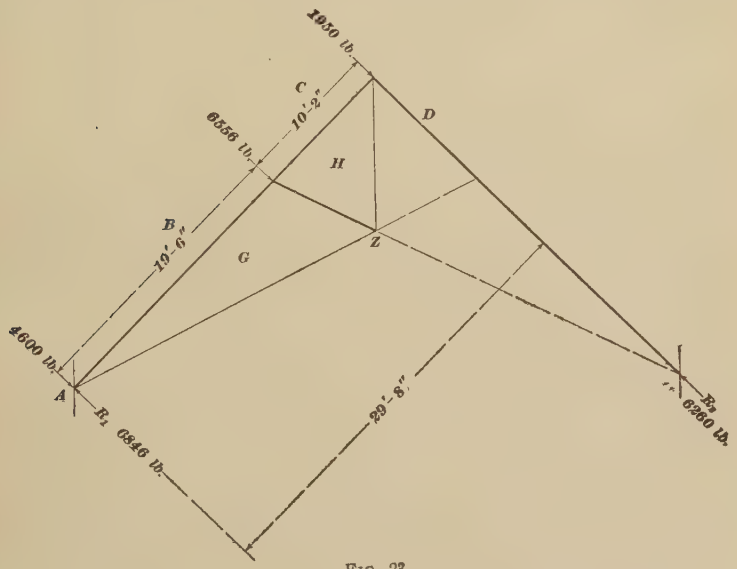


FIG. 23

of the truss being shown on the frame diagram. As the heels of the trusses are fixed, the reactions act in lines parallel to the wind pressure.

To estimate the magnitude of the reactions R_1 and R_2, consider the left-hand rafter member as a beam, and R_1

and R_2 as the reactions supporting it. The moments due to the wind pressure BC and CD acting about R_1 are:

At BC,	$6{,}556 \times 19.5$	$= 127{,}842$ foot-pounds
At CD,	$1{,}950 \times 29.66$	$= 57{,}837$ foot-pounds
Total,		185,679 foot-pounds

The lever arm with which R_2 resists the wind pressure acting at the joints, since the apex of the truss happens in

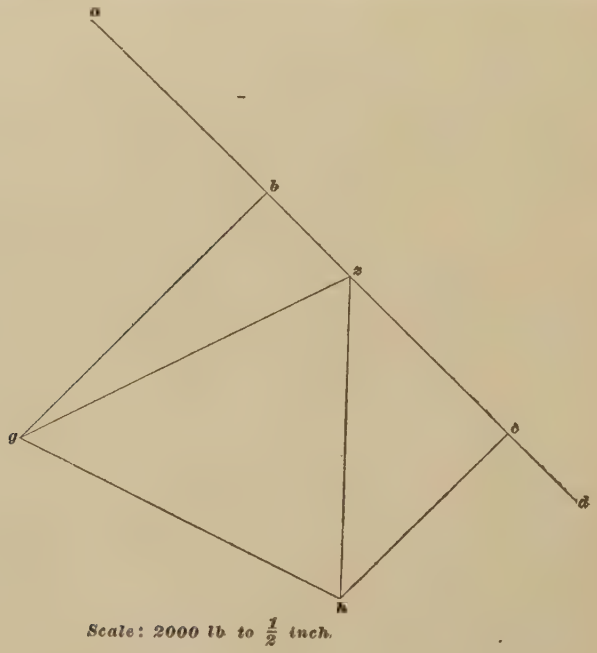


FIG. 24

this case to be a right angle, is 19 feet 6 inches + 10 feet 2 inches = 29 feet 8 inches, so $185{,}679 \div 29.66 = 6{,}260$ pounds, the amount of the reaction at R_2. The sum of the loads being

$$4{,}600 + 6{,}556 + 1{,}950 = 13{,}106 \text{ pounds,}$$

the reaction at R_1 is $13{,}106 - 6{,}260 = 6{,}846$ pounds.

First, draw the load line $a\,d$ in the stress diagram, Fig. 24; this line is parallel to the direction of the wind pressure and the reactions. Now lay off to the scale to which the stress

diagram is drawn—in this case 2,000 pounds to every ½ inch—the force $a\,b$ equal to $A\,B$ in the frame diagram; then lay off $b\,c$ equal to $B\,C$ and $c\,d$ equal to $C\,D$. From d lay off the magnitude of the reaction R_2, or $d\,z$, which determines the point z, and the distance $z\,a$, according to scale, represents the left-hand reaction R_1. The polygon of external forces is, then, from a to b, from b to c, from c to d, from d to z, and from z back again to a, the starting point. This polygon, as may be readily seen, is a straight line, as in all cases so far analyzed.

Continue the stress diagram, Fig. 24, by going around the joint $A\,B\,G\,Z$; from the point b draw the line $b\,g$, and from the point z draw the line $z\,g$ parallel to the corresponding members in the frame diagram, the point where the two lines intersect being g. The polygon of forces around this point is from a to b, from b to g, from g to z, and from z back again to a.

The next joint is $B\,C\,H\,G$; $b\,c$ has been already obtained; then from the point c draw the line $c\,h$ parallel to $C\,H$, and from g draw the line $g\,h$ parallel to $H\,G$, h being the point where these two lines cross. Disregard the two members in the frame diagram shown in dotted lines, which do nothing toward sustaining the wind pressure. Now work around the joint $H\,C\,D\,Z$; $c\,d$ and $d\,z$ are known; draw from z the line $z\,h$, parallel with $Z\,H$ in the frame diagram; if this closing line of the diagram passes through the point h, the diagram has been drawn accurately. The polygon of forces around the joint $B\,C\,H\,G$ is from b to c, from c to h, from h to g, and from g back again to b. The polygon of forces around the joint $C\,D\,Z\,H$ is from c to d, from d to z, from z to h, and from h back again to c. The polygon of forces around the joint $Z\,G\,H$ is from z to g, from g to h, and from h to z, the starting point.

The stress diagram for both the dead and the wind load being complete, to obtain the stress in each member of the truss, it is required to determine, by scale, the stress due to both the dead and wind loads in each member, adding the two together for the maximum load in the member. To

determine, for instance, the stress in the strut HG, measure the length of the line hg in the stress diagram, Fig. 22, for the dead load; then measure the same line hg in the stress diagram, Fig. 24, for the wind load, add the two measurements together, and determine the maximum stress in the strut hg from the assumed scale of the drawing.

It must be remembered that while the wind acting on one side of a truss does not create stresses in all the members on the opposite side, these members should be proportioned in like manner as the other members, because the wind is quite as likely to blow on this side of the roof and reverse the conditions.

20. Howe Truss With an 80-Foot Span.—Fig. 25 is the frame diagram for an 80-foot span Howe roof truss It is desired to draw the dead-load diagram and the wind-stress diagram.

Draw the frame diagram shown in Fig. 25 and mark the dead load coming on the different panel points, or joints, in the truss. The truss being symmetrically loaded, the reactions R_1 and R_2 are each equal to one-half the load on the truss.

Draw the stress diagram, Fig. 26, for the dead load, say to the scale of 4,000 pounds to $\frac{1}{2}$ inch. Draw the vertical load line aj, and determine the point z, having previously located on the line all the loads. Then draw the stress diagram by the methods previously given. Only one-half of the diagram need be drawn, as the stresses obtained on one side of the center of the truss apply to the other side. For instance, fq is the same as pe. Having completed one-half of the stress diagram for the dead load, the frame diagram should be redrawn as shown in Fig. 27. The direction and amount of the wind pressure at the several panel points, or joints, of the truss, are shown in the frame diagram.

As both ends of the truss are secured against sliding, the reactions act in a direction parallel to the wind pressure. If the left-hand side of the truss is secured, the right-hand side

T
20 ft.
1750 lb.
R_2 14000 lb.
I
3500 lb.
A
H
U
3500 lb.
G
S
T
3500 lb.
F
Q
R
3500 lb.
Z
80 ft.
E
P
O
3500 lb.
D
N
M
3500 lb.
C
L
3500 lb.
B
K
1750 lb.
R_1 14000 lb.
V

FIG. 23

being on rollers, as is sometimes the case with iron or structural-steel trusses, to allow for expansion, then the right-hand reaction, instead of being parallel to the direction of the wind, will be vertical. This makes considerable difference in the stress diagram, as will be explained further on.

To determine the magnitude of the reactions R_1 and R_2, let R_1, Fig. 27, be the center around which the moment

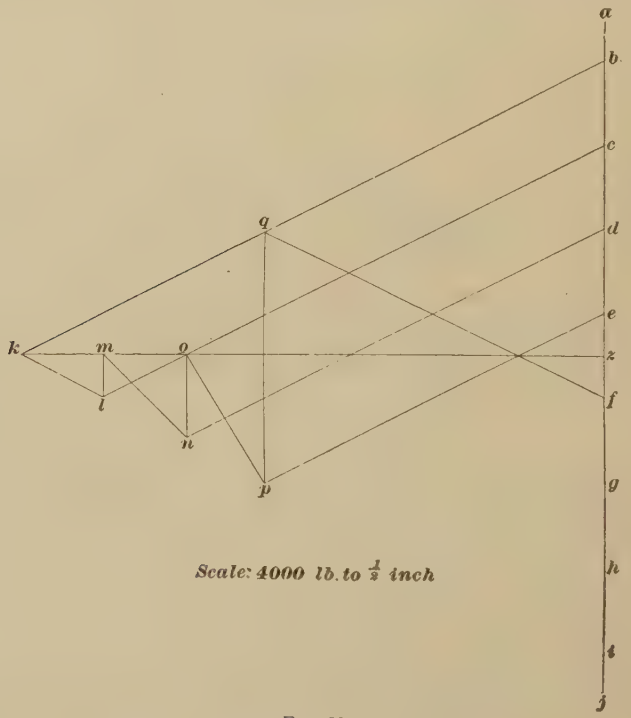


FIG. 26

of R_2 is taken; then the perpendicular distance between the line of action of R_2 and the point R_1 will be 71.22 feet. Extend the left-hand rafter until it cuts the line of action of the force R_2 at the point y'. Regard this extension and the rafter as a beam, and calculate the magnitude of the reactions R_1 and R_2 by the methods given for beams.

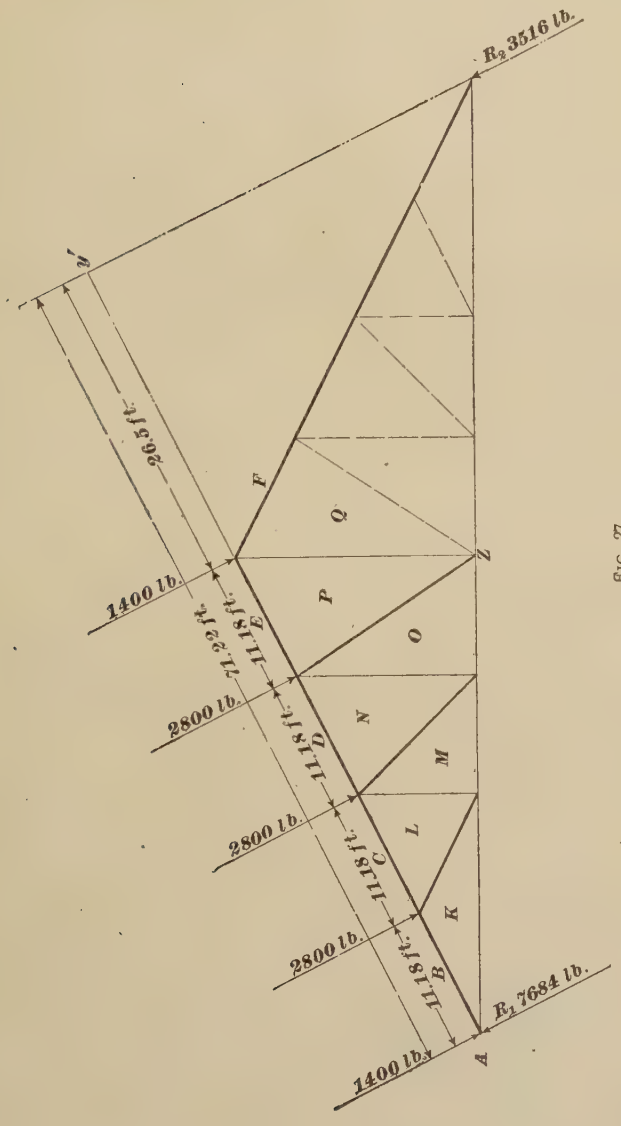

R_2 3516 lb.
a'
26.5 ft.
1400 lb.
71.22 ft.
11.18 ft.
E
2800 lb.
11.18 ft.
D
2800 lb.
11.18 ft.
C
2800 lb.
11.18 ft.
B
1400 lbs
A
R_1 7684 lb.
F
Q
Z
P
O
N
M
L
K

FIG. 27

The moments about R_1 are as follows:

$2{,}800 \times 11.18 =$	3 1 3 0 4	foot-pounds
$2{,}800 \times 22.36 =$	6 2 6 0 8	foot-pounds
$2{,}800 \times 33.54 =$	9 3 9 1 2	foot-pounds
$1{,}400 \times 44.72 =$	6 2 6 0 8	foot-pounds
Total,	2 5 0 4 3 2	foot-pounds

and $250{,}432 \div 71.22 = 3{,}516$ pounds, the reaction R_2. Having found R_2, find R_1 by subtracting R_2 from the sum of the loads. The sum of the loads is

$1{,}400 + 2{,}800 + 2{,}800 + 2{,}800 + 1{,}400 = 11{,}200$ pounds

Then, $11{,}200 - 3{,}516 = 7{,}684$ pounds, the reaction R_1.

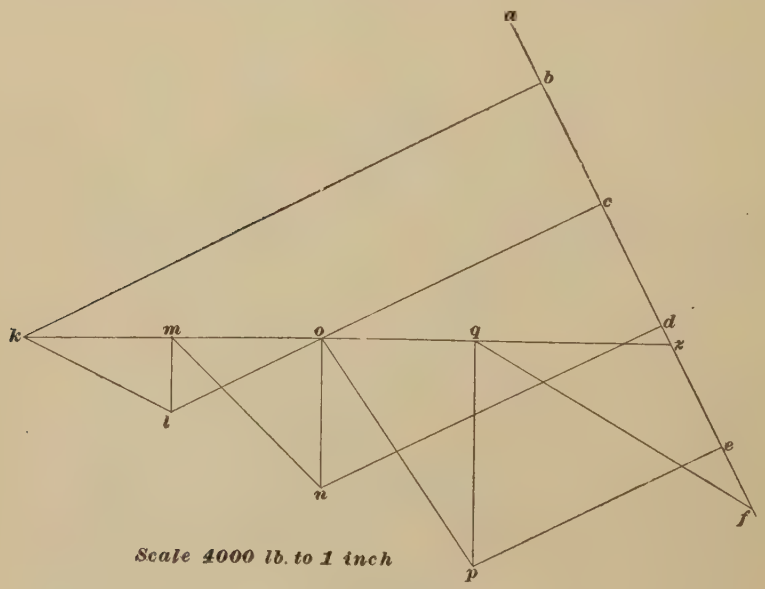


FIG. 28

Next, lay out the wind diagram, Fig. 28, by drawing the road line $a\,f$ parallel to the direction of the wind in the frame diagram, Fig. 27. Lay off to the scale (in this case 4,000 pounds to 1 inch) the forces $a\,b$, $b\,c$, $c\,d$, $d\,e$, and $e\,f$ equal to AB, BC, CD, and so on, in the frame diagram. Then, from a lay off $a\,z$ equal to the reaction ZA, or R_1. If the other forces or loads have been laid off accurately,

fz should, on measurement, be found equal to the right-hand reaction R_z.

The first joint to analyze is $ABKZ$. Start at b and draw the line bk parallel to BK in the frame diagram; then from z draw zk parallel to KZ, where the two lines intersect being the point k. Then the polygon of forces around this joint is from a to b, from b to k, from k to z, and from z back again to the starting point a.

The next joint to analyze is $BCLK$. From the point c draw the line cl parallel to CL in the frame diagram, and from k draw the line kl parallel to the member LK, the point of intersection being l. The polygon of forces around this joint is from b to c, from c to l, from l to k, and from k back again to b.

To analyze the joint $KLMZ$: kl being already known, the next number is LM; therefore, from the point l draw the line lm parallel to LM in the frame diagram. As the next member around this joint is MZ, to which mz in the stress diagram is parallel, the point m is located where the line lm intersects the line mz; this completes this joint, the polygon of forces around it being from k to l, from l to m, from m to z, and from z back again to k.

To determine the stresses in the members around the joint $CDNML$, draw from the point d the line dn, parallel to the member DN in the frame diagram; then, from the point m draw mn parallel to NM. The polygon of forces around this joint is from c to d, from d to n, from n to m, from m to l, and from l back again to c.

To analyze the forces around the joint $MNOZ$, draw from n the line no upwards, parallel to NO in the frame diagram; as the next member OZ is horizontal, the point o must be at the intersection no and oz. This completes this joint, and the polygon of forces around it is from m to n, from n to o, from o to z, and from z back again to m, the starting point.

Now analyze the joint $DEPON$. From e draw the line ep parallel to EP in the frame diagram, and from o draw the line op parallel to PO. The intersection of these two lines determines the point p, and the polygon of forces around

this joint is from d to e, from e to p, from p to o, from o to n, and from n back again to d, the starting point.

The analysis of the joint $OPQZ$ is made by drawing the vertical line pq from the point p; the point where pq intersects qz is q. Then the polygon of forces around this joint is from o to p, from p to q, from q to z, and from z back again to o. The members shown in dotted lines do not sustain any stresses from the pressure of the wind, when it blows on the left-hand side of the truss.

The final joint to consider, thus completing the stress diagram, is $EFQP$. There is only one unknown force around this joint, and that is the stress in the member FQ. A line drawn from f in the stress diagram, parallel with the member FQ, should pass through the point q; if it does not, the diagram has been inaccurately drawn. This is always a test of the accuracy of the stress diagram, and if the last line in this diagram does not close on the proper point, when drawn parallel to the member it represents, there is something so radically wrong as to demand that the stress diagram be redrawn, to determine whether the loads and reactions have been laid out correctly, and whether any of the joints or members in the structure have been passed over.

It can be readily seen that there are no stresses produced in the internal members of the truss on the side opposite to that from which the wind is blowing. First, examine the joint at the right-hand reaction; the stresses in FQ and QZ can be found. Next, examine the first joint on the rafter above the right-hand reaction; it will be seen that one internal member terminates at this joint. This member can have no stress in it because the other two members of the joint are in the same line and there is no fourth member to oppose that part of the stress of the internal member that is at right angles to the rafter and thus keep the joint in equilibrium. As there is no stress in the first internal member, by like reasoning about the first joint of the lower chord, it can be proved that there is no stress in the second internal member, nor in the third, and so on to the member PQ.

21. The two diagrams completed, measure the different lines and obtain the stresses in the various members, tabulating the results as follows:

Member	Dead-Load Diagram Pounds	Wind-Load Diagram Pounds	Total of Both Pounds	Kind of Stress
BK	27,000	12,000	39,000	Compressive
CL	23,500	10,000	33,500	Compressive
DN	19,500	7,600	27,100	Compressive
EP	16,000	5,680	21,680	Compressive
KZ	24,000	13,300	37,300	Tensile
MZ	21,000	10,000	31,000	Tensile
OZ	17,500	7,000	24,500	Tensile
LK	4,000	3,500	7,500	Compressive
NM	5,000	4,400	9,400	Compressive
PO	6,500	5,400	11,900	Compressive
ML	1,600	1,500	3,100	Tensile
ON	3,500	3,000	6,500	Tensile
QP	10,600	4,500	15,100	Tensile
FQ	16,000	6,600	22,600	Compressive

The values in this table represent the stress in round numbers, on the various members in the truss, as obtained from the dead-load and wind diagrams.

The size of the timber and tension rods in the truss may now be calculated.

22. Trusses With One End Free. — On account of expansion, due to changes in temperature, in steel trusses of long spans, one end of the truss is often placed on rollers so as to allow for this expansion. This prevents the truss from pushing out the walls of the building, as would be the case if the truss were rigidly fixed at both ends. Of course, it is necessary that at least one end of the truss should be made fast, otherwise, in a wind storm, the entire roof might slide off the walls. The expansion and contraction of a steel truss is comparatively small, but there

is very little elasticity to masonry and it is necessary to remember this feature when building over long spans. Now, the end of the truss that is on rollers, known as the *free end*, cannot take any lateral thrust and for this reason the reaction at this end must be vertical. Therefore, all the horizontal thrust due to the wind must be resisted by the fixed end. Of course, since the dead load acts vertically, the analysis of the stresses in a truss with one end free, for such a load, is exactly the same as in any other truss, since both of the reactions are also vertical. Therefore, it is only in the case of the wind diagram that a different method of analysis must be employed. There are two cases to consider, one

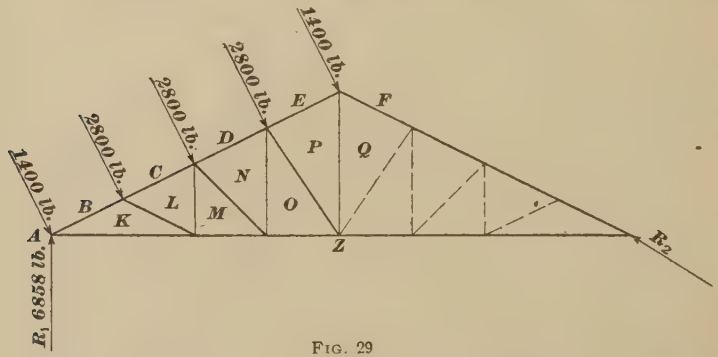


FIG. 29

when the wind acts on the free side and the other when the wind acts on the fixed side.

First, consider the case of the wind on the free side. A truss will be taken exactly like the one shown in Fig. 27 and with the same loading. As has been said, the analysis given in Fig. 26 for the dead load holds good for this truss also. The truss is redrawn in Fig. 29 and the analysis of the stresses is shown in Fig. 30. In this case, it is known that R_1 is vertical, but its magnitude is not known; both the direction and the magnitude of R_2 are unknown. To find the value of R_1, since all the forces acting on the truss are in equilibrium, moments may be taken about R_2, or the fixed end of the frame. The loads due to the wind all exert negative moments. The arm of each of these forces can be readily

found from the dimensions given on Fig. 27. Thus, the arm of EF is 26.5 feet, the arm of DE is $26.5 + 11.18 = 37.68$ feet, and soon. Adding these negative moments together, it is

$1400 \times 26.50 =$ 3 7 1 0 0 foot-pounds
$2800 \times 37.68 =$ 1 0 5 5 0 4 foot-pounds
$2800 \times 48.86 =$ 1 3 6 8 0 8 foot-pounds
$2800 \times 60.04 =$ 1 6 8 1 1 2 foot-pounds
$1400 \times 72.22 =$ 1 0 1 1 0 8 foot-pounds
5 4 8 6 3 2 foot-pounds

Now, the arm of R_1 about the fixed end is 80 feet. Therefore, the value of R_1 is $548{,}632 \div 80 = 6{,}858$ pounds.

The stress diagram can now be laid out. Start, for instance, at the point Z and move around the figure in

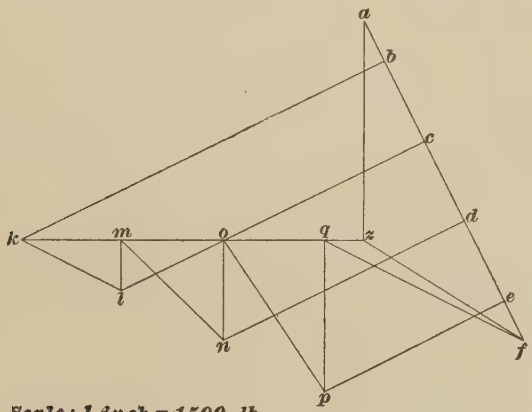


FIG. 30

the direction of the hands of a clock. Lay off, in Fig. 30, za vertical and upwards and equal to the reaction ZA, to the scale of, say, 1,500 pounds to $\frac{1}{4}$ inch. From a lay off ab equal in magnitude and direction to AB in Fig. 29; then, bc equal to BC, and so on to ef. Then, the only force that remains to be drawn is FZ or R_2, and, as this is the force that holds the truss in equilibrium, it closes the figure. The line from f to z will therefore represent R_2, both in magnitude and in direction. The direction of R_2 can now be drawn in Fig. 29, and we are in a position to analyze the stresses in the

different members of the truss. First, take the joint $A\,B\,K\,Z$. From b, Fig. 30, draw $b\,k$ parallel to $B\,K$; and from z draw $k\,z$ parallel to $K\,Z$, and so on. The only difference between this

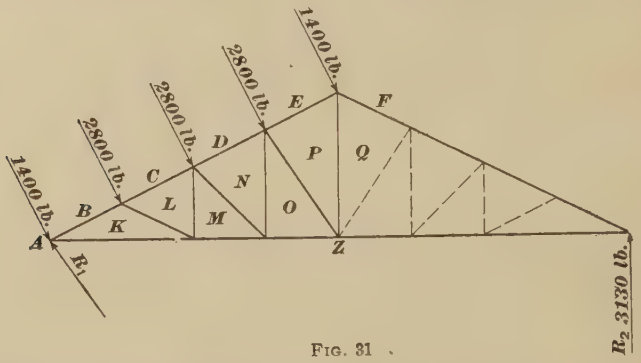


FIG. 31

analysis and the one when both ends of the truss are fixed, as shown in Fig. 28, is that z is not on the line $a\,f$.

The other case is when the wind blows on the fixed side. The truss is again redrawn in Fig. 31, and the stress diagram laid out in Fig. 32.

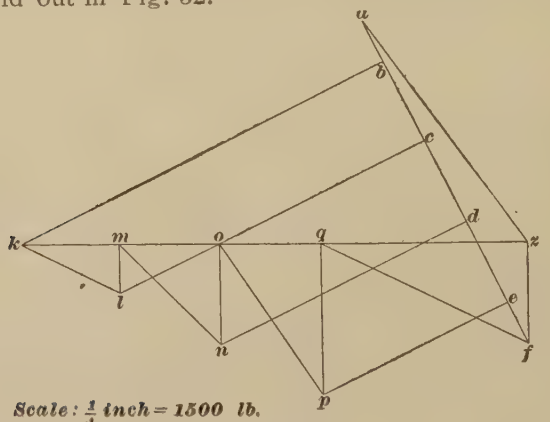


FIG. 32

In this case also, moments are taken about the fixed end. The sum of the moments of the wind loads has already been found in Art. **20** to be 250,432 foot-pounds. This quantity divided by 80, the span of the truss, gives the value

of R_2, which is $R_2 = 250{,}432 \div 80 = 3{,}130$ pounds. Lay off the wind loads in Fig. 32 to their correct direction and scale from a to f. From f, draw fz equal in magnitude and direction to R_2. Draw za; then za is equal in magnitude and direction to R_1 and can be so laid off in Fig. 31. The stress diagram can now be completed. From b, draw bk parallel to BK in Fig. 31. From z, draw kz parallel to KZ, Fig. 31. Proceed to find the other stresses in the truss members in the regular way. It will again be noted that z is not on the line ab, but is now on the side opposite to where it was in Fig. 30.

The rule to follow in solving such problems as the foregoing is this: First, find the reaction at the free end by taking moments about the fixed end. Lay off the wind loads and the reaction at the free end in their proper order and direction, all to one scale. By joining the two ends of this diagram (as z and a, Fig. 32) the magnitude and direction of the reaction at the fixed end is found. Then proceed as in any other stress diagram.

DETERMINATION OF STRESSES IN THE FINK TRUSS

23. The **Polonceau**, or **Fink, truss**, shown in Fig. 33 (*a*), is a favorite form of truss. It is often built of

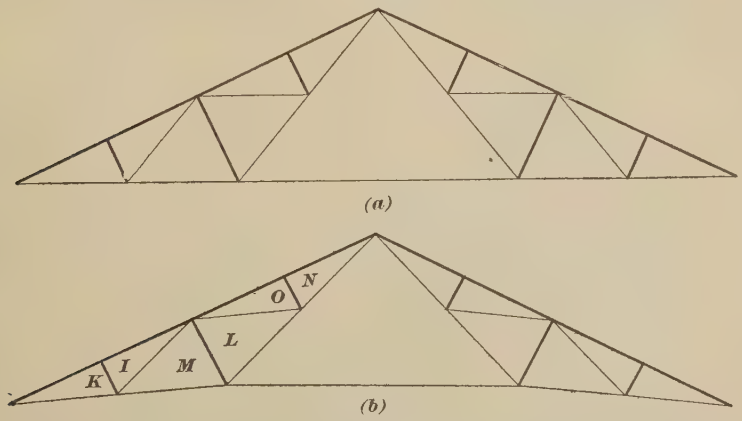
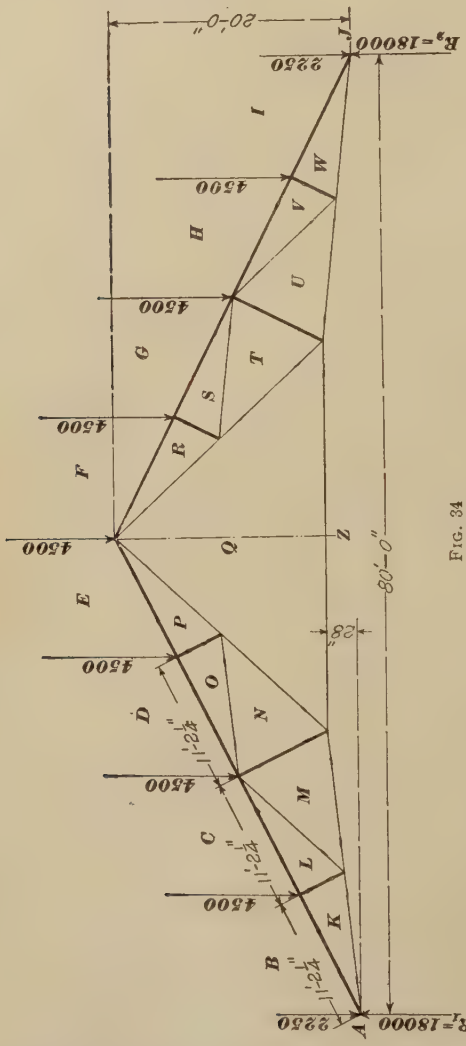


FIG. 33

rolled-steel sections; it is also built with wooden rafter

members, structural-steel struts, and wrought-iron tension members. Frequently, the lower chord of the truss is **cambered** (raised at the center), as shown in Fig. 33 (*b*). Cambering the lower chord in this manner gives greater headroom under the truss at the center, and somewhat improves its appearance; but it increases the stresses on all the members except *KI*, *ML*, and *ON*.


20'-0"
R_2=18000
22250
4500
4500
4500

only one-half of the stress diagram need be drawn as the other side will be exactly the same, and consequently the loads only as far as $e f$ need be laid off on the vertical load line. The reactions R_1 and R_2 are each equal to one-half of the total load, or 18,000 pounds; hence, the point z is located midway between e and f, and $z a$ represents the reaction R_1, 18,000 pounds.

The stresses around the joint $A B K Z$ may be drawn in the stress diagram by commencing at b and drawing $b k$ parallel with $B K$, and from z, a line parallel with $K Z$, intersecting the first line at k. The polygon of forces around this joint will then be from a to b, from b to k, from k to z, and from z back again to a, the starting point. From the direction of these forces, the kind of stress on the member is observed.

The next joint in the truss to analyze is $B C L K$. In the stress diagram begin at c and draw $c l$ parallel with $C L$ in the frame diagram, then from k, draw a line parallel with $L K$ in the frame diagram, until it intersects the line drawn from c at the point l. The polygon of forces around the joint is from b to c, from c to l, from l to k, and from k back to b, the starting point.

Around the joint $K L M Z$, the stresses are obtained by drawing from l a line parallel with $L M$ in the frame diagram, until it intersects the line $z k$ at the point m. The polygon of forces around this joint is from k to l, from l to m, from m to z, and from z back again to k, the starting point.

Difficulty will be encountered on attempting to analyze either the joint $C D O N M L$ or $M N Q Z$, for at each of these joints there are three unknown forces, or stresses. It is not easy, by the graphic method, to solve the stresses around a joint in a structure where there are more than two unknown forces, consequently some other method of solution must be found.

On inspecting the frame diagram, Fig. 34, it will be seen that the joint $D E$ is similar to the joint $B C$; the panel load, 4,500 pounds in each case, is supported by two forces, $B K$ and $K L$, $D O$ and $O P$, respectively. Since the

directions of the forces are respectively parallel, and the panel loads are equal, the stresses in KL and OP are equal, these stresses being due only to a component of their respective panel loads. In a similar manner, it can be shown that the stresses in KL and OP are each held in equilibrium by the pairs of forces, KZ and LM, ON and PQ, and that the stresses in LM and NO are produced by equal components of the equal stresses in KL and OP; therefore, the stresses in LM and NO are equal. This gives the magnitude of the stress ON, its equal LM having already been

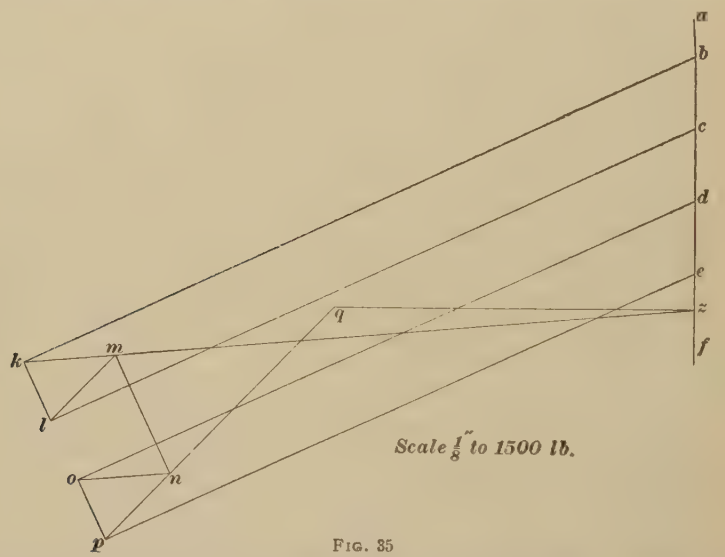


FIG. 35

determined, and the diagram can be completed as follows: Draw mn parallel to MN of the frame diagram, then draw do parallel with DO, and of such a length that when on is drawn, from its extremity o, the point n will meet the line mn midway between the lines do and cp. This construction makes the length on equal to lm, which is in accordance with the condition that the stresses in LM and ON are equal. Then the polygon of forces around this joint will be from c to d, from d to o, from o to n, from n to m, from m to l, and from l to c, the starting point.

The remaining joints, when taken in their usual order, offer no difficulty, and the other half of the diagram need not be drawn unless it is desired to check the half of the diagram just completed.

The polygon of forces that has just been traced around the joint $CDONML$ affords a good illustration of the rule that the forces that meet at a joint must make a closed polygon in the stress diagram.

One of the peculiarities of the stress diagram, Fig. 35, and one that is worthy of note, as it will materially assist in drawing the diagram, is that the triangles lkm and pon are equal to each other and similar to the larger one whose base is mn.

25. The wind-stress diagram may now be drawn. First, the frame diagram is redrawn, and on it, as shown in Fig. 36, are designated the wind loads acting at the several joints of the truss, in a direction normal to the slope of the roof.

The reactions R_1 and R_2 may be calculated by the principle of moments. Since the truss is securely fastened at both ends, neither end being free to move in a lateral direction, these reactions will be parallel with the action of the wind on the roof, that is, normal to the slope. The reaction R_2 may first be obtained by extending its line of direction until it intersects the extension of the left-hand rafter member at the point a'. Then, by taking the center of moments at the left-hand reaction R_1, the magnitude of the reaction R_2 may be computed.

For convenience, reduce to feet and decimals the distance from each panel point to the point of rotation R_1; the moments about this point will then be as follows:

$$\begin{aligned}
5{,}625 \times 11.188 &= 62932.50 \text{ foot-pounds} \\
5{,}625 \times 22.375 &= 125859.38 \text{ foot-pounds} \\
5{,}625 \times 33.563 &= 188791.88 \text{ foot-pounds} \\
2{,}813 \times 44.750 &= 125881.75 \text{ foot-pounds} \\
\text{Total,} \qquad & 503465.51 \text{ foot-pounds}
\end{aligned}$$

The distance of the center of moments of the line of action of the reaction R_2 is $44.75 + 27 = 71.75$ feet, and

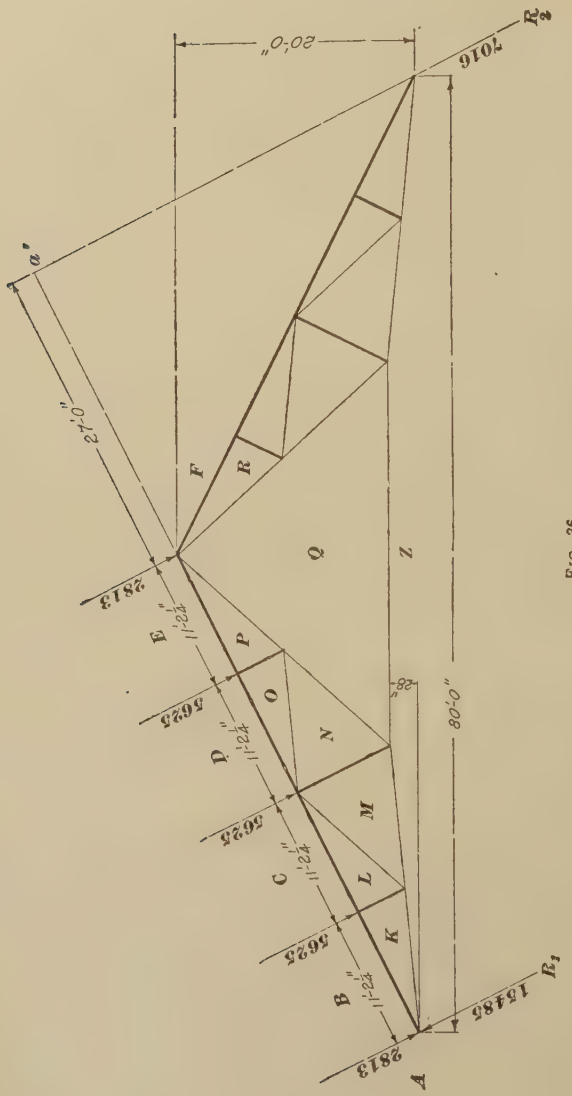

20'-0"
7016
R_2
a'
21'-0"
2813
F
R
Q
Z
E 11-22"
P
5625
O
28'-0"
80'-0"
D 11-22"
N
5625
M
C 11-22"
L
5625
K
B 11-22"
15486
R_1
2813
A

Fig. 36

503,465.51 ÷ 71.75 = 7,016, the magnitude of the reaction R_2, in pounds.

Since the sum of the wind loads is 22,501 pounds, which is equal to the sum of the reactions, the reaction due to the wind at R_1 is 22,501 − 7,016 = 15,485 pounds.

The wind-stress diagram, Fig. 37, may now be drawn. First draw the load line a to f parallel to the reactions and direction of the wind loads at the several joints. Then lay off on the load line the loads ab, bc, cd, de, and ef,

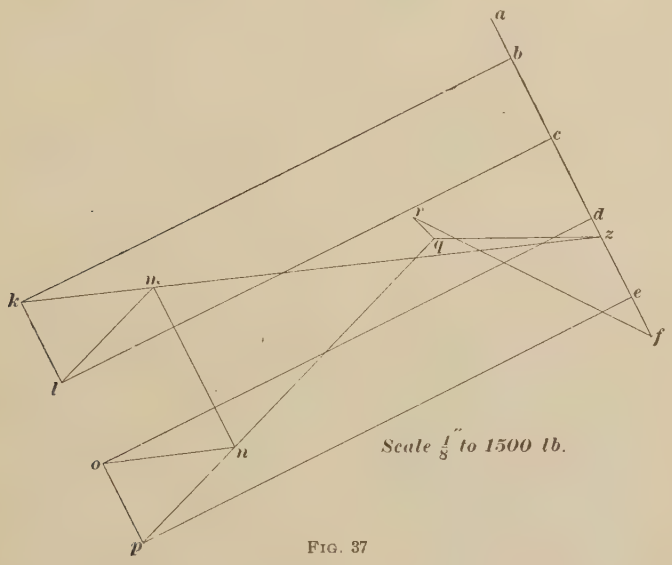


FIG. 37

which are respectively equal to the corresponding loads AB, BC, CD, DE, and EF in the frame diagram. Having located the point f, the magnitude of the reaction R_2, represented by fz, may be laid off upwards (the direction in which it acts), and the point z is thus located; then by scaling za, it should be found equal to 15,485 pounds, the reaction R_1.

The polygon of external forces will then be from a to b, from b to c, from c to d, from d to e, from e to f, from f to z, and from z to a, the starting point.

The joint $CDONML$ may be solved in this stress diagram in the same manner as it was solved in the vertical-load stress diagram.

The analysis of the stresses around the last joint $EFRQP$ in the frame diagram is interesting from the fact that the stress qr closes the diagram, and, if the diagram is correctly drawn, it must be parallel to the member QR.

Having drawn both the wind and the vertical-load stress diagrams, the stresses in the several members in the truss may be obtained by scaling, and their magnitudes may be tabulated as follows, compression in this table being indicated by the plus sign and tension by the minus sign:

Member	Stress Due to Vertical or Dead Load Pounds	Stress Due to Wind Load Pounds	Total Stress Pounds
BK	+ 46,000	+ 34,500	+ 80,500
CL	+ 44,000	+ 34,500	+ 78,500
DO	+ 42,000	+ 34,500	+ 76,500
EP	+ 40,000	+ 34,500	+ 74,500
KL	+ 4,000	+ 5,500	+ 9,500
MN	+ 8,000	+ 11,250	+ 19,250
OP	+ 4,000	+ 5,500	+ 9,500
ZK	− 41,500	− 36,750	− 78,250
ZM	− 35,500	− 28,500	− 64,000
ZQ	− 21,000	− 11,000	− 32,000
QN	− 15,500	− 18,500	− 34,000
QP	− 21,500	− 26,000	− 47,500
FR		+ 17,000	
RQ		− 2,000	
NO	− 5,750	− 8,000	− 13,750
LM	− 5,750	− 8,000	− 13,750

STRENGTH OF RIVETS AND PINS

26. Rivets and pins are the elements by which the different sections and members of a steel truss are bound, or tied, together. Pins also are sometimes used in the better class of timber construction, in which case, however, the tension members are usually made of steel.

Where plates or rolled shapes are joined by either pins or rivets, as in Fig. 38, there is more or less friction between the several parts, which acts to prevent them from being pulled apart. This is especially true when rivets are driven close against the plates while hot; in cooling, they contract between the heads and bind the plates tightly together.

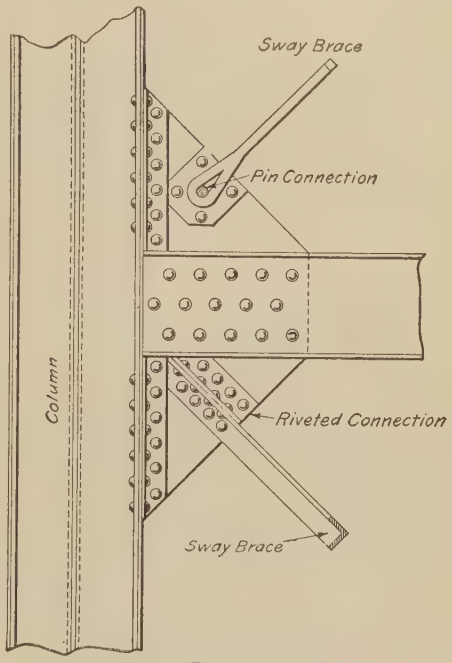


FIG. 38

The friction between plates bound together by rivets and pins, however, is not taken into account by American engineers when calculating the strength of the connections.

27. Methods of Failure.—Riveted joints may fail either by the shearing of the rivet or the crimping of the plates that they connect. When a rivet shears, the tendency is to cut straight through it across its section, as shown in

Fig. 39 (*a*) and (*b*). Where there are only two plates connected, as shown at (*a*), the tendency is to cut the rivet on the single plane *a b*. A rivet in this position is said to be in

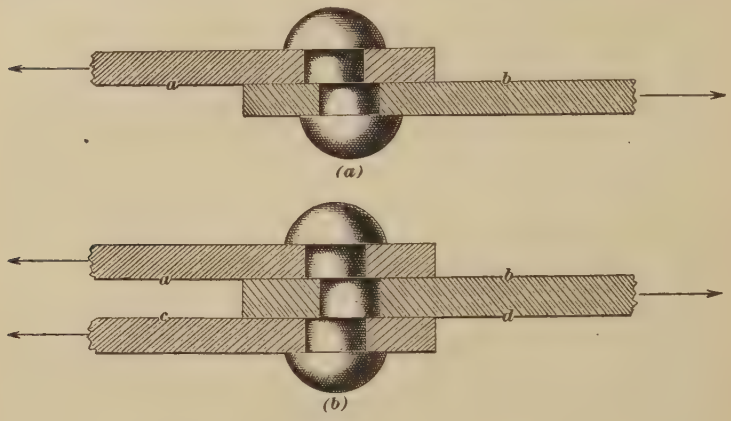


Fig. 39

single shear. At (*b*), the tendency is to cut through the rivet on both the planes *a b* and *c d*; under these conditions the rivet is in **double shear,** and it is evident that, since the rivet will shear across at two places, it will be twice as strong as where the tendency is to shear through only one section.

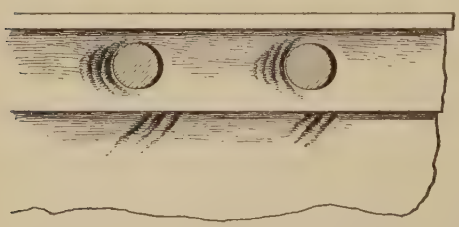


Fig. 40

When the diameter of the rivet is large in proportion to the thickness of the plate, the joint or connection is liable to fail by the crimping of the plate, as shown in Fig. 40. This occurs when the resistance of the plate to crushing or crimping is less than the resistance of the rivet to shear.

28. Bearing Value of Rivets.—The condition of the plates in the joint shown in Fig. 41 (*a*) is called **ordinary bearing,** while the plate *m* connected as shown at (*b*) is said to be in **web bearing.** This distinction is important,

because the bearing value of a web-plate is greater than that of an outside plate, the value for web bearing being about one-third greater than for ordinary bearing.

As the tensile strength of iron and steel used in the manufacture of rivets, pins, and plates for structural work is more easily determined by tests, and, therefore, better known than either its compressive or shearing strength, it is customary to use this as a basis from which to calculate the bearing value of plates and the shearing strength of rivets and pins.

Good practice assumes that the compressive strength of steel or high-test iron is about thirteen-fifteenths of its tensile strength; that is, if the safe tensile strength of the material per square inch of section is 15,000 pounds, the safe compressive strength may be taken as thirteen-fifteenths of 15,000, or 13,000 pounds.

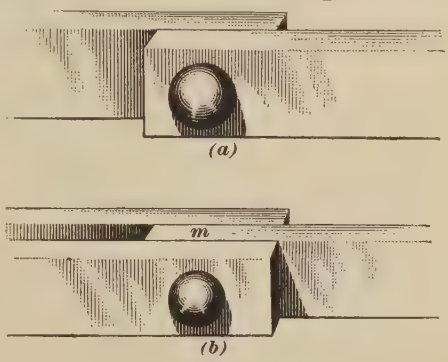


FIG. 41

The shearing strength is considered to be five-sixths of the compressive strength. For example, if, as stated, the tensile strength is 15,000 pounds and the compressive strength 13,000 pounds, the shearing strength becomes five-sixths of 13,000, or 10,833 pounds per square inch of section.

In order to determine the bearing value of the plates around a rivet hole, it is necessary to consider the bearing area of the rivet on the plate. This is always assumed to be the product of the diameter of the rivet multiplied by the thickness of the plate. Owing, however, to the support that the material around the hole receives from the rest of the plate, it will safely sustain a pressure greater than the compressive strength of the material when not so supported. The safe bearing strength of the rivet on the plate, for ordinary bearing, is therefore assumed to be $1\frac{1}{2}$ times its

compressive strength, and for web bearing the safe bearing strength is assumed as double the compressive strength of the material.

In deducting the rivet holes, to ascertain the net section of a riveted plate, the diameter of the hole is taken as $\frac{1}{8}$ inch larger than the diameter of the rivet.

EXAMPLE 1.—Two pieces of structural steel are joined by rivets, as shown in Fig. 42. If the tensile strength of the steel is 60,000 pounds

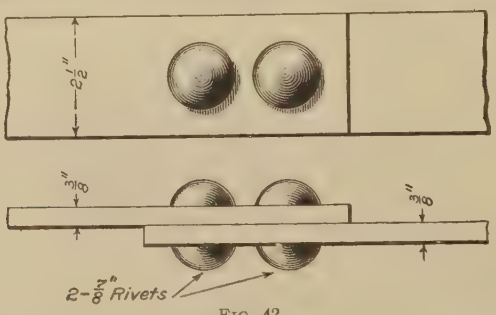


FIG. 42

per square inch and the factor of safety of 4 is used, what is the safe strength of this joint?

SOLUTION.—The safe tensile strength of the steel is $60{,}000 \div 4 = 15{,}000$ lb. per sq. in. The width of the pieces connected is $2\frac{1}{2}$ in., from which is to be deducted 1 in. for the rivet hole, leaving a net width of $1\frac{1}{2}$ in., which, multiplied by the thickness of the plate, gives a net area of $1\frac{1}{2} \times \frac{3}{8} = .5625$ sq. in. Then, $.5625 \times 15{,}000 = 8{,}437$ lb., the strength of the plate.

Now, to determine whether the strength of the rivets is equal to the net section of the plate: Taking the compressive value of the plate as thirteen-fifteenths of 15,000 lb., or 13,000 lb. per sq. in., and the rivets being in ordinary bearing, the safe bearing strength is $13{,}000 \times 1\frac{1}{2} = 19{,}500$ lb. per sq. in. of bearing area. The bearing area is $\frac{7}{8} \times \frac{3}{8} = .328$ sq. in.; therefore, the safe bearing strength for one rivet is $19{,}500 \times .328 = 6{,}396$ lb., and for the two it is $2 \times 6{,}396 = 12{,}792$ lb.

The next point to consider is the resistance of the rivets to shear: The shearing strength of the steel is five-sixths of $13{,}000 = 10{,}833$ lb. per sq. in. The area of a $\frac{7}{8}$-in. rivet is .601 sq. in., which, multiplied by 10,833, gives 6,510 lb., the shearing strength of one rivet. The total resistance to shear of the rivets in the joint is therefore $6{,}510 \times 2 = 13{,}020$ lb.

The safe resistance of the three elements entering into the strength of the joint is therefore as follows: Resistance of net section of the

plate = 8,437 lb.; bearing value of the plate = 12,792 lb.; shearing strength of the two rivets = 13,020 lb.; from which it is easily seen that the strength of the joint is that of the net section of the plate, 8,437 lb. Ans.

Since the bearing value of the plate and the shearing strength of the rivets are considerably in excess of this amount, it appears that the rivets are large for the joint, and it is probable that $\frac{3}{4}$-in. rivets will give better results.

EXAMPLE 2.—One of the tension members in a structure is connected as shown in Fig. 43. The tension bars are made of structural

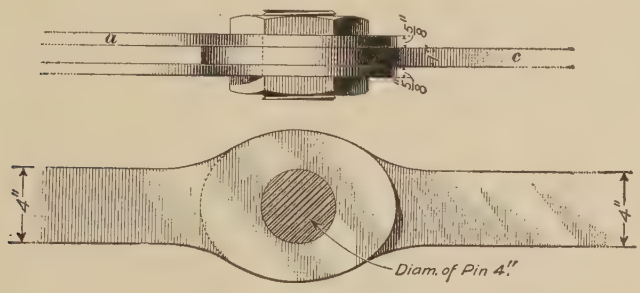


FIG. 43

steel with a safe tensile strength of 15,000 pounds per square inch. (*a*) What is the bearing value of the bar *c*? (*b*) What is the bearing value of the two bars *a*?

SOLUTION.—(*a*) The safe compressive strength of the material is thirteen-fifteenths of 15,000 lb., or 13,000 lb. Then the bearing value of the bar *c*, which may be considered as a web, is $2 \times 13{,}000 = 26{,}000$ lb. per sq. in. The bearing area of the pin in the bar *c* is $4 \times 1 = 4$ sq. in.; therefore, the bearing strength of the bar is

$$26{,}000 \times 4 = 104{,}000 \text{ lb.} \quad \text{Ans.}$$

(*b*) As the piece *a* is in ordinary bearing, its bearing value is $13{,}000 \times 1\frac{1}{2} = 19{,}500$ lb. per sq. in. The bearing area of the two bars is $2 \times 4 \times \frac{5}{8} = 5$ sq. in.; their combined bearing strength is therefore

$$19{,}500 \times 5 = 97{,}500 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 3.—Determine the safe strength of the riveted joint shown in Fig. 44, in which the plates and rivets each have a safe tensile strength of 16,000 pounds per square inch.

SOLUTION.—The safe tensile strength of the material being 16,000 lb., the safe compressive strength is thirteen-fifteenths of 16,000 lb., or 13,867 lb., and the shearing strength of the rivets is five-sixths of 13,867, or 11,556 lb. per sq. in. of section. The area of the section of a

$\frac{7}{8}$-in. rivet is .601 sq. in.; therefore, the total shearing strength of the three rivets, each of which is in double shear, is

$$2 \times .601 \times 11{,}556 \times 3 = 41{,}670 \text{ lb.}$$

The two outside plates are in ordinary bearing, and their bearing value is $1\frac{1}{2} \times 13{,}866 = 20{,}800$ lb. per sq. in. There are three rivet holes in each plate, each with a bearing area of $\frac{3}{8} \times \frac{7}{8} = .328$ sq. in.; the total bearing strength of the two plates is therefore

$$20{,}800 \times .328 \times 3 \times 2 = 40{,}934 \text{ lb.}$$

The bearing value of the central, or web-bearing, plate at one rivet hole is $2 \times 13{,}867$ lb. $= 27{,}734$ lb. per sq. in., and the bearing area is

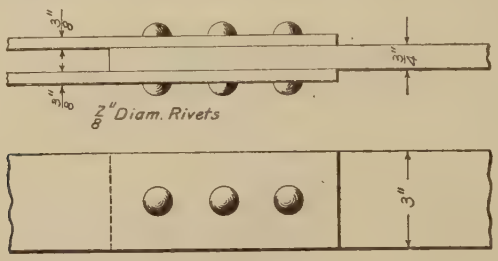


FIG. 44

$\frac{3}{4} \times \frac{7}{8} = .656$ sq. in.; the total bearing strength for the three rivets is therefore

$$27{,}734 \times .656 \times 3 = 54{,}580 \text{ lb.}$$

The safe tensile strength of the central plate is equal to its net section multiplied by 16,000, the safe unit tensile strength of the material. The net width of the plate is $3 - 1 = 2$ in., and its net area, $2 \times \frac{3}{4} = 1.5$ sq. in.; therefore, the safe strength is $1.5 \times 16{,}000 = 24{,}000$ lb. The strength of the two outside plates, calculated in the same manner, is found to be 24,000 lb. also; therefore, it is evident that, since the strength of the net section of the plate is much less than either the strength of the rivets or the bearing value of the plates, it determines the strength of the joint, which is therefore 24,000 lb. Ans.

29. Table of Bearing Values of Rivets.—In order to avoid the necessity of calculating the shearing value of the rivets and the bearing value of the riveted plates and rolled sections, Table III, showing the values of rivets, has been prepared. It will be noticed that the areas and shearing values for both double and single shear are given for rivets from $\frac{1}{2}$ inch to $\frac{7}{8}$ inch in diameter.

The least distance that rivets may be placed from the end or side of a plate is given in the third and fourth vertical columns from the left-hand side of the table. By referring

to Fig. 45, the terms *end distance* and *side distance* used in the table will be readily understood.

The minimum allowable distance from the end and side of plate, as given in the table, is readily ascertained by the following: The end distance equals the thickness of the plate plus one-half the diameter of the rivet plus $\frac{1}{2}$ inch, while the side distance equals one-half the thickness of the plate plus one-half the diameter of the rivet plus $\frac{1}{8}$ inch.

The values under the heading Allowable Stress per Square Inch on Soft Steel refer to the tensile strength of the

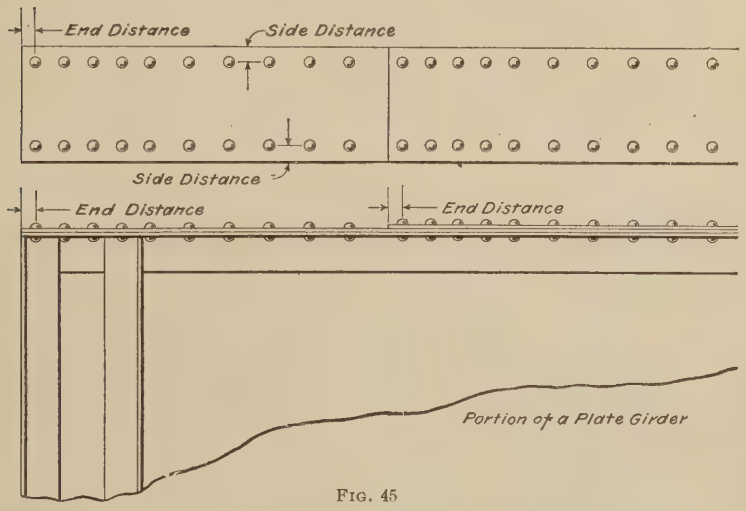


FIG. 45

material. If an allowable stress of 15,000 pounds is desired, use the value of the plates and rivets given under this head. Although not always done, the allowable tensile stress when calculating rivet values should be kept below 15,000 pounds. A stress of 10,000 or 12,000 pounds can be generally used.

If the requirements of the structure are such that values other than those given in the table are desired, the required values may be obtained by finding the sum or the difference of two of the given columns. For example, a stress of 11,000 pounds per square inch is to be allowed, and the ordinary bearing value of a $\frac{1}{4}$-inch plate at a $\frac{5}{8}$-inch rivet hole is

TABLE III

VALUES OF RIVETS

Rivet Section		Minimum Distance From Edge of Plate		Ordinary Bearing					Web Bearing				
Diameter Inch	Area Square Inch	End Inches	Side Inch	Plate Thickness	Allowable Stress per Square Inch on Soft Steel				Plate Thickness	Allowable Stress per Square Inch on Soft Steel			
					1,000	10,000	12,000	15,000		1,000	10,000	12,000	15,000
		$\frac{7}{8}$	$\frac{7}{16}$	$\frac{1}{8}$-inch plate	81.25	813	975	1,219	$\frac{1}{8}$-inch plate	108.34	1,083	1,300	1,625
		$1\frac{5}{16}$	$\frac{15}{32}$	$\frac{3}{16}$-inch plate	121.28	1,219	1,463	1,828	$\frac{3}{16}$-inch plate	162.50	1,625	1,950	2,438
$\frac{1}{2}$	.196	1	$\frac{1}{2}$						$\frac{1}{4}$-inch plate	216.67	2,167	2,600	3,250
		$1\frac{1}{16}$	$\frac{17}{32}$						$\frac{5}{16}$-inch plate	270.84	2,708	3,250	4,063
				Single shear	141.56	1,416	1,699	2,123	Double shear	283.12	2,831	3,397	4,247
		$1\frac{5}{16}$	$\frac{1}{2}$	$\frac{1}{8}$-inch plate	101.56	1,016	1,219	1,523	$\frac{1}{8}$-inch plate	135.42	1,354	1,625	2,031
		1	$\frac{17}{32}$	$\frac{3}{16}$-inch plate	152.34	1,523	1,823	2,285	$\frac{3}{16}$-inch plate	203.13	2,031	2,438	3,047
		$1\frac{1}{16}$	$\frac{9}{16}$	$\frac{1}{4}$-inch plate	203.13	2,031	2,437	3,046	$\frac{1}{4}$-inch plate	270.84	2,708	3,250	4,063
$\frac{5}{8}$	.307	$1\frac{1}{8}$	$\frac{19}{32}$						$\frac{5}{16}$-inch plate	338.55	3,386	4,063	5,078
		$1\frac{3}{16}$	$\frac{5}{8}$						$\frac{3}{8}$-inch plate	406.26	4,063	4,875	6,094
				Single shear	221.73	2,217	2,661	3,326	Double shear	443.46	4,435	5,322	6,652

		1	$\frac{9}{16}$	$\frac{1}{8}$-inch plate	121.88	1,219	1,463	1,828	$\frac{1}{8}$-inch plate	162.50	1,625	1,950	2,438
		$1\frac{1}{16}$	$\frac{19}{32}$	$\frac{3}{16}$-inch plate	182.81	1,828	2,193	2,742	$\frac{3}{16}$-inch plate	243.75	2,438	2,925	3,656
		$1\frac{1}{8}$	$\frac{5}{8}$	$\frac{1}{4}$-inch plate	243.75	2,438	2,925	3,656	$\frac{1}{4}$-inch plate	325.00	3,250	3,900	4,875
$\frac{3}{4}$	.442	$1\frac{3}{16}$	$\frac{21}{32}$	$\frac{5}{16}$-inch plate	304.69	3,047	3,656	4,570	$\frac{5}{16}$-inch plate	406.25	4,063	4,875	6,094
		$1\frac{1}{4}$	$\frac{11}{16}$						$\frac{3}{8}$-inch plate	487.50	4,875	5,850	7,313
		$1\frac{5}{16}$	$\frac{23}{32}$						$\frac{7}{16}$-inch plate	568.75	5,688	6,825	8,531
				Single shear	319.24	3,192	3,830	4,788	Double shear	638.47	6,385	7,662	9,577
		$1\frac{1}{16}$	$\frac{5}{8}$	$\frac{1}{8}$-inch plate	142.19	1,422	1,706	2,133	$\frac{1}{8}$-inch plate	189.58	1,896	2,275	2,844
		$1\frac{1}{8}$	$\frac{21}{32}$	$\frac{3}{16}$-inch plate	213.28	2,133	2,559	3,199	$\frac{3}{16}$-inch plate	284.37	2,844	3,413	4,266
		$1\frac{3}{16}$	$\frac{11}{16}$	$\frac{1}{4}$-inch plate	284.37	2,844	3,413	4,266	$\frac{1}{4}$-inch plate	379.17	3,792	4,550	5,687
		$1\frac{1}{4}$	$\frac{23}{32}$	$\frac{5}{16}$-inch plate	355.47	3,555	4,266	5,332	$\frac{5}{16}$-inch plate	473.96	4,740	5,687	7,109
$\frac{7}{8}$	.601	$1\frac{5}{16}$	$\frac{3}{4}$	$\frac{3}{8}$-inch plate	426.57	4,266	5,119	6,398	$\frac{3}{8}$-inch plate	568.75	5,688	6,825	8,531
		$1\frac{3}{8}$	$\frac{25}{32}$						$\frac{7}{16}$-inch plate	663.54	6,635	7,962	9,953
		$1\frac{7}{16}$	$\frac{13}{16}$						$\frac{1}{2}$-inch plate	758.33	7,583	9,100	11,375
		$1\frac{1}{2}$	$\frac{27}{32}$						$\frac{9}{16}$-inch plate	853.12	8,531	10,237	12,796
				Single shear	434.07	4,341	5,209	6,511	Double shear	868.15	8,682	10,418	13,022

NOTE.—Ordinary bearing = $1\frac{1}{2}$ × unit compressive stress × bearing area. Web bearing = 2 × unit compressive stress × bearing area. Shear = $\frac{5}{8}$ × unit compressive stress × section. Compression = $\frac{13}{15}$ × allowable tensile stress.

required. By adding the value given for an allowable stress of 1,000 pounds to that given for 10,000 pounds, the required bearing value may be determined; in this case it is 203.13 + 2,031 = 2,234.13, which is the allowable bearing value of a $\frac{1}{4}$-inch plate in ordinary bearing around a $\frac{5}{8}$-inch diameter rivet, where the safe tensile stress of the materials is taken at 11,000 pounds per square inch.

30. Pins Subjected to Bending Stresses.—Pins, when used to connect the several members of a structure, besides being subjected to shearing in the same manner as rivets, may be required to resist heavy bending stresses; they may then be regarded as solid cylindrical beams and

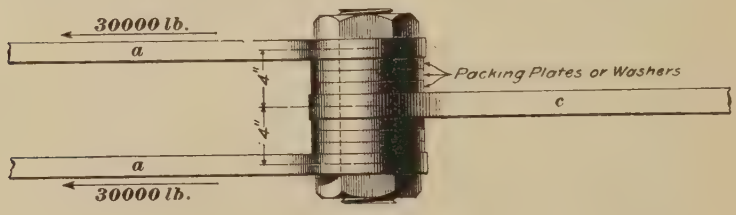


FIG. 46

calculated to resist the greatest bending moment that may come on them.

Take, for example, the pin in Fig. 46 which connects the three tension bars *a*, *a*, and *c*; the pull of the two bars *a*, *a*, both acting in the same direction, is transmitted to the bar *c*

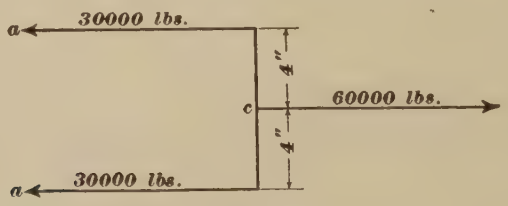


FIG. 47

by means of the pin. The stress on each bar *a* is 30,000 pounds, consequently the stress on the bar *c* must be 60,000 pounds. Assuming a maximum unit stress of 15,000 pounds per square inch, it is desired to find what diameter of pin

TABLE IV

RESISTING MOMENTS OF PINS

With Extreme Fiber Stresses Varying From 15,000 to 25,000 Pounds per Square Inch

Diameter of Pin Inches	Area of Pin Square Inches	Moments, in Inch-Pounds, for Fiber Strains of			
		15,000 lb. per Square Inch	20,000 lb. per Square Inch	22,000 lb. per Square Inch	25,000 lb. per Square Inch
1	.785	1,470	1,960	2,160	2,450
1 1/8	.994	2,100	2,800	3,080	3,500
1 1/4	1.227	2,880	3,830	4,220	4,790
1 3/8	1.485	3,830	5,100	5,620	6,380
1 1/2	1.767	4,970	6,630	7,290	8,280
1 5/8	2.074	6,320	8,430	9,270	10,500
1 3/4	2.405	7,890	10,500	11,570	13,200
1 7/8	2.761	9,710	12,900	14,240	16,200
2	3.142	11,800	15,700	17,280	19,600
2 1/8	3.547	14,100	18,800	20,730	23,600
2 1/4	3.976	16,800	22,400	24,600	28,000
2 3/8	4.430	19,700	26,300	28,900	32,900
2 1/2	4.909	23,000	30,700	33,700	38,400
2 5/8	5.412	26,600	35,500	39,000	44,400
2 3/4	5.940	30,600	40,800	44,900	51,000
2 7/8	6.492	35,000	46,700	51,300	58,300
3	7.069	39,800	53,000	58,300	66,300
3 1/8	7.670	44,900	59,900	65,900	74,900
3 1/4	8.296	50,600	67,400	74,100	84,300
3 3/8	8.946	56,600	75,500	83,000	94,400
3 1/2	9.621	63,100	84,200	92,600	105,200
3 5/8	10.321	70,100	93,500	102,900	116,900
3 3/4	11.045	77,700	103,500	113,900	129,400
3 7/8	11.793	85,700	114,200	125,600	142,800

TABLE IV—(*Continued*)

Diameter of Pin Inches	Area of Pin Square Inches	Moments, in Inch-Pounds, for Fiber Strains of			
		15,000 lb per Square Inch	20,000 lb. per Square Inch	22,000 lb. per Square Inch	25,000 lb. per Square Inch
4	12.566	94,200	125,700	138,200	157,100
4⅛	13.364	103,400	137,800	151,600	172,300
4¼	14.186	113,000	150,700	165,800	188,400
4⅜	15.033	123,300	164,400	180,800	205,500
4½	15.904	134,200	178,900	196,800	223,700
4⅝	16.800	145,700	194,300	213,700	242,800
4¾	17.721	157,800	210,400	231,500	263,000
4⅞	18.665	170,600	227,500	250,200	284,400
5	19.635	184,100	245,500	270,000	306,800
5⅛	20.629	198,200	264,300	290,700	330,400
5¼	21.648	213,100	284,100	312,500	355,200
5⅜	22.691	228,700	304,900	335,400	381,100
5½	23.758	245,000	326,700	359,300	408,300
5⅝	24.850	262,100	349,500	384,400	436,800
5¾	25.967	280,000	373,300	410,600	466,600
5⅞	27.109	298,600	398,200	438,000	497,700
6	28.274	318,100	424,100	466,500	530,200
6⅛	29.465	338,400	451,200	496,300	564,000
6¼	30.680	359,500	479,400	527,300	599,200
6⅜	31.919	381,500	508,700	559,600	635,900
6½	33.183	404,400	539,200	593,100	674,000
6⅝	34.472	428,200	570,900	628,000	713,700
6¾	35.785	452,900	603,900	664,200	754,800
6⅞	37.122	478,500	638,000	701,800	797,500
7	38.485	505,100	673,500	740,800	841,900
7⅛	39.871	532,700	710,200	781,200	887,800
7¼	41.282	561,200	748,200	823,000	935,300
7⅜	42.718	590,700	787,600	866,300	984,500

TABLE IV—(*Continued*)

Diameter of Pin Inches	Area of Pin Square Inches	Moments, in Inch-Pounds, for Fiber Strains of			
		15,000 lb. per Square Inch	20,000 lb. per Square Inch	22,000 lb. per Square Inch	25,000 lb. per Square Inch
$7\frac{1}{2}$	44.179	621,300	828,400	911,200	1,035,400
$7\frac{5}{8}$	45.664	652,900	870,500	957,500	1,088,100
$7\frac{3}{4}$	47.173	685,500	914,000	1,005,300	1,142,500
$7\frac{7}{8}$	48.707	719,200	958,900	1,054,800	1,198,700
8	50.265	754,000	1,005,300	1,105,800	1,256,600
$8\frac{1}{8}$	51.849	789,900	1,053,200	1,158,500	1,316,500
$8\frac{1}{4}$	53.456	826,900	1,102,500	1,212,800	1,378,200
$8\frac{3}{8}$	55.088	865,100	1,153,400	1,268,800	1,441,800
$8\frac{1}{2}$	56.745	904,400	1,205,800	1,326,400	1,507,300
$8\frac{5}{8}$	58.426	944,900	1,259,800	1,385,800	1,574,800
$8\frac{3}{4}$	60.132	986,500	1,315,400	1,446,900	1,644,200
$8\frac{7}{8}$	61.862	1,029,400	1,372,500	1,509,800	1,715,700
9	63.617	1,073,500	1,431,400	1,574,500	1,789,200
$9\frac{1}{8}$	65.397	1,118,900	1,491,900	1,641,100	1,864,800
$9\frac{1}{4}$	67.201	1,165,500	1,554,000	1,709,400	1,942,500
$9\frac{3}{8}$	69.029	1,213,400	1,617,900	1,779,600	2,022,300
$9\frac{1}{2}$	70.882	1,262,600	1,683,400	1,851,800	2,104,300
$9\frac{5}{8}$	72.760	1,313,100	1,750,800	1,925,900	2,188,500
$9\frac{3}{4}$	74.662	1,364,900	1,819,900	2,001,900	2,274,900
$9\frac{7}{8}$	76.590	1,418,100	1,890,800	2,079,900	2,363,500
10	78.540	1,472,600	1,963,500	2,159,900	2,454,400
$10\frac{1}{4}$	82.520	1,585,900	2,114,500	2,325,900	2,643,100
$10\frac{1}{2}$	86.590	1,704,700	2,273,000	2,500,200	2,841,200
$10\frac{3}{4}$	90.760	1,829,400	2,439,300	2,683,200	3,049,100
11	95.030	1,960,100	2,613,400	2,874,800	3,266,800
$11\frac{1}{4}$	99.400	2,096,800	2,795,700	3,075,400	3,494,800
$11\frac{1}{2}$	103.870	2,239,700	2,986,300	3,284,800	3,732,800
12	113.100	2,544,700	3,392,900	3,732,200	4,241,200

is required to resist the bending moment produced by the stresses exerted on it.

In calculating the bending moment on a pin, the forces acting on it through the several members are considered as being applied at the center of the bearings. In Fig. 46, the distance between the centers of the bearings of the members is 4 inches, and by referring to the diagram, Fig. 47, it is seen that the greatest bending moment is at c, and is equal to $30{,}000 \times 4 = 120{,}000$ inch-pounds.

Having found the greatest bending moment on the pin, it is necessary to determine its diameter, in order that its resisting moment may equal the moment of the bending stresses.

By referring to *Design of Beams*, we find the section modulus of a circular section to be

$$K = \frac{AD}{8} = \frac{.7854\,D^2 \times D}{8} = .0982\,D^3$$

The allowable unit stress on the material is $S = 15{,}000$ pounds per square inch, and the bending moment is $M = 120{,}000$ inch-pounds; substituting these values in the formula for a beam, we have $M = SK$, or $120{,}000 = 15{,}000 \times .0982\,D^3$, from which we get

$$D^3 = \frac{120{,}000}{15{,}000 \times .0982} = 81.46$$

The diameter of the pin is therefore

$$D = \sqrt[3]{81.46} = 4.335 = 4\tfrac{3}{8} \text{ inches, nearly}$$

31. Table of Resisting Moments of Pins.—To avoid the necessity of calculating the resisting moment of pins, Table IV will be found convenient. This table gives the resisting moments of pins from 1 to 12 inches in diameter, calculated for allowable fiber stresses of 15,000, 20,000, 22,000, and 25,000 pounds per square inch.

32. Resultant Moment of Several Stresses.—When a pin is used at a joint at which several members, extending in different directions, meet, as shown in Fig. 48, it is necessary to combine the stresses so as to find the resultant that gives the greatest bending moment. This is conveniently

done by first resolving the stresses on each of the different members into vertical and horizontal components, and calculating the bending moments produced in each of these directions by all the corresponding components. The maximum bending moment is then given by the resultant of these two bending moments.

The details of the method for finding the maximum bending stress on a pin will be made clear by a study of the following illustrative examples:

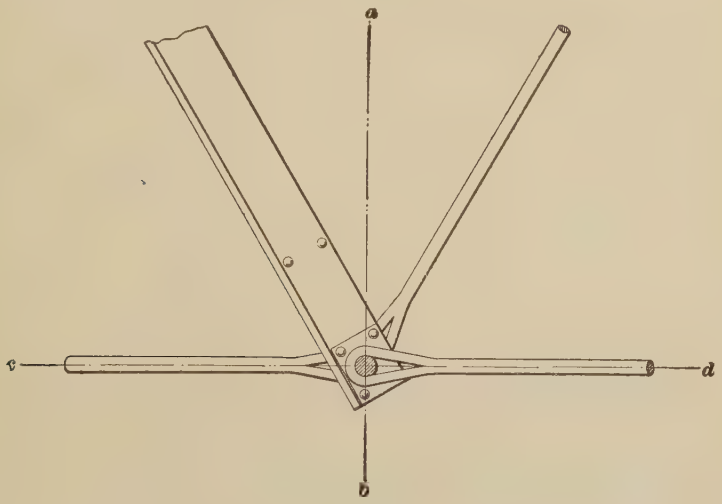


FIG. 48

Fig. 49 shows one of the lower joints of a roof truss. At this joint there are four sets of members, two of which act in a horizontal, while one acts in a vertical, direction; since they already act in the directions of the required components, these forces need not be resolved. There is, however, one inclined member in which there is a compressive stress of 40,000 pounds, and this stress must be resolved into its vertical and horizontal components. Draw the line *a b* parallel to the strut and of such a length as to represent the magnitude of the stress. From *a*, draw the horizontal line *a c* intersecting the vertical line at the point *c*. The direction of the forces around the triangle is shown by

the arrows. On measuring the line ac, the horizontal component of the stress in ab is found to be 20,000 pounds, while the vertical component of the stress is found to be 34,650 pounds.

Having determined these components, a diagram showing all the horizontal stresses acting on the pin and tending to

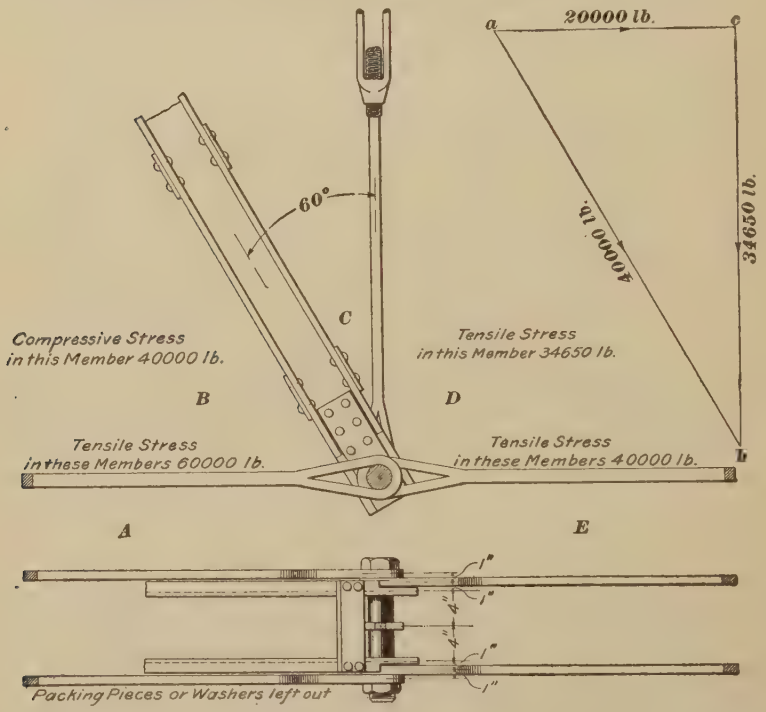


Fig. 49

bend it, and also another showing all the vertical forces, should be drawn as illustrated in Fig. 50 (a) and (b), the distance from center to center of the members being taken from the detail plan of the joint, Fig. 49. It must always be remembered that, in accordance with the principles of equilibrium, the sum of the resultants of the forces acting on the pin in any one direction must equal the sum of all the resultants acting in the opposite direction; otherwise,

the pin would move in the direction of the greater sum, and the structure would fall. Thus, from Fig. 49, it is readily seen that the vertical component of the stress in BC acts in an opposite direction to the stress in the member CD, while the horizontal component acts in opposition to the stresses in the member AB, and in the same direction as the stress in DE. This makes the algebraic sum of all the components in either the horizontal or vertical direction equal to zero, and fulfils the condition of equilibrium.

The resultant of the vertical and horizontal bending moments may also be calculated by the rule for finding the length of the hypotenuse of a right triangle; for example, in this case the lengths of the sides are represented by the horizontal bending moment of 40,000 inch-pounds, and the vertical bending moment of 69,300 inch-pounds; the resultant bending moment is therefore

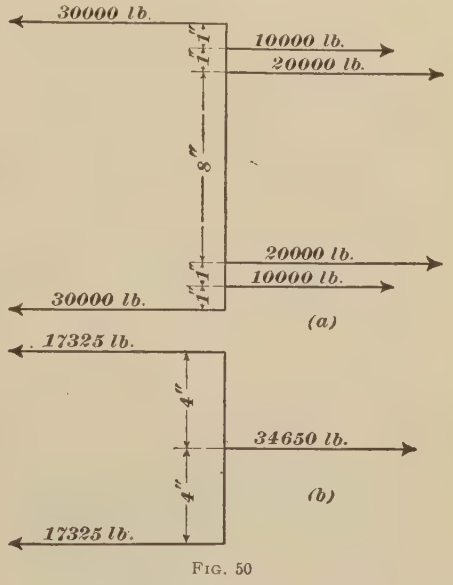


FIG. 50

$$\sqrt{40{,}000^2 + 69{,}300^2} = 80{,}015 \text{ inch-pounds.}$$

In order to determine the required size of pin for this joint, assume a safer fiber stress of 15,000 pounds per square inch, then, by referring to Table IV, it is seen that a pin $3\frac{7}{8}$ inches in diameter, under a fiber stress of 15,000 pounds per square inch, has a resisting moment of 85,700 inch-pounds, which is very nearly the value required by the conditions.

It is necessary to remember, however, that in all cases the pin should be examined for shear as well as bending stresses.

EXAMPLES FOR PRACTICE

1. What are the safe strengths of $\frac{7}{8}$-, $\frac{3}{4}$-, and $\frac{5}{8}$-inch rivets, in double shear, and also in single shear, assuming that the safe tensile strength of the material used in their manufacture is 15,000 pounds per square inch of section?

	Diameter of Rivet Inch	Double Shear Pounds	Single Shear Pounds
Ans.	$\frac{7}{8}$	13,022	6,511
	$\frac{3}{4}$	9,577	4,788
	$\frac{5}{8}$	6,652	3,326

2. What pulling force will two pieces of $\frac{3}{8}'' \times 2\frac{1}{2}''$ bar safely resist, provided that they are connected at the end by two $\frac{3}{4}$-inch diameter rivets, as shown in Fig. 51? The safe tensile strength of the material in rivet and bar is 15,000 pounds. Ans. 9,140 lb.

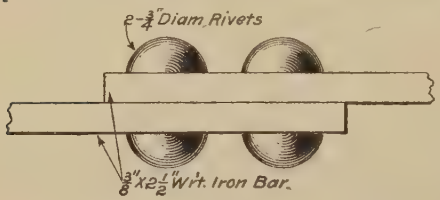


Fig. 51

3. What is the safe resisting moment of a pin 5 inches in diameter, if the safe unit strength of the material is 20,000 pounds? Ans. 245,500

4. In Fig. 52 is shown a pin connection, the pull on the tension

Fig. 52

bar *a* being 140,000 pounds. If the safe shearing strength of the

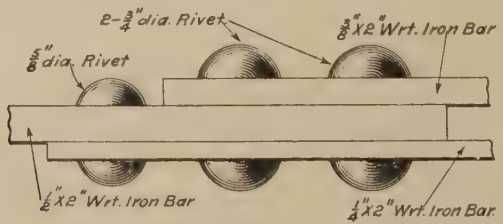


Fig. 53

material in the pin is 10,000 pounds per square inch, and the safe fiber stress in bending is 15,000 pounds per square inch: (*a*) what size of

pin will be required to resist the shear? (*b*) what size will be required to resist the bending?

Ans. { (*a*) 3 in. in diameter
(*b*) $4\frac{1}{8}$ in. in diameter

5. It is necessary to construct the connection of a tension member as shown in Fig. 53. What is the safe load that this member will carry, if the safe tensile strength of the material in both the rivets and bars is 18,000 pounds per square inch? Ans. 11,250 lb.

TRUSS DESIGN

33. Designing the Members of a Truss.—Let the truss described in Art. **20** be considered, as it is a quite common type. In Art. **21** is given the stress in the various members. The first vertical tension member is ML, and has a total pull on it of 3,100 pounds. This rod will be made of soft steel, the tensile strength of which is 56,000 pounds. If a factor of safety of 4 is used, the safe load will be $56{,}000 \div 4 = 14{,}000$ pounds per square inch. A rod $\frac{3}{4}$ inch in diameter has an area at the root of the screw thread, according to Table V, of .301 square inch. Then, the safe strength of a rod $\frac{3}{4}$ inch in diameter, threaded at the ends, is $.301 \times 14{,}000 = 4{,}214$ pounds. This member being required to support 3,100 pounds only, a soft-steel rod $\frac{3}{4}$ inch in diameter is amply strong. The tension member ON is subjected to a stress of 6,500 pounds. A rod 1 inch in diameter has an area at the root of the thread, according to Table V, of .55 square inch. Hence, its safe strength is $.55 \times 14{,}000 = 7{,}700$ pounds, which is slightly in excess of that required. The member QP is subjected to a stress of 15,100 pounds. The area at the root of the thread of a rod $1\frac{1}{2}$ inches in diameter is 1.29 square inches, and the safe strength is $1.29 \times 14{,}000 = 18{,}060$ pounds, which exceeds the strength required.

To calculate the size of timber demanded for the rafter member, note that it is usual in timber trusses to make each rafter member of one size and of one length. This truss requires, then, a large stick of timber, say about 45 feet long, for the rafter members, and can no doubt be obtained

TABLE V

STANDARD SCREW THREADS AND NUTS

Diameter of Rod or Bolt	Threads per Inch	Diameter at Root of Thread	Area of Rod or Bolt at Root of Thread	Short Diameter of Nuts	Long Diameter, Hexagon Nuts	Long Diameter, Square Nuts	Thickness of Nuts
$\frac{1}{4}$	20	.185	.026	$\frac{1}{2}$	$\frac{37}{64}$	$\frac{7}{10}$	$\frac{1}{4}$
$\frac{5}{16}$	18	.240	.045	$\frac{19}{32}$	$\frac{11}{16}$	$\frac{10}{12}$	$\frac{5}{16}$
$\frac{3}{8}$	16	.294	.067	$\frac{11}{16}$	$\frac{51}{64}$	$\frac{63}{64}$	$\frac{3}{8}$
$\frac{7}{16}$	14	.344	.092	$\frac{25}{32}$	$\frac{7}{8}$	$1\frac{7}{64}$	$\frac{7}{16}$
$\frac{1}{2}$	13	.400	.125	$\frac{7}{8}$	1	$1\frac{15}{64}$	$\frac{1}{2}$
$\frac{9}{16}$	12	.454	.161	$\frac{31}{32}$	$1\frac{1}{8}$	$1\frac{23}{64}$	$\frac{9}{16}$
$\frac{5}{8}$	11	.507	.201	$1\frac{1}{16}$	$1\frac{7}{32}$	$1\frac{1}{2}$	$\frac{5}{8}$
$\frac{3}{4}$	10	.620	.301	$1\frac{1}{4}$	$1\frac{7}{16}$	$1\frac{49}{64}$	$\frac{3}{4}$
$\frac{7}{8}$	9	.731	.419	$1\frac{7}{16}$	$1\frac{21}{32}$	$2\frac{1}{32}$	$\frac{7}{8}$
1	8	.837	.550	$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{19}{64}$	1
$1\frac{1}{8}$	7	.940	.693	$1\frac{13}{16}$	$2\frac{3}{32}$	$2\frac{9}{16}$	$1\frac{1}{8}$
$1\frac{1}{4}$	7	1.065	.890	2	$2\frac{5}{16}$	$2\frac{53}{64}$	$1\frac{1}{4}$
$1\frac{3}{8}$	6	1.160	1.056	$2\frac{3}{16}$	$2\frac{17}{32}$	$3\frac{3}{32}$	$1\frac{3}{8}$
$1\frac{1}{2}$	6	1.284	1.294	$2\frac{3}{8}$	$2\frac{3}{4}$	$3\frac{23}{64}$	$1\frac{1}{2}$
$1\frac{5}{8}$	$5\frac{1}{2}$	1.389	1.515	$2\frac{9}{16}$	$2\frac{31}{32}$	$3\frac{5}{8}$	$1\frac{5}{8}$
$1\frac{3}{4}$	5	1.491	1.746	$2\frac{3}{4}$	$3\frac{3}{16}$	$3\frac{57}{64}$	$1\frac{3}{4}$
$1\frac{7}{8}$	5	1.616	2.051	$2\frac{15}{16}$	$3\frac{13}{32}$	$4\frac{5}{32}$	$1\frac{7}{8}$
2	$4\frac{1}{2}$	1.712	2.301	$3\frac{1}{8}$	$3\frac{5}{8}$	$4\frac{27}{64}$	2
$2\frac{1}{4}$	$4\frac{1}{2}$	1.962	3.023	$3\frac{1}{2}$	$4\frac{1}{64}$	$4\frac{61}{64}$	$2\frac{1}{4}$
$2\frac{1}{2}$	4	2.176	3.718	$3\frac{7}{8}$	$4\frac{1}{2}$	$5\frac{31}{64}$	$2\frac{1}{2}$
$2\frac{3}{4}$	4	2.426	4.622	$4\frac{1}{4}$	$4\frac{29}{32}$	6	$2\frac{3}{4}$
3	$3\frac{1}{2}$	2.629	5.428	$4\frac{5}{8}$	$5\frac{3}{8}$	$6\frac{17}{32}$	3
$3\frac{1}{4}$	$3\frac{1}{2}$	2.879	6.509	5	$5\frac{13}{16}$	$7\frac{1}{16}$	$3\frac{1}{4}$
$3\frac{1}{2}$	$3\frac{1}{4}$	3.100	7.547	$5\frac{3}{8}$	$6\frac{7}{64}$	$7\frac{39}{64}$	$3\frac{1}{2}$
$3\frac{3}{4}$	3	3.318	8.641	$5\frac{3}{4}$	$6\frac{21}{32}$	$8\frac{1}{8}$	$3\frac{3}{4}$
4	3	3.567	9.993	$6\frac{1}{8}$	$7\frac{3}{32}$	$8\frac{41}{64}$	4

by special order. As the maximum stress on the rafter, made of one piece of timber throughout, is at BK, it will be necessary to calculate its size at this point only. Assume that the truss supports heavy purlins only at the panel points, or joints, of the truss, common rafters being laid up and down the roof, resting on these purlins, as shown in Fig. 54. This concentrates all the load on the truss at the panel points, and BK is consequently not to be estimated as a beam, sustaining as it does only the compressive stress as obtained by the diagram.

The total compressive stress in BK is 39,000 pounds, and its length is 11.18 feet = 134 inches, nearly. Using an $8'' \times 8''$ yellow-pine timber, the ultimate compressive strength, parallel to the grain, as given in *Design of Beams*, is 5,000 pounds per square inch. By a formula given in *Design of Columns*, the ultimate compressive breaking strength of BK as a column is

$$S = U - \left(\frac{U \times L}{100\,D}\right) = 5{,}000 - \left(\frac{5{,}000 \times 134}{100 \times 8}\right)$$

$$= 5{,}000 - 838 = 4{,}162 \text{ pounds per square inch}$$

If, on account of its uncertain nature, a factor of safety of 5 is used for the timber, the safe bearing value of the column is then $4{,}162 \div 5 = 832$ pounds per square inch. The area of an $8'' \times 8''$ column is 64 square inches, which multiplied by 832 gives a safe load of 53,248 pounds. While considerably in excess of the 39,000 pounds required, this is the nearest even-sized timber that can be used for this member.

To determine the size of timber required for the tie-member, bear in mind that the greatest pull, or tensile stress, on this member is at KZ, and amounts to 37,300 pounds. This member, made of yellow-pine timber, must be spliced, say, at the center. It is safe to assume that, in making the connection and joints, one-third of this timber will be cut away. The ultimate tensile strength of yellow pine is given in *Design of Beams* as 12,000 pounds per square inch. This divided by the factor of safety of 10 equals 1,200 pounds, the safe stress that a square inch will sustain.

For convenience in construction, a $6'' \times 8''$ timber is selected. The area of this timber is 48 square inches; two-thirds of this is 32 square inches. Then, $32 \times 1{,}200 = 38{,}400$ pounds. A $6'' \times 8''$ timber is thus amply strong, but on drawing the truss and laying out the detail of its connections, it may be found necessary to use a timber 8 in. $\times$ 8 in. In determining the size of the member PO, bear in mind that the stress on this member is 11,900 pounds. The member PO in the frame diagram, Fig. 27, measures about 18 feet. The formula for wooden columns shows that this member may be made of a $6'' \times 8''$ timber, which is amply strong. A $4'' \times 8''$ timber would probably carry the load, but the length of the strut being more than 45 times the width of the least side, the next larger stock size of timber is adopted, namely, 6 in. $\times$ 8 in. The timbers, to facilitate making the connections, should all be of one thickness.

In this case, for instance, the face of all the members in the truss are flush, for all are of the same thickness. The other struts, KL and MN, will be found, in like manner, to be amply strong, if made of $4'' \times 8''$ timber. The tie-member is of such a length as to be made up of at least two pieces, spliced at the center. The shear of the wood parallel to the grain determines the strength of the splice, as explained in *Design of Beams*. Bolts may be used to hold the splice together, but should not be depended on to withstand any of the pull on the tie-member.

In estimating the strength of the tie-member at the heel, it may be difficult to obtain a sufficient section of wood at the end of the tie to resist the shear parallel to the grain on the line ab. In this instance, the pull on the member is 37,300 pounds. The area of the shearing section on the line ab is $8 \times 26 = 208$ square inches. According to *Design of Beams*, the ultimate shearing strength of yellow pine per square inch, parallel to the grain, is 600 pounds. If a factor of safety as low as 4 is used, the safe strength per square inch will be 150 pounds. Then, $208 \times 150 = 31{,}200$ pounds. Now, as previously stated, the stress is 37,300 pounds, and

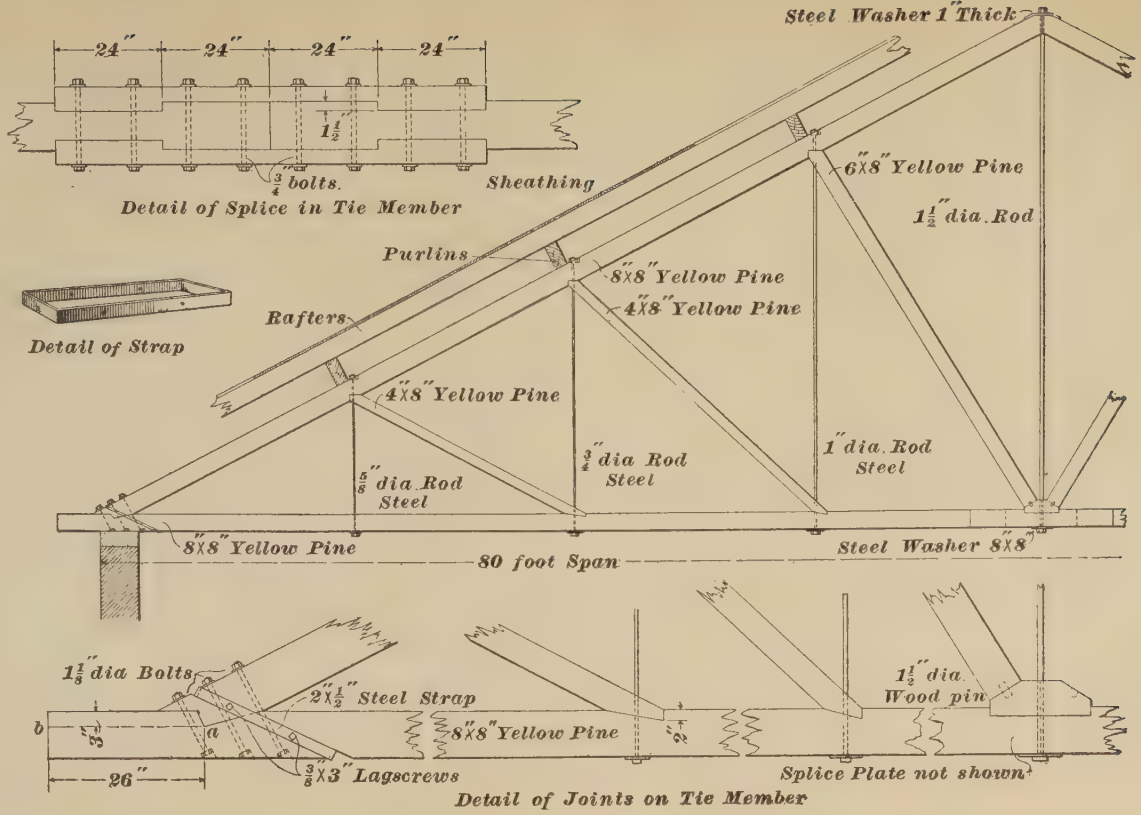

24″
24″
24″
24″
1½″
¾″ bolts.
Detail of Splice in Tie Member
Steel Washer 1″Thick
6″X8″ Yellow Pine
1½″ dia. Rod
Sheathing
Purlins
8″X8″ Yellow Pine
4″X8″ Yellow Pine
Rafters
Detail of Strap
4″X8″ Yellow Pine
⅝″ dia. Rod Steel
¾″ dia Rod Steel
1″ dia. Rod Steel
8″X8″ Yellow Pine
80 foot Span
Steel Washer 8″X8″
1⅛″ dia Bolts
2″X½″ Steel Strap
b
a
8″X8″ Yellow Pine
1⅛″ dia. Wood pin
26″
⅜″X3″ Lagscrews
Splice Plate not shown
Detail of Joints on Tie Member

FIG. 54

the remaining stress of 6,100 pounds will have to be taken up by securely bolting the joint and by strapping it, as shown in Fig. 54. It is seldom found that sufficient shear can be obtained in the wood to resist the stress at this point, it being always necessary to depend more or less on the bolts. In a large truss, like that under consideration, the best practice demands that the steel strap be so proportioned as to be capable of resisting all the stress not borne by the shear of the wood on the line *a b*, no reliance whatever being placed on the bolts. The lagscrews simply retain the straps in position. One notch in the tie, to receive the rafter, is

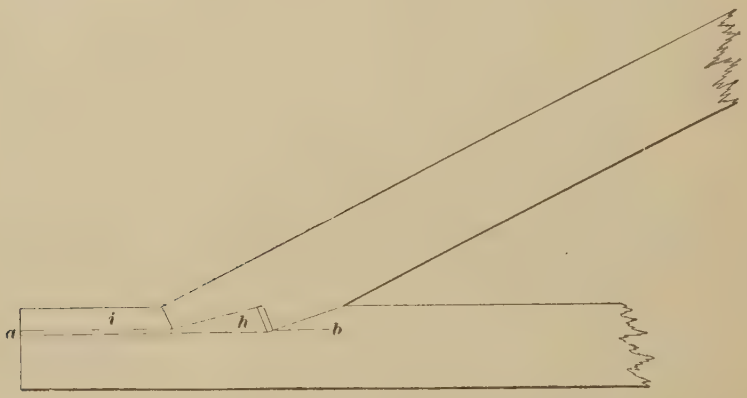


FIG. 55

always preferable to two, as shown in Fig. 55. Nothing is gained in strength by using two, because one of the joints will open from shrinkage, and consequently either the piece *h* or *i* will shear off before any stress is brought on the other.

Care should be taken that the washers are of sufficient area to prevent them from cutting into the wood. In *Design of Beams* will be found a table giving the allowable compression perpendicular to the grain that is used in determining the sizes of the washers.

The tension member $\frac{5}{8}$ inch in diameter, Fig. 54, is not needed to resist any stress, but is useful in the truss to hold the strut in position and prevent trouble from any shrinkage that may occur. No further details need be given in the

design of wooden roof trusses, good judgment and foresight on the part of the designer securing, with the principles and practice already laid down, the desired result, namely, maximum strength in the structure with a minimum amount of material.

THE DESIGN OF A COMPOSITE PIN-CONNECTED ROOF TRUSS

34. One of the forms most generally used for a composite pin-connected truss is the Fink truss, previously described. A truss of this pattern is usually made of wooden rafter members, structural-steel struts, and steel tension members. Where the wooden rafter member is oiled, or otherwise finished, and where the steel members are tastily painted, these trusses make a good appearance from the interior of the room or building that they span, and are adapted for supporting the covering, or roof, for such buildings as mess halls of barracks or asylums, armory drill halls, railroad stations, etc.

In order to illustrate the method of designing the members and joints in a truss of this character, we will now consider the design of the truss, shown in Fig. 34, the stresses in which were determined by the diagrams in Figs. 35 and 36, and tabulated in Art. **25.**

35. The **rafter member** has the greatest compressive stress on it between the points AB and BC, Fig. 34. This stress is represented in the table of stresses by the stress of 80,500 pounds in the member BK.

In addition to this compressive stress, there is a bending stress in the rafter due to the construction of the roof, which is composed of sheathing laid on joists spaced about 14 inches center to center along each rafter. The wind and vertical loads are, by this construction, transmitted to the panel points by the transverse strength of the rafter between these points; it will therefore be necessary, before proportioning the rafter member, to calculate the stresses due to the bending moment produced by the dead and wind loads.

The wind load acts normally to the rafter, but the dead or vertical load does not. If great accuracy were required, it might be advisable to resolve the vertical load so as to determine its component normal to the roof, and add this amount to the wind load to obtain the entire load normal to the rafter; such exactness, however, will not be necessary here, and the vertical load and wind load will be added together directly and considered as a uniformly distributed load on the rafter member between the panel points.

The sum of the wind load and the dead load for a panel is 5,625 + 4,500 = 10,125 pounds; therefore, the bending moment, according to the formula $M = \frac{WL}{8}$, is

$$M = \frac{10{,}125 \times 11.188}{8} = 14{,}160 \text{ foot-pounds}$$

or 14,160 × 12 = 169,920 inch-pounds.

Assuming that the rafter member is made of yellow pine, it will be seen by referring to *Design of Beams*, that the modulus of rupture is 7,000; hence, if a factor of safety of 6 is adopted, the safe working transverse stress in the material will be 7,000 ÷ 6 = 1,167 pounds. Since the required section modulus K may be obtained by dividing the bending moment M, in inch-pounds, by the safe unit stress S of the material, as expressed by the formula $K = \frac{M}{S}$, we have

$$K = \frac{169{,}920}{1{,}167} = 146$$

the section modulus required to resist the transverse stress on the rafter member.

Assume that a depth of 14 inches is adopted for the rafter member; then, by transposing the formula $K = \frac{b\,d^2}{6}$ to $b = \frac{K \times 6}{d^2}$, the breadth, or width, of the rafter member required to resist the transverse stress, by substituting the values of K and d, is

$$b = \frac{146 \times 6}{14^2} = 4\tfrac{1}{2} \text{ inches}$$

Thus, it is seen that a section of $4\frac{1}{2}'' \times 14''$ yellow pine is sufficient to take care of the transverse stress on the rafter member. But, in addition to this transverse stress, there is a direct compressive stress of 80,500 pounds, as shown by the stress diagram, which must be provided for in the same member, by adding material in the direction of its width.

From *Design of Beams*, the ultimate compressive strength of yellow pine parallel to the grain, is found to be 5,000 pounds per square inch, and as a factor of safety of 5 is used, the safe compressive strength will be $5{,}000 \div 5 = 1{,}000$ pounds per square inch.

Since the distance from joint to joint is not great, being only about 11 feet, the rafter member between the joints need not be considered as a long column, but it may be designed to resist direct compression and the full allowable compressive strength of the material parallel to the grain may be used in calculating the cross-section required. Therefore, $80{,}500 \div 1{,}000 = 80.5$ square inches is the sectional area of the material that must be added to the rafter member to resist the compressive stress. The depth of the rafter being 14 inches, and the sectional area required 80.5 square inches, the width to add to the rafter will be $80.5 \div 14 = 5\frac{3}{4}$ inches.

By combining, the total width of the timber required to resist the two stresses is $4\frac{1}{2} + 5\frac{3}{4} = 10\frac{1}{4}$, say 10 inches. The rafter member will therefore be made of one piece of $10'' \times 14''$ yellow-pine timber, which will extend from the heel to the apex of the truss.

36. The **tension rods** should be made of soft steel with eyes formed on the ends and provided with turnbuckles where required; their sectional area should be calculated to safely carry the loads indicated in the table of stresses.

It is well to make all the tension bars in the lower chord in pairs; the stresses on ZK, ZM, and ZQ, Fig. 34, will thus each be taken care of by two bars, one on each side of the rafter member and strut.

Since the stress in the pair of members ZK is 78,250 pounds, each of the two rods composing this part of the

truss is proportioned so as to sustain one-half of 78,250, or 39,125 pounds. We will assume that a quality of steel is used whose ultimate tensile strength is 56,000 pounds per square inch, and, owing to the reliability of the material, a factor of safety of 4 is sufficient in the steel tension members; the allowable tensile strength of the soft steel is, therefore, $56{,}000 \div 4 = 14{,}000$ pounds per square inch, and the required sectional area of one rod is $39{,}125 \div 14{,}000 = 3$ square inches. If square rods are used, as they will be in this case, a $1\frac{3}{4}'' \times 1\frac{3}{4}''$ rod will be found adequate, as it has a sectional area of $1.75 \times 1.75 = 3.06$ square inches.

The size of the other tension rods may be found in a similar manner, but as the stresses on LM and NO, Fig. 34, are light, these members may be made of a single tension bar; these bars and also QZ must be provided with turnbuckles, which will be required to tighten the whole structure and take care of any slack in the truss due to inaccuracy in construction. In designing the members provided with turnbuckles, care should be taken to see that the rods are upset on the ends, so as to realize a sectional area at the root of the screw thread equal to the required sectional area of the rod.

37. The **steel struts** may be proportioned by the formula for calculating the strength of structural-steel columns with one end hinged and one end fixed (see formula given in *Design of Columns*); all the values in the formula are known except the square of the radius of gyration, which may be obtained from the moment of inertia. Assume some convenient section that will be thought to have the required resistance. In this case, the section shown in Fig. 56 is assumed, it being convenient for making the various connections to the tension rods and wooden rafter member.

In order to calculate the least value of R^2, it will be necessary to calculate the least moment of inertia of the section, which will be done in accordance with the principles given in *Design of Columns*.

The properties of this section are given in the tables in *Design of Columns* and the moment of inertia of one of these

channels, with respect to the axis $a\,b$, is 13. Since the neutral axis $a\,b$ of the section passes through the center of gravity of the two channels, the moment of inertia of the section, with respect to this axis, is equal to the sum of the moment of inertia of the channels, with respect to the same axis, that is, to $13 \times 2 = 26$.

The moment of inertia I' of the channel, with respect to an axis through its center of gravity parallel to $c\,d$, is .7, the area of the section of the channel is 2.38 square inches, and the distance of its center of gravity from the back of the web is .52 inch. The distance of the axis through the center of gravity of the channel, from the neutral axis $c\,d$ of the column section is, therefore, $5\frac{1}{2} \div 2 + .52 = 3.27$ inches. By applying the formula given in *Design of Columns*, the moment of inertia of one of the channels, with respect to the axis $c\,d$, is

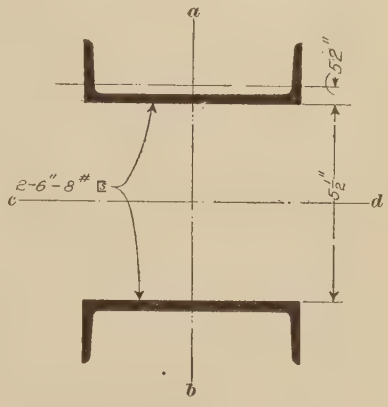


FIG. 56

$$I' = .7 + 2.38 \times 3.27^2 = 26.15$$

For the entire section, the moment of inertia, with respect to $c\,d$, is equal to the sum of the moments of its parts, that is, to $26.15 \times 2 = 52.3$.

From these calculations, it is evident that the least moment of inertia is on the axis $a\,b$, and is equal to 26. Then, by substituting the values of the least moment of inertia and the total area of the section in the formula in *Design of Columns*, the square of the least radius of gyration is

$$R^2 = \frac{26}{2 \times 2.38} = 5.46$$

According to *Design of Beams*, the ultimate compressive strength of structural steel is 64,000 pounds per square inch; the length of the longest column in the truss is 8 feet, or

96 inches. By substituting in the formula in *Design of Columns*, the ultimate compressive resistance of the section is

$$S = \frac{64{,}000}{1 + \left(\frac{64{,}000 \times 96^2}{10 \times \frac{3}{4} \times 29{,}000{,}000 \times 5.46}\right)} = 54{,}890 \text{ lb. per sq. in.}$$

Since the area of the section is 4.76 square inches, the ultimate resistance of the column will be $54{,}890 \times 4.76 = 261{,}276$ pounds; if a factor of safety of 4 is used in this column, its safe resistance will be $261{,}276 \div 4 = 65{,}319$ pounds.

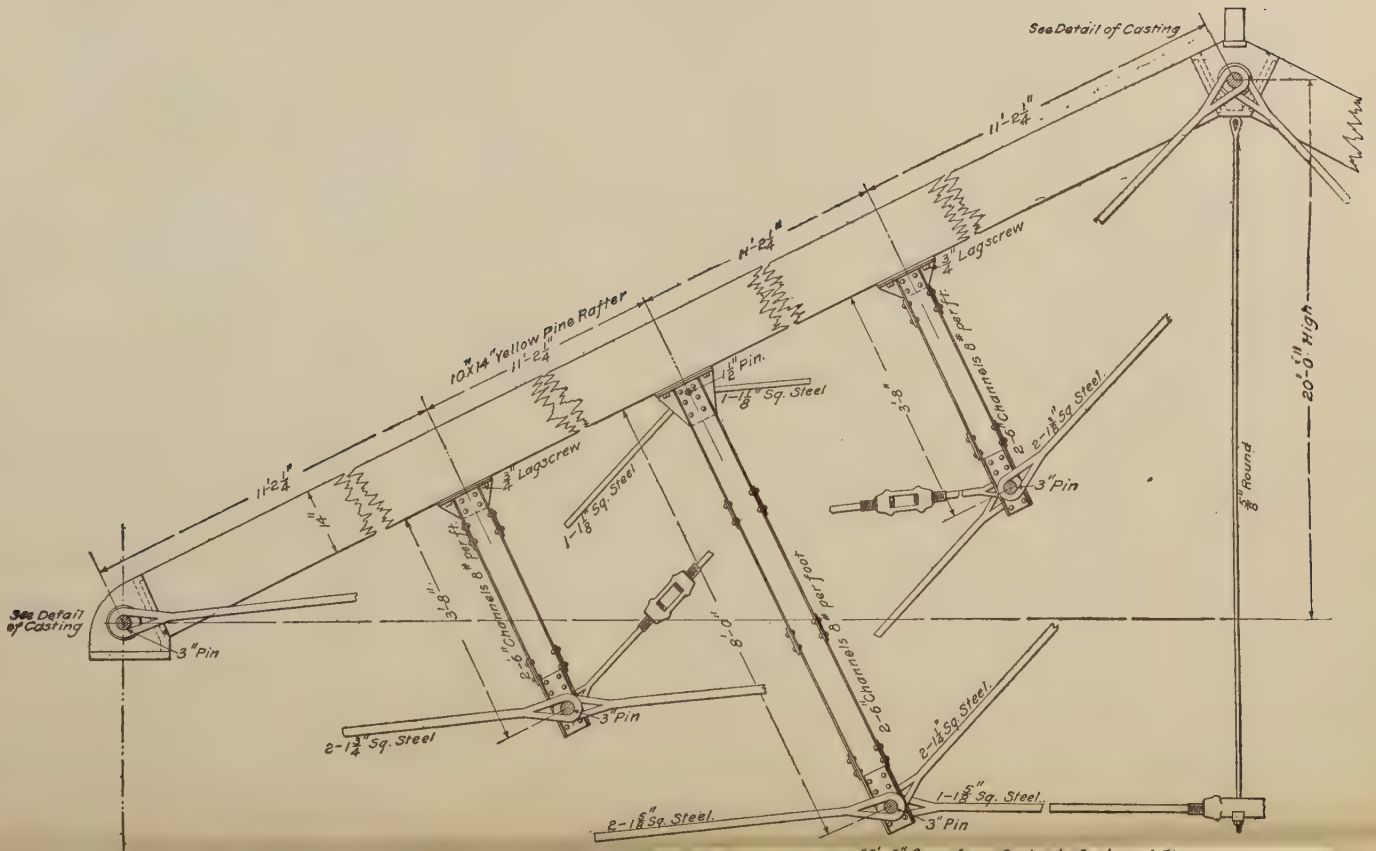
Since the stress in this long strut, or column, is only 19,250 pounds, it can readily be seen that it has several times the required strength. It is, however, deemed advisable to use this size of column, as the detailing where it joins the rafter member and also the pin connections at the lower end demand that channels of this size be used.

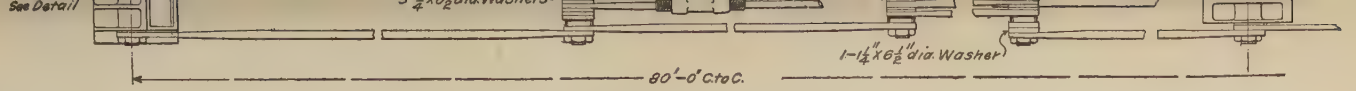
38. On account of the facilities in making the connections, and because, in a case of this kind, it is generally advisable to adopt the same rolled sections throughout, wherever possible, the short struts should be made of the same rolled sections as the long strut; this method saves labor in the shop and facilitates assembling and erection in the field.

39. The size of the pins is yet to be determined. This was so thoroughly treated in Arts. **30** to **32** that no further explanation will be required here. It is sufficient to say that on thorough examination of the several pinned joints, it was decided that a 3-inch pin would be sufficiently strong to resist any bending, shearing, or bearing stresses that would be applied to them.

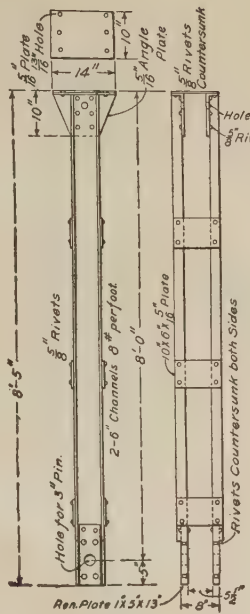
The correct design of the castings at the heel and apex of the truss is more a matter of experience and good judgment than of calculation.

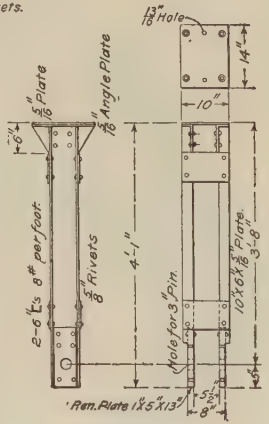
40. The truss details shown in Fig. 57 should be carefully studied. They have been designed according to the preceding diagrams and calculations. It is well to observe how all the connections are made and especially the details of the pin connections and the castings at the apex and heel of the truss.

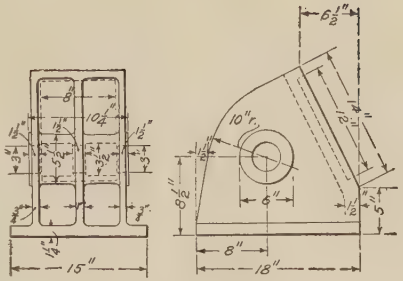

See Detail of Casting
11'-2 1/4"
11'-2 1/4"
10"X14" Yellow Pine Rafter
11'-2 1/4"
11'-2 1/4"
3/4" Lagscrew
3/4" Lagscrew
1 1/2" Pin.
1-1 1/8" Sq. Steel
1-1 1/8" Sq. Steel
2-6" Channels 8 # per ft.
2-6" Channels 8 # per ft.
2-1 1/8" Sq. Steel
3" Pin
3'-8"
1'-4"
3'-8"
See Detail of Casting
3" Pin
2-6" Channels 8 # per ft.
2-6" Channels 8 # per foot
8'-0"
3" Pin
2-1 3/4" Sq. Steel
2-1 1/8" Sq. Steel
1-1 1/8" Sq. Steel
2-1 1/8" Sq. Steel
3" Pin
3/4" Round
20'-0" High



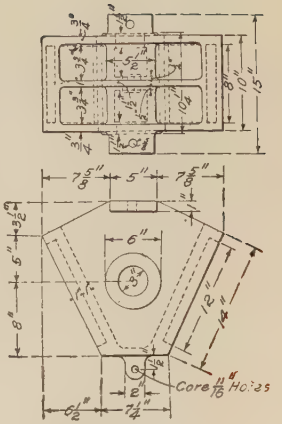
Plan of Tension Members







Detail of Foot Castings.



Detail of Peak Casting

FIG. 57

The light rod at the center of the truss has no stress on it, but is used simply to support the lower central tie-member and prevent it from sagging.

In designing compression members made up of two channels tied together with plates, as shown in Fig. 57, the ties should not be farther apart than sixteen times the width of the flange; for example, in this case, the width of the flange of the channels composing the column is about $1\frac{3}{4}$ inches, and sixteen times this width is 28 inches; hence, the distance between the two pieces or ties should not, in this column, be over 28 inches. An inch one way or the other, however, would make very little difference.

THE DESIGN OF A STRUCTURAL-STEEL ROOF TRUSS

41. The material most generally used in the construction of roof trusses for the support of the roof covering of modern buildings is structural steel. The rolled sections

FIG. 58

chiefly used in their construction are angles and plates, though any of the other steel sections may be adapted to special cases.

When a structural-steel roof truss is made of angles and plates, the angles are usually connected in pairs with the plate between them, as shown in Fig. 58. When this construction is adopted, the joints and connections of the several members may be made quite conveniently.

Assume that it is desired to construct the Fink roof truss, previously described and designed as a pin-connected truss, of structural steel. The stresses will be the same as given in the table, Art. **25**, and the general dimensions as given in Fig. 34.

The frame diagram may be redrawn and the stresses marked on the several members, as shown in Fig. 59.

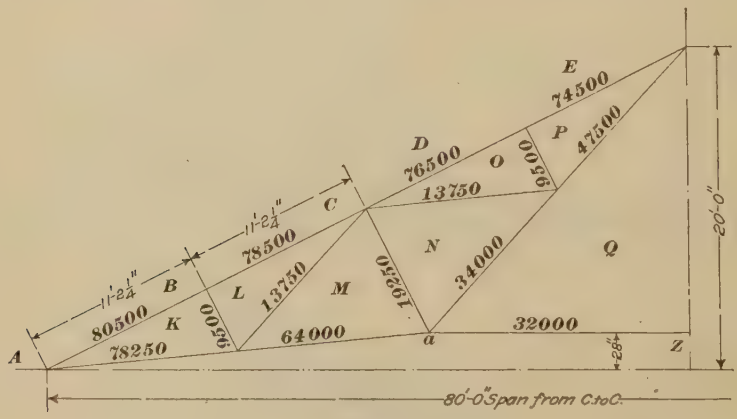


FIG. 59

42. The rafter member should be made in one length from the heel to the apex of the truss and proportioned to suit the greatest stress on it, which in this case is 80,500 pounds. Generally, in steel construction, the roof covering is supported on purlins placed at the panel points. There is, therefore, no bending stress in the rafter, and it is subjected to compression only.

In this case, the rafter is in compression only, and the portion between each panel point will be regarded as a column whose length is equal to the distance from center to center of the joints. Assume the size of a pair of angles that judgment and experience dictate as being adequate to

support the stress on the member; then by the short formula for structural-steel columns with fixed ends, determine the strength of the assumed section. If its strength, as found by the formula, is equal to, or slightly in excess of, the strength required to resist the stress in the member, the section may be adopted; provided, of course, that suitable connections can be made to it.

In this case it was decided to assume a section made of two $5'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angles, placed back to back with the long legs vertical, and about $\frac{5}{16}$ inch apart. The length of the column is 11 feet $2\frac{1}{4}$ inches, say 134 inches; and from *Design of Columns*, the radius of gyration of a section composed of two $5'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angles, with the long legs back to back and $\frac{1}{2}$ inch apart, is found to be 1.51, a value sufficiently exact for our purpose. Assume that the angles are of medium steel. Now, 134 is more than fifty times 1.51. The formula to use from the *Design of Columns* is, therefore, $S = 15{,}000 - 57\frac{L}{R}$.

Substituting the value for $\frac{L}{R}$, it is

$$15{,}000 - 57\frac{134}{1.51} = 15{,}000 - 5{,}058 = 9{,}942 \text{ pounds per square inch of section}$$

The area of a $5'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angle, according to the table given in *Design of Columns*, is 4 square inches; the area of the entire section is, therefore, $4 \times 2 = 8$ square inches, and the allowable strength of the member $9{,}942 \times 8 = 79{,}536$ pounds. Since the stress on the member is only 80,500 pounds, it is evident that the assumed section will approximately fulfil the requirements, and may be used for the rafter member.

43. The main strut, or the member MN, is the next compression member of any considerable size; its length is 8 feet 7 inches, or 103 inches; it is assumed that a section composed of two $3'' \times 2\frac{1}{2}'' \times \frac{1}{4}''$ angles, placed back to back at a distance apart of $\frac{5}{16}$ inch, will suit this position in the truss. If the two long legs are placed back to back, the least radius of gyration is .95 and is independent of the distance

apart that the angles are placed. This value of R is less than $\frac{1}{60} \times 103$. Using the same formula as before,

$$S_t = 15{,}000 - 57\frac{103}{.95} = 15{,}000 - 6{,}180 = 8{,}820 \text{ pounds per square inch of area}$$

Since the area of the two $3'' \times 2\frac{1}{2}'' \times \frac{1}{4}''$ angles is 2.88 square inches, the allowable strength of the section will be 8,820 $\times$ 2.88 = 25,402 pounds. Since the stress in the member is only 19,250 pounds, it is evident that this section will be sufficiently strong.

44. The stress on the short struts KL and OP is so small that any calculation of their required section would only result in obtaining a section composed of such small shapes that their use would not be practical in a truss of this size and character.

Here it is well to call attention to the practical principle that in selecting the section of a member where the stresses are light, care must be taken to choose such a section as will best fulfil the requirements of the construction, disregarding the fact that it will be stronger than is actually required to sustain the load. For example, it is considered good practice to make the leg of the angle held by a rivet not less in width than three times the diameter of the rivet; accordingly, a $\frac{3}{4}$-inch rivet should not be used in an angle leg whose width is less than three times $\frac{3}{4}$ inches, or $2\frac{1}{4}$ inches.

In the case of the truss under consideration, it was deemed advisable to use two $2\frac{1}{2}'' \times 2'' \times \frac{1}{4}''$ angles for each of the struts KL and OP.

45. The tension members in the truss are to be composed of rolled sections similar to those used in the struts; that is, two angles back to back and far enough apart to allow the $\frac{5}{16}$-inch **gusset plate**, forming the means of connection at the joints, to slip between them.

So far as practicable, tension members, in common with the struts, are made up of one continuous section; thus, irrespective of the fact that the stress in ZK is greater

than in ZM, these members should both be made of one continuous pair of angles; by so doing, the labor is minimized, thus effecting a saving that would more than offset the cost of the superfluous material in the member ZM, besides producing a much more pleasing appearance.

In the truss under consideration, ZK and ZM will be made of the same pair of angles; QN and QP will also be composed of a single pair of angles, care being taken to proportion the section to withstand the greater stress.

The stress in the member ZK is 78,250 pounds; the allowable tensile strength of structural steel, if the ultimate stress is taken at 60,000 pounds, and a factor of safety of 4 is used, will be $60{,}000 \div 4 = 15{,}000$ pounds per square inch. Then $78{,}250 \div 15{,}000 = 5.21$, the sectional area in square inches that will be required in the pair of angles forming the member ZK after deducting the sectional area cut out by one rivet hole in each angle.

From *Design of Columns*, a $3'' \times 3'' \times \frac{9}{16}''$ angle is seen to have a sectional area of 3.06 square inches; two angles will then have a sectional area of twice 3.06, or 6.12 square inches. Since the thickness of these angles is $\frac{9}{16}$ inch and a $\frac{3}{4}$-inch rivet is to be used, the area of the section to be deducted for one rivet hole is $.875 \times .5625 = .49$ square inch, and the net sectional area in the two angles is therefore

$$6.12 - 2 \times .49 = 5.14 \text{ square inches}$$

This is slightly under the sectional area demanded by the calculation, but, as a low ultimate tensile strength was assumed, it will be safe to use this section for the members ZK and ZM.

The other tension members in the truss may be proportioned in the same manner, care being taken to deduct from the section area of the member the sectional area removed for rivet holes; also, that the rolled sections adopted will satisfy the practical demands of the construction.

46. The number of rivets required at the several joints in the truss should be carefully calculated. Since the method of calculation is the same for all the joints, only one will

be considered here, though the designer should analyze the others for himself.

Fig. 60 represents a detail of the joint at *a*, Fig. 59. The stress in the member *MN* is 19,250 pounds, and a sufficient rivet section must be provided at the end of this member to resist this stress. By referring to Table III, and using an allowable stress per square inch of 15,000 pounds, the least value of a $\frac{3}{4}$-inch rivet in this connection is found to be the web bearing of a $\frac{5}{16}$-inch plate, which is 6,094 pounds; then 19,250 ÷ 6,094 = 3.15, or say 4, the number of rivets required in this connection.

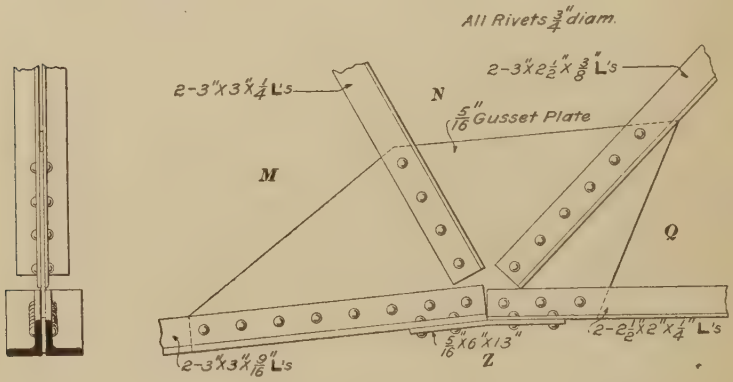
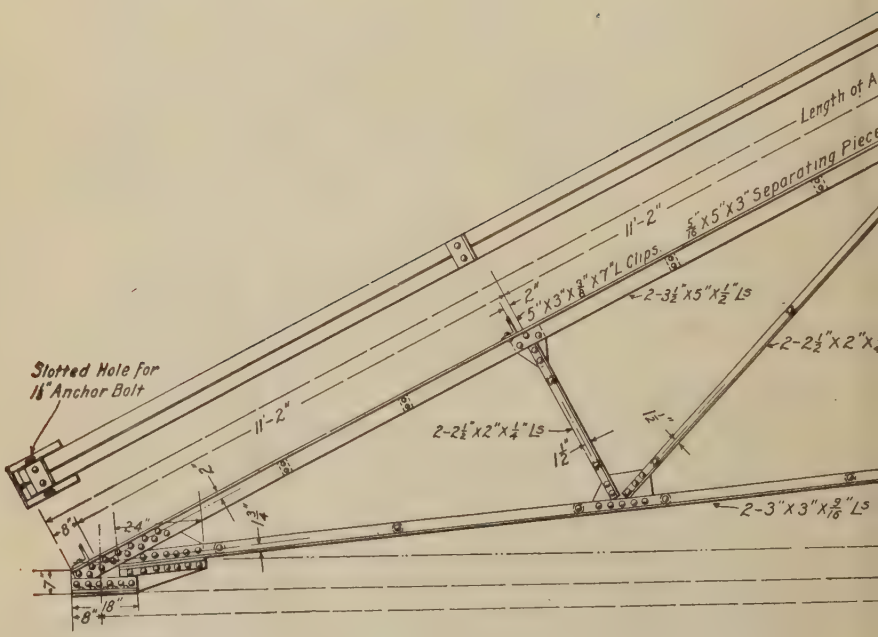


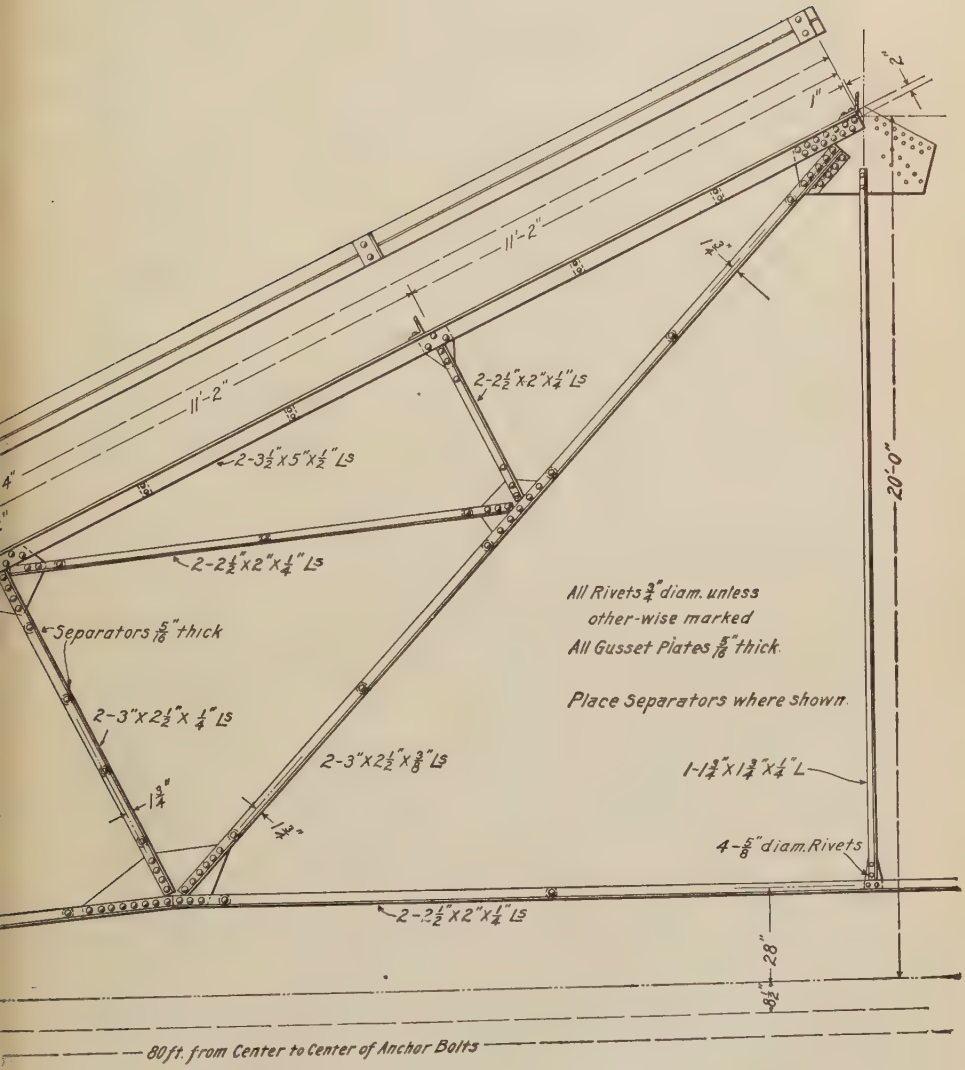
FIG. 60

The member *NQ* is subjected to a stress of 34,000 pounds, the least value for the rivets is the same as before, and the number required in the end of this member is 34,000 ÷ 6,094 = 5.56, say 6 rivets.

The stress in the member *ZM* is 64,000 pounds, which will require a large number of rivets. Using eight rivets in the vertical legs and gusset plate, as shown in Fig. 60, the value of each is 6,094 pounds and the value of eight is 6,094 × 8 = 48,752 pounds. This deducted from 64,000 pounds, the entire stress in the member, leaves 15,248 pounds yet to be provided for by the use of a splice plate attached to the horizontal legs of the angles. The value of one rivet in the horizontal legs of the angles forming the member *QZ* is the ordinary bearing value of a $\frac{3}{4}$-inch rivet in a $\frac{1}{4}$-inch


Slotted Hole for
1¼" Anchor Bolt
11'-2"
11'-2"
2"
5"x3"x⅜"x7" L Clips.
5/16"x5"x3" Separating Piece
2-3½"x5"x½" Ls
2-2½"x2"x¼" Ls
2-3"x3"x5/16" Ls
8"
24"
18"
8"

156-I L T 11D § 9


11'-2"
11'-2"
2"
1"
20'-0"
2-2½"x2"x¼" Ls
2-3½"x5"x½" Ls
2-2½"x2"x¼" Ls
Separators 5/16" thick
All Rivets ¾" diam. unless other-wise marked
All Gusset Plates 5/16" thick.
Place Separators where shown.
2-3"x2½"x¼" Ls
2-3"x2½"x⅜" Ls
1-1½"x1¾"x¼" L
4-⅝" diam. Rivets
2-2½"x2"x¼" Ls
28"
80 ft. from Center to Center of Anchor Bolts

plate, which, according to Table III, is 3,656 pounds, with an allowable stress of 15,000 pounds. The value of four rivets will then be $3{,}656 \times 4 = 14{,}624$ pounds, a little less than the amount to be taken care of, but the difference is so small that it may be disregarded and for the sake of symmetry the same number of rivets will be used in connecting the splice plate to the member MZ. The number of rivets required to connect the member ZQ to the gusset plate may be readily obtained.

47. Fig. 61 shows the usual shop drawing of the truss just designed. Particular attention should be paid to the details of the connections. **Separators** should be placed in both the tension and the compression members; they are placed in the tension members to prevent the angles from striking against each other when the trusses are subjected to vibrations, and also to join the two angles of a member, so that the work will arrive at a point of erection in a convenient form, ready to be put together. The separators are placed in the compression members so as to insure against any tendency to bend them apart, so that they will act in unison. The spacing of these separators is more a matter of judgment on the part of the designer than anything else, though any spacing over eight times the least dimension of the member is not to be recommended. Separators should always be placed near the end of a pair of angles to be connected to a gusset plate, as it will hold them the right distance apart in shipment and facilitate erection in the field. The $1\frac{3}{4}'' \times 1\frac{3}{4}'' \times \frac{1}{4}''$ angle, joining the apex of the truss and its lower chord, is required only to support the chord angles and prevent them from sagging. When there is considerable stress in this member, sagging is unlikely to occur, but it is the usual practice and a good one, to introduce some support for this long and usually light member. It will be noticed that one size of rivets is used throughout this truss wherever practicable; this is a point in economical construction that should always be observed. The ends of the angles and members, when practicable, should be cut off square,

and the gusset plates should be designed so that they may be formed with as few cuts as possible, and unless it is desired to make the truss somewhat fanciful, these cuts should never be other than straight lines.

In designing roof trusses, and, in fact, all structural-steel constructions, care should be taken to see that the several sections into which each may be divided are not so large that they cannot be shipped by railroad or other transportation at hand; this is important and should be carefully considered.

GENERAL NOTES REGARDING THE DESIGN OF A ROOF TRUSS

48. Lateral Bracing.—Trusses forming the principal support of a roof, if of any considerable size, should be braced together in the planes of the rafters so as to secure them against any tendency of the wind, when blowing in a direction perpendicular to the gable ends, to produce lateral movement. If the roof sheathing is laid close and is well nailed, it will sufficiently stiffen trusses of moderate span. The heels of trusses are sometimes fastened securely to the walls, especially in those buildings where the wind is liable to get under the roof. When so secured there is a tendency for the wind to reverse the stresses in the members of a roof provided with a light covering, and this reversal should be taken care of in the design of the truss.

49. Factors of Safety.—Since due allowance must be made for unforeseen and unknown defects of material and workmanship, and for unknown stresses that are liable to occur, it is necessary to proportion the several parts of a structure so that they will be able to resist, without failure, much larger forces than those obtained from the stress diagram.

In roof trusses, however, the stresses can be calculated with more certainty than in the case of a bridge or a machine, and the application of these stresses is more steady in its nature, and, therefore, not so severe on the material. For this reason, it is permissible to allow unit stresses, in the

design of roof trusses, some 50 per cent. in excess of those considered allowable in first-class bridges.

50. Tension Members.—The strength of a tension member is that of the smallest cross-section. It is therefore good practice to *upset*, or enlarge, the ends of long bolts or tension rods with screw ends, so that the cross-section at the root of the thread will be at least equal to the sectional area of the main part of the rod.

The central axis of the cross-section of ties and struts should coincide with the line of action of the thrust or pull, as otherwise the piece will be greatly weakened and dangerous bending moments will be developed in the structure. To calculate the net sectional area required in any tension member, the force or load on it should be divided by the safe working tensile strength of the material, and allowance must be made in the area of the cross-section for the cutting of bolt and rivet holes.

51. Compression members whose lengths do not exceed six times the least dimensions may be proportioned by the method followed for the tie as just explained, but, when the length of the truss is increased, there is a tendency to yield sidewise when compressed, and the sectional area must be increased or the unit stress diminished. Hence, the column formulas given in *Design of Columns* must be used. Pieces subjected to alternate compression and tension should have a materially larger section than would be required for either stress alone. Cast iron is seldom used in the best work for anything but short compression pieces, packing blocks, and pedestals.

52. Members in Trusses Subjected to Transverse Stresses.—In determining the resisting moment of a member subjected to transverse stresses, or in calculating the section required at the point of maximum bending moment, due allowance must be made for portions cut away on the tension side in attaching fastenings, or in making connections; similar allowance must also be made on the compression side, unless the holes are completely filled by the rivets,

in which case no deduction need be made from the sectional area of the member.

53. Members in Trusses Subjected to Both Transverse and Direct Stresses.—The rafter members in a roof truss, likewise members in other structures, are often called on to resist both a bending and a direct stress. Such pieces must first be designed to safely resist the bending moment, and then their transverse dimensions must be increased so that the added material will have sectional area sufficient to resist the direct pull or thrust. Should the direct force be a compressive stress, it will be well, after the member is designed, in order to check up its strength, to test by the proper column formula.

54. Pins and Eyes.—In proportioning the pins and eyes of tension bars, the diameter of the pin should be from three-fourths to four-fifths of the width of the bar in flats, and one and one-fourth times the diameter of the bar in rounds. The sectional area of the metal around the eye should be 50 per cent. in excess of that of the rod or bar. When flat bars are used, their thickness should be not less than one-fourth of their width; this will secure a good bearing surface on the pin. The size of a pin is usually decided by the bending moment on it, consequently the assembled pieces on a pin should be packed close together, and opposing members should be brought as nearly in line with one another as possible.

55. Details.—The designer should carefully examine each joint and connection in the structure, and consider such practical points as means of shipment, erection in the field, etc. Care should be taken in designing all connections, to so place the rivets and bolts as to realize their full strength, and at the same time not cut away too much of the material of the members connected. All members and joints should be examined for tension, compression, shearing, and bending, and proportioned accordingly. The strength of the joints and connections between the members of a structure are of as much importance as the strength of the members

themselves; the strength of any structure depends on the strength of its weakest point, and its failure at a joint or connection is as fatal as the failure of any of its members.

The designer will find the handbooks issued by the various steel mills of great value in the prosecution of his work. They contain many useful tables giving the properties of rolled sections, with information as to their use and application.

DESIGN OF PLATE GIRDERS

CONSTRUCTION AND DESIGN

GENERAL CONSTRUCTION

1. A plate girder is a beam built up of a number of plates and angles securely riveted together.

The names given to the different parts of a plate girder may be understood by referring to Fig. 1, in which *a* is the **flange plate,** of which there may be one or more on each flange, depending on the strength required. Flange plates are the principal elements for resisting the bending stresses in the girder. The **flange angles** *b*, *b* are the means of connecting the flange plates and the **web-plate** *c*. When the load on the girder is small, the flange plates may be omitted, in which case the flange angles are the members that chiefly act to resist the bending stresses.

FIG. 1

2. Stiffness.—On account of the construction of a plate girder, there is very little stiffness in the web-plate; consequently, there is always a strong tendency for it to fail by buckling and twisting under the load imposed on the girder. This tendency to buckle is greatest at the supports or abutments of the girder and at points where concentrated loads are applied. Because of this

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buckling tendency, it becomes necessary to reinforce the girder by riveting at stated intervals **stiffeners** generally made of angles.

The most common and cheapest form of stiffener is shown in Fig. 2 (*a*). This is simply a straight piece of angle riveted to the web-plate and to the flange angles. The space between the stiffeners and web-plate, due to the thickness of the flange angles, is filled with a piece of bar iron or plate, as shown at *d*; this is called a **filler**, or **packing piece.**

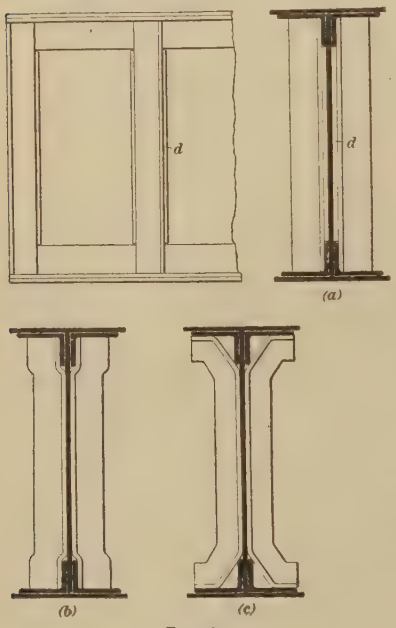


FIG. 2

A more expensive form of stiffener for plate girders is shown in Fig. 2 (*b*). The angle is swaged out, to allow it to fit over the flange angles, and is riveted directly to the web-plate, thus doing away with the filler, or packing piece. This construction does not require so much material as the filler shown at (*a*), but, where only a small number of girders of the same dimensions is to be built, the extra amount of labor required to make the fillers will increase the cost considerably. However, where a large number of girders of the same dimensions is to be built, dies for swaging the ends, in connection with a power or hydraulic press, may be used, thereby reducing the cost.

The stiffeners shown at (*c*) are sometimes used, but are subject to the same general criticism in regard to cost of manufacture as those shown at (*b*). Stiffeners of this shape possess a possible advantage in the fact that they stiffen the flanges considerably more than either of the other styles.

3. Usual Forms of Sections.—The four principal sections used in plate-girder construction are shown in Fig. 3. A simple plate girder with a web-plate and two flange angles, but with no flange plate, is shown at (*a*); this section is used for short spans or light loads. At (*b*) is shown a similar girder with one flange plate; this girder is used to support heavier loads and to clear longer spans than the girder shown at (*a*). The girder at (*c*), which may have two or more flange plates at each flange, may, if the conditions require, be made as heavy as is necessary in order to carry great loads over long spans. In fact, the strength of a girder of this character may be increased almost indefinitely by adding flange plates. The section (*d*) is a plate girder of

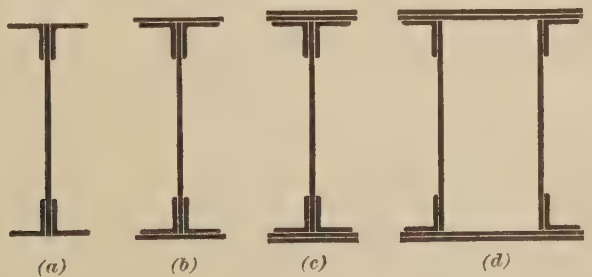


FIG. 3

box section; it is stiffer laterally than the forms shown at (*a*), (*b*), and (*c*), but the difficulty of reaching the interior for painting and inspecting, and the excessive amount of labor required in its construction, are such serious objections that it is much less used than the other three sections. On account of their open construction, the girders shown at (*a*), (*b*), and (*c*) are especially good forms to use in buildings where the objection in regard to lateral stiffness does not hold good, as when the girder is used in the position in which it is usually found, being generally prevented from deflecting laterally by the floorbeams; any lack of stiffness in comparison with the box girder is more than compensated by the simplicity of construction and easy access on all sides for painting and inspecting.

PRINCIPLES OF DESIGN

STRESSES

4. The external forces, loads, and reactions produce the same kind of stresses in a plate girder as in an ordinary beam, but, on account of its special construction, the distribution of these stresses in the girder is assumed to be somewhat different from that in a beam made of a single piece. In the girder, the shear is generally assumed to be borne wholly by the web-plate, while the bending moment is assumed to be resisted by the stresses in the flange members. The method of calculating the magnitude of the shear and bending moment is the same as that for beams, already discussed in *Design of Beams;* owing, however, to the different assumption in regard to the distribution of these forces, a different method of calculation is used in determining the relations between them and the stresses in the girder.

5. Shearing Stresses in Web-Plate.—In discussing the methods of calculating the dimensions of a plate girder for a given purpose, it will be necessary to first consider the shear, which is the principal factor that determines the thickness of the web-plate and the number and size of stiffeners required. In *Design of Beams*, it was shown that the greatest shear in a beam occurs at the point of support at which the reaction is greatest, and that the magnitude of the shear is equal to the reaction at that point; consequently, in a simple plate girder, the greatest shear occurs at a point of support, and is equal in amount to the reaction at that point.

WEB-PLATES AND STIFFENERS

6. Depth of Girder.—Having calculated the shear, the depth of the girder is assumed in accordance with practical rules that fix the relation between the depth and the span. According to the best practice, the depth should not be less than one-fifteenth of the span, though some authorities consider one-twentieth as being ample. The latter proportion,

however, gives an exceedingly shallow girder, and cannot be recommended except where the loads are very light and the span short, or where it is absolutely necessary that an extremely shallow girder be used, on account of decorative features, or lack of space in regard to headroom, in which case the girder should be so proportioned that when fully loaded its deflection will not be excessive.

7. Thickness of Web-Plate.—Knowing the depth of the girder and the shear at the points of support, the thickness of the web-plate is proportioned so as to give it sufficient area to resist the maximum shear. It is always necessary to stiffen the plates over the supports, as shown in Fig. 5. The stiffeners are riveted to the plate and transfer the shearing stress from it to the supports.

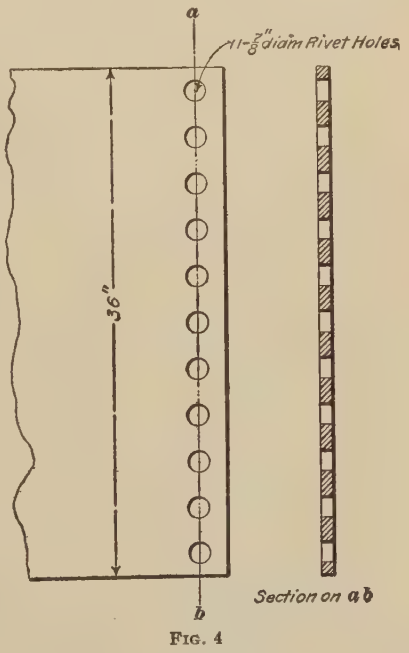


FIG. 4

A considerable portion of the plate is cut away by the holes for the rivets by which it is fastened to the stiffeners; hence, the least strength of the plate is along the line of the rivet holes. It can readily be seen, by referring to Fig. 4, which shows the end of a plate with the holes punched for riveting to the stiffener, that the net or efficient depth of the plate is equal to the actual depth minus the sum of the diameters of the rivet holes.

The following rule may be used for calculating the thickness of a web-plate so that it will have sufficient strength to resist the shearing stress:

Rule.—*From the total depth of the web-plate, deduct the sum of the diameters of the rivet holes, which will give the net or efficient depth of the web-plate; multiply the net depth by the safe resistance of the material to shear, and divide the maximum shear, in pounds, by the product; the quotient will be the required thickness of the metal in the web of the girder.*

This rule may be expressed by the formula

$$T = \frac{R}{D \times S}$$

in which T = thickness of web-plate;

R = greatest reaction or maximum shear;

S = safe shearing resistance of material per square inch;

D = net depth of the web-plate after all rivet holes have been deducted.

The safe resistance of the material to shear is, of course, governed by the factor of safety required in the girder. For example, the ultimate shearing strength of structural steel being about 50,000 pounds per square inch, if a factor of safety of 4 is required, the safe resistance of the metal will be 50,000 ÷ 4 = 12,500 pounds, while if a factor of safety of 5 is desired, the safe strength will be 50,000 ÷ 5 = 10,000 pounds.

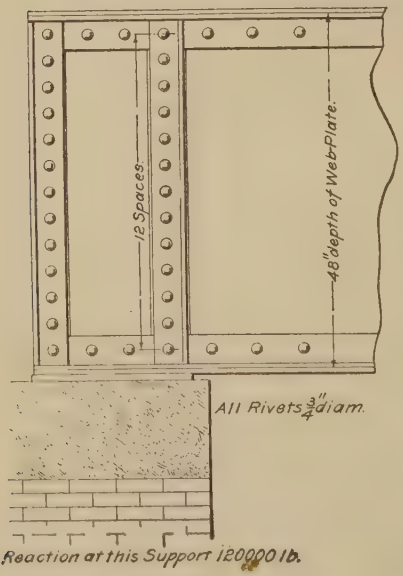


FIG. 5

In deducting the metal for the rivet holes, in order to ascertain the net depth of the web-plate, the holes should always be considered as being $\frac{1}{8}$ inch larger than the nominal diameter of the rivet; this allowance is made because

the holes are always made $\frac{1}{16}$ inch larger in diameter than the rivet so that the rivet may be inserted easily, and another $\frac{1}{16}$ inch should also be allowed in the diameter of the hole to compensate for any injury that the metal immediately around it may suffer from the punch.

It will often be found that the calculated thickness of the web-plate is less than is allowable for practical reasons. The thinnest plate that should be used for any case is $\frac{5}{16}$ inch.

EXAMPLE.—Fig. 5 shows the end of a plate girder in which the greatest reaction is 120,000 pounds. The girder is made of structural steel, the safe fiber stress of which is assumed to be 11,000 pounds per square inch for shear. What should be the thickness of the web-plate?

SOLUTION.—The width of the plate is 48 in., and there are 13 holes punched for $\frac{3}{4}$-in. rivets. The deduction to be made for each rivet hole is $\frac{1}{8}$ in. $+ \frac{3}{4}$ in. $= \frac{7}{8}$ in.; therefore, the net depth of the plate is

$$48 - 13 \times \tfrac{7}{8} = 48 - 11\tfrac{3}{8} = 36\tfrac{5}{8} \text{ in.}$$

Applying the formula, the thickness of the plate is

$$T = \frac{120{,}000}{36\frac{5}{8} \times 11{,}000} = .297 \text{ in.}$$

In no case, however, should the web-plate of a girder be less than $\frac{5}{16}$ in. in thickness. Hence, as .297 is less than $\frac{5}{16}$, the thickness of the web-plate in this girder should be $\frac{5}{16}$ in. Ans.

8. Buckling of Web-Plate and Distribution of Stiffeners.—The shearing stresses in a web-plate, in addition to their tendency to shear the plate, as just described, are likely to cause it to fail by buckling; therefore, in order to properly resist the vertical shearing stresses and prevent them from buckling the web-plate before its full shearing strength is realized, it is necessary to provide the stiffeners, as has been stated.

It may be seen in *Design of Beams* that the shearing stresses in a simple beam are always greatest at the points of support, and diminish toward the center of the beam until a point is reached between which and the support the sum of the loads is equal to the reaction; at such a point, the shear is said to change sign. The natural inference from this fact is that the stiffeners should be more numerous at the points of greatest vertical shear, decreasing in number as the shear

decreases. Theoretically, this would be a correct method of locating the stiffeners, but practically they are spaced at equal distances along the length of the girder, except at the points of support, where several are placed near one another in order to give the end of the girder more nearly the character of a column and enable it to successfully resist the great vertical shear, due to the reaction at this point. In no case should the stiffeners at the end of a plate girder be omitted, even if the conditions make the intermediate ones unnecessary.

It is also good practice to place stiffeners directly under any concentrated load that may be placed on the girder. These stiffeners are necessary not only to stiffen the web-plate at the point of application of the load, and thus prevent buckling, but also, through the medium of the rivets, to assist in distributing the load on the web-plate and other members of the girder.

The end stiffeners of a plate girder may be considered as columns subjected to a compressive stress equal to the reaction, and calculated by the rules and formulas for columns. For safety, the stress on the end stiffeners should never exceed 15,000 pounds per square inch of section.

Practice in regard to the placing of stiffeners on plate girders varies considerably, being more a matter of judgment and experience than of calculation. Some engineers determine the resistance of the web-plate to buckling by the formula

$$B = \frac{11{,}000}{1 + \frac{d^2}{3{,}000\, t^2}}$$

in which B = safe resistance of web to buckling, in pounds per square inch;

d = depth of web-plate, in inches;

t = thickness of web-plate, in inches.

If the value of B given by this formula is less than the unit shearing stress, the girder should be stiffened.

Example.—The allowable shearing stress on the web of a plate girder is 11,000 pounds per square inch. The stiffeners at the end supports are riveted by nine $\frac{7}{8}$-inch rivets to the web-plate, which is

36 inches wide. The end reaction on the girder is 100,000 pounds. (*a*) What should be the thickness of the web-plate? (*b*) Will it be sufficiently strong without the addition of stiffeners?

SOLUTION.—(*a*) Since $\frac{7}{8}$-in. rivets are used, the allowance to be made for one rivet hole is $\frac{7}{8} + \frac{1}{8} = 1$ in., and the effective width of the plate along the line of rivets is $36 - 9 \times 1 = 27$ in. Applying the formula of Art. **7**, the thickness of the plate is found to be

$$T = \frac{100{,}000}{27 \times 11{,}000} = .33 \text{ in.}$$

The thickness of the nearest standard-size plate above this is $\frac{3}{8}$ in., which will be the thickness used. Ans.

(*b*) By the formula, the safe unit resistance of the plate to buckling is

$$B = \frac{11{,}000}{1 + \frac{32^2}{3{,}000 \times \frac{3^2}{8}}} = 2{,}700 \text{ lb. per sq. in.}$$

which, since it is much less than the unit shearing stress on the web-plate, shows that stiffeners are required. Ans.

9. Practical Rule for Spacing Stiffeners.—It is not the general practice to make the foregoing calculations to determine whether stiffeners are required; according to the best engineering practice, stiffeners should be provided, unless the thickness of the web-plate is at least one-fiftieth of the clear distance between the vertical legs of the flange angles.

EXAMPLE.—Assume the girder in the previous example to be provided with $6'' \times 6''$ flange angles; the depth and thickness of the plate, as previously shown, are 36 inches and $\frac{3}{8}$ inch, respectively. According to the foregoing statement, does this girder require stiffeners?

SOLUTION.—The unsupported depth of the plate between the flange angles is $36 - 2 \times 6 = 24$ in.; $\frac{1}{50}$ of 24 in. = .48, say, $\frac{1}{2}$ in. As the thickness of the web-plate is only $\frac{3}{8}$ in., the girder must be provided with stiffeners. Ans.

The spacing and size of stiffeners to be used on a plate girder is almost entirely a matter of experience and judgment. As a general rule, it may be said that stiffeners should be provided at the ends of all plate girders over the supports or abutments, and they should be so proportioned that they will take care of the entire reaction at these points. They should also be placed under large concentrated loads. The stiffeners between the abutments or supports should be of such a size that they will best suit the general

requirements of the design of the girder. The practice in spacing intermediate stiffeners is to make the distance between their center lines equal to the depth of the girder, thus dividing the girder into equal square panels provided there are no large concentrated loads. Under no conditions, however, should stiffeners be placed more than 5 feet apart from center to center of line of rivets.

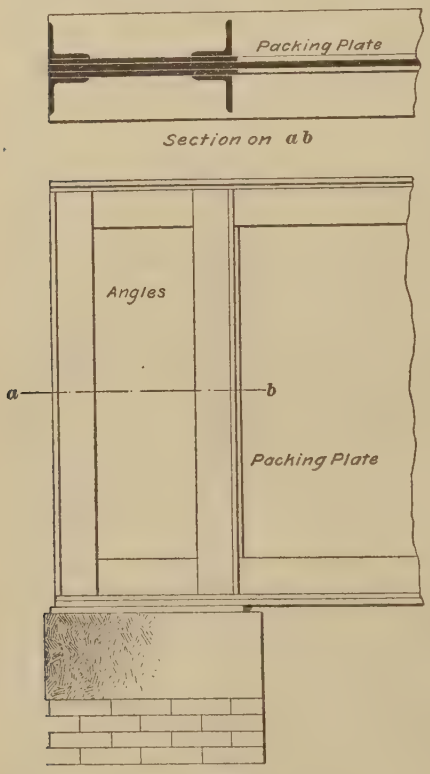


FIG. 6

Having proportioned the stiffeners at the abutments to take the entire reaction, it is good practice, when possible, to make the intermediate stiffeners the same size as the end ones. In general, the angles used for stiffeners should not be less than 3 in. $\times$ 3 in. $\times \frac{5}{16}$ in., though on shallow girders, with extremely light loads, it might be economical to use angles as light as $2\frac{1}{2}$ in. $\times$ $2\frac{1}{2}$ in. $\times \frac{5}{16}$ in. Sizes smaller than this should certainly never be used as stiffeners.

Stiffeners should always extend over the vertical legs of the flange angles; they should always be either swaged out to fit over the flange angles or be provided with a filling piece as illustrated in Fig. 2.

EXAMPLE.—The end reaction on a plate girder is 300,000 pounds. If a compressive fiber stress of 13,000 pounds per square inch is allowed on the stiffeners, and four stiffeners are used, as shown in Fig. 6, what should be the size of the angles?

SOLUTION.—The reaction or greatest shear being 300,000 lb., and the allowable stress 13,000 lb. per sq. in., the area of stiffeners required must be $300{,}000 \div 13{,}000 = 23$ sq. in.; this sectional area, divided among four angles, gives $23 \div 4 = 5.75$ sq. in. as the area required for each angle.

By referring to the table of angles with even legs, in *Design of Columns*, it is seen that a $4'' \times 4'' \times \frac{13}{16}''$ angle has a sectional area of 5.84 square inches; and, in the table of areas of the uneven angles, a $5'' \times 3'' \times \frac{13}{16}''$ is also shown to have an area of 5.84 sq. in.; therefore, either of these angles may be used. Ans.

FLANGES

10. The flanges of a riveted girder include all the metal at the top and bottom of the girder, and are sometimes called the *top chords* and *bottom chords*, though the term is more frequently applied to lattice, or open, girders, such as are more often used for railroad and highway bridges.

In building construction, it is customary to include in the flange the two flange angles, the flange plates, and one-sixth of the web-plate included between the flange angles. The building ordinances of some of the large cities of the United States, however, disregard this last item, and will not allow any portion of the web-plate to be included as part of the flange. In this Section, the web-plate will not be considered in calculating the flange area.

If, on account of economic considerations, one-sixth of the web-plate must be included as part of the flange, it must be remembered that the plate should never be spliced near the center, when the girder is uniformly or symmetrically loaded, or directly under the point of greatest bending moment, when the load on the girder is unsymmetrically placed. Especial care must also be taken to insure that any splice made on the length of such a web-plate is so designed as to furnish the greatest possible percentage of strength of the solid plate included within one-sixth of the depth of the web.

11. Flange Stresses.—In a simple girder, the top flange is subjected to compression and the bottom flange to tension. Nevertheless, it is customary in practice to make

the two flanges equal in area and of the same size rolled plates and angles.

In proportioning the flanges of a plate girder, the lower flange is calculated for tension; the areas of the rivet holes cut out of the flanges are deducted from the total area, so as to give the net or actual area of the flange at the point of least strength.

The stresses in the flanges are assumed to be produced wholly by the bending moment on the girder, and the moments of these stresses are assumed to be equal to the moments of the external forces.

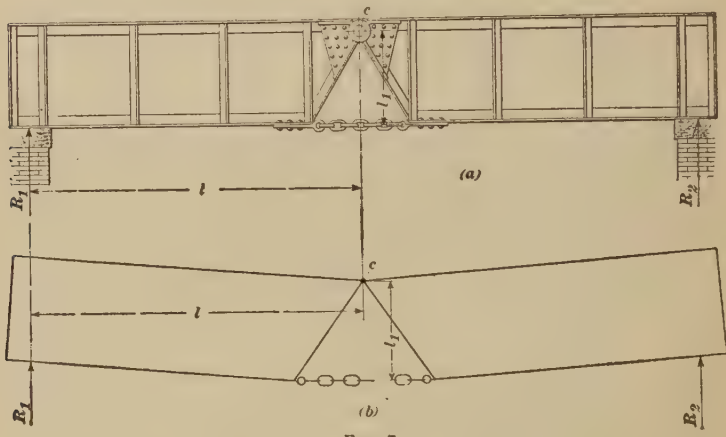


FIG. 7

The principles on which the flange stresses of a plate girder are calculated will be made clear by referring to Fig. 7, which shows a girder in two sections joined by a hinge pin c at the upper flange, and a chain at the lower flange. The resultant moment of the loads and reactions tends to produce rotation about the center c, which, however, for convenience of illustration, is taken at a point in the upper flange instead of on the neutral axis, as was done in the case of the beam composed of a single section. The stress in the chain, which represents the lower chord or flange of the girder, resists this tendency to rotate about the center c, with a lever arm l_1, which is the perpendicular distance

from the chain to the point *c*. It is evident, then, that the strength of the girder depends on two factors, the tensile strength of the lower chord, and its distance from the center of the hinge *c*, the latter of which represents the depth of the girder. The compressive stress in the top flange of the girder must be equal to the tensile stress in the bottom flange, for if not, each section of the beam would tend to move sidewise.

12. Proportioning the Flanges.—Having determined the principles on which the bending strength of a plate girder is calculated, it remains to show a method for proportioning the metal in the flanges. The usual process is as follows:

First, calculate the maximum bending moment on the girder; this may be done by the principles and rules given in *Design of Beams*. In calculating the bending moment on a plate girder, however, it is customary to express the moment in foot-pounds, the depth of a girder being generally given in feet and not in inches, as in solid beams of shallow depth. If, however, the depth of the girder is expressed in inches, the bending moment must be calculated in inch-pounds.

Having found the maximum bending moment on the girder, it is necessary to assume an allowable fiber stress for the material of which the flanges are composed. The following rule may then be used to calculate the sectional area of either flange:

Rule.—*To find the sectional area of a flange, divide the bending moment on the girder, in foot-pounds, by the product obtained by multiplying the depth of the girder, in feet, by the safe fiber stress.*

The safe fiber stress for a given case is obtained by dividing the ultimate fiber stress per square inch of the material by the factor of safety required in the girder.

The rule may be expressed by the formula

$$A = \frac{M}{D \times S}$$

in which A = net area of one flange, in square inches;
D = depth of girder, in feet;
S = safe fiber stress per square inch of the material;
M = bending moment on girder, in foot-pounds.

EXAMPLE.—The depth of a plate girder is 6 feet, the span is 80 feet, and the load on the girder is 3,000 pounds per lineal foot. (*a*) What will be the required net flange area for structural steel, if safe fiber stress of 1,500 pounds is used? (*b*) Of what size rolled sections should the flange be composed?

SOLUTION.—(*a*) The span being 80 ft. and the load 3,000 lb. per lin. ft., the entire load on the girder will be $80 \times 3{,}000 = 240{,}000$ lb.

Substituting in the formula $M = \frac{WL}{8}$ for the bending moment on a simple beam,

$$M = \frac{240{,}000 \times 80}{8} = 2{,}400{,}000 \text{ ft.-lb.}$$

The net area of the flange, from the formula in this article, is

$$A = \frac{2{,}400{,}000}{6 \times 15{,}000} = 26.7 \text{ sq. in.} \quad \text{Ans.}$$

(*b*) In order to determine the size of the flange plates and angles, it is useful to assume some particular size of angle and plates, and make a detail sketch of the flange, as shown in Fig. 8, marking on it the size of the respective plates and angles that have been assumed. The rivets should also be shown, so that the metal cut out of the rivet holes may be deducted from the sectional area of the flange in order to determine that area.

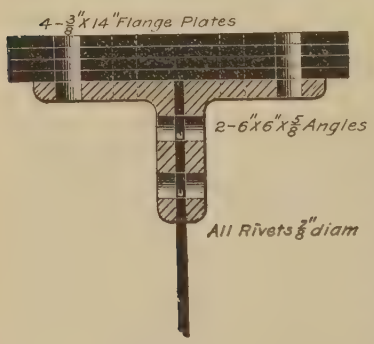


FIG. 8

It is assumed in the section under consideration that there are two rows of rivets through the vertical legs of the angles, each pair of these rivets being placed in the same vertical plane, in consequence of which the amount to be deducted from the net section is double the area cut out for one rivet. The rivets in the two rows through the horizontal legs of each angle are staggered, and consequently only one rivet hole in each horizontal leg affects the area of the flange section.

According to *Design of Columns*, the area of a $6'' \times 6'' \times \frac{5}{8}''$ angle is 7.11 sq. in.; therefore, the total area of the metal in the flange is

two $6'' \times 6'' \times \frac{5}{8}''$ angles, $7.11 \times 2 = 14.22$ sq. in.
four $14'' \times \frac{3}{8}''$ plates, $4 \times 14 \times \frac{3}{8} = 21.00$ sq. in.
Total, 35.22 sq. in.

From the total area of the flange, it is necessary to deduct the metal cut out for the rivet holes. As $\frac{7}{8}$-in. rivets are used, the rivet holes are considered to be $\frac{1}{8}$ in. larger, or 1 in. in diameter.

There are four 1-in. holes through $\frac{5}{8}$ in. of metal to be deducted from the vertical legs of the angles, and in the plates and the horizontal legs of the angles there are two 1-in. holes through $2\frac{1}{8}$ in. of metal. The areas to be deducted for the rivet holes are, therefore,

four 1-in. holes through $\frac{5}{8}$ in. of metal $= 2\frac{1}{2}$ sq. in.
two 1-in. holes through $2\frac{1}{8}$ in. of metal $= 4\frac{1}{4}$ sq. in.
Total, $6\frac{3}{4}$ sq. in.

and the net area of the flange is $35.22 - 6.75 = 28.47$ sq. in. Since the calculations showed that the net area required in this flange is 26.7 sq. in., it is evident that the assumed flange is amply strong. Ans.

13. Lengths of Flange Plates.—Since the bending moment in a simple beam varies along the entire length of the beam, the location of the maximum bending moment depending on the distribution of the load, it would seem that, in order to design an economical girder, the area of the flange should vary with the bending moment. Where flange plates are used, this condition may be partially fulfilled by

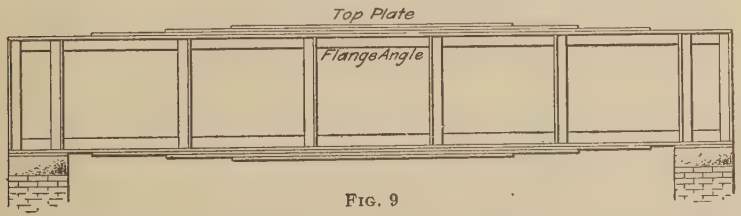


FIG. 9

the use of plates of different lengths, each extending only as far as may be required in order to provide the flange section demanded by the bending moment.

Reference to Fig. 9 will make this construction more clear. In this figure, it is seen that the topmost plate of the top flange is the shortest, and extends over a small portion

of the girder only, each successive plate under this one being longer than the one above it. The third plate from the top is the longest and extends nearly the full length of the girder, while the angles extend from end to end.

Where the beam is *uniformly loaded*, the following method may be used to obtain the theoretical length of each of the flange plates:

Commencing with the outside plate of the flange, find the sum of all the net areas, in square inches, of the plates to and including the plate in question. Thus, in Fig. 10, if it is required to obtain the length of the third plate from the outside, find the sum of the areas of the first, second, and third plates. If the length of the second plate is required, then the sum of the areas of the first and second plates is to be taken. Divide the area so obtained by the net area of the whole flange, in square inches, and multiply the square root of this quotient by the length of the girder, in feet; the product will be the theoretical length of the plate, in feet.

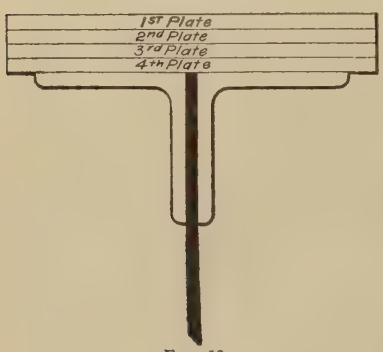


FIG. 10

The theoretical length of the plate having been determined, 12 to 16 inches should now be added to each end. This is done in order that the plate in question may be carried past the point of bending moment governing the area of the flange at its ends, far enough to insure of its being securely riveted to the plates and angles that make up the flange from the point of bending moment to the abutment.

The method for determining the length of flange plates where the beam is uniformly loaded may be expressed by the formula

$$l = L\sqrt{\frac{a}{A}}$$

in which l = theoretical length, in feet, of plate in question;
L = length of girder, in feet;
a = net area of all plates to and including plate in question, beginning with outside plate;
A = total net area of entire flange.

EXAMPLE.—In Fig. 11 is shown a section through the flange of a plate girder the span of which is 60 feet. What is the theoretical length of each of the three flange plates?

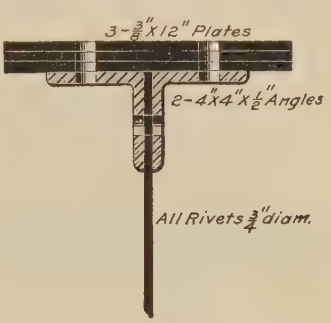


FIG. 11

SOLUTION.—The area of a $4'' \times 4'' \times \frac{1}{2}''$ angle according to *Design of Columns* is 3.75 sq. in. The area of each plate is $\frac{3}{8} \times 12 = 4.5$ sq. in. The diameter to be deducted for the rivet holes is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in. The area cut out by a $\frac{7}{8}$-in. hole through a $\frac{3}{8}$-in. plate is $.875 \times .375 = .328$ sq. in. Then, as there are two rivet holes in each plate, its net area is

$$4.5 \text{ sq. in.} - (.328 \text{ sq. in.} \times 2) = 3.844 \text{ sq. in.}$$

The net area of the angles is

$$(3.75 \text{ sq. in.} \times 2) - (.4375 \text{ sq. in.} \times 4) = 5.75 \text{ sq. in.}$$

Therefore, the net area of the flange section is

three plates	3.844×3	= 11.532 sq. in.
two angles		= 5.75 sq. in.
Total,		17.282 sq. in.

Now calculate the length of the outside plate. Substituting in the formula, we have

$$l = 60\sqrt{\frac{3.844}{17.282}} = 28.29 \text{ ft.}$$

By substituting the proper values, the theoretical length of the second, or middle, plate, is

$$l = 60\sqrt{\frac{7.688}{17.282}} = 40 \text{ ft.}$$

The length of the third, or last, plate in the flange, that is, the one next to the flange angles, is next to be calculated, though some engineers prefer to run this the entire length of the girder, as it stiffens the girder laterally and assists in preventing any tendency toward side deflection. The theoretical length of this plate is

$$l = 60\sqrt{\frac{11.532}{17.282}} = 49.12 \text{ ft.}$$

14. Graphic Method of Determining Length of Flange Plates.—The graphic method of determining the theoretical length of flange plates in built-up girders is sometimes more convenient than the analytic method previously given. In order to explain this method, it will be necessary to assume that a section through the flange of a plate girder at the point of maximum bending moment has already been obtained, and then find the lengths of the several flange plates.

Fig. 12 shows a section through the flange of a girder, built up of four $\frac{1}{2}'' \times 14''$ flange plates, the span of the girder being 90 feet. It will be noticed that there are two rows of rivets in the flange, and two rows in the vertical leg of the angles, but as the latter are staggered, there will be but one rivet hole to be deducted from the vertical leg of each angle.

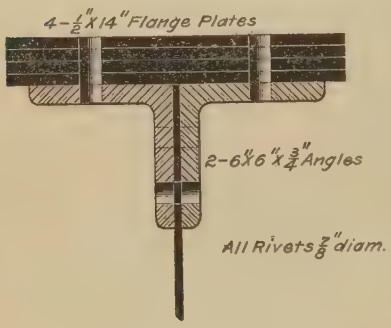


FIG. 12

The sectional area of a $6'' \times 6'' \times \frac{3}{4}''$ angle is found, in *Design of Columns*, to be 8.44 square inches; from this deduct $1\frac{1}{2}$ square inches, the area cut out by the two rivet holes, making the net area of each flange angle 6.94 square inches. The sectional area of a $\frac{1}{2}'' \times 14''$ flange plate is 7 square inches, from which there is to be deducted 1 square inch for the sectional area cut out by the two rivet holes. Hence, the net area of one flange plate is $7 - 1 = 6$ square inches.

The net area of the entire flange will therefore be

two $6'' \times 6'' \times \frac{3}{4}''$ angles	= 13.88 sq. in.
four $\frac{1}{2}'' \times 14''$ flange plates	= 24.00 sq. in.
Total,	37.88 sq. in.

Now proceed with the diagram, which is shown in Fig. 13. Since the load is assumed uniformly distributed, the flange

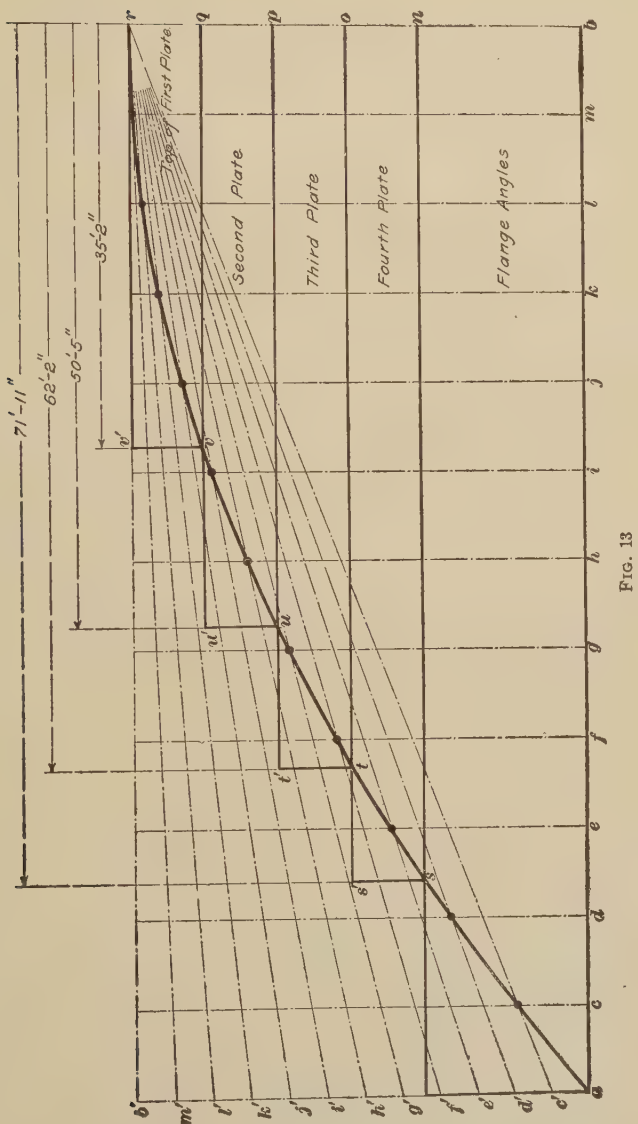

71'-11"
68'-2"
50'-5"
35'-2"
Top of First Plate
Second Plate
Third Plate
Fourth Plate
Flange Angles

FIG. 13

plates extend equally on each side of the center; consequently, the diagram for only one-half of the girder will be drawn. Draw, to any scale, a horizontal line $a\,b$, Fig. 13, equal to one-half of the span; divide this line into any number of equal parts (in the figure, twelve parts have been used). Upwards from the points of division, draw indefinite perpendicular lines. On the perpendicular from b, lay off to some scale a distance that represents the entire net section of the flange, thus locating the point r. For example, the net area of the flange in this case is 37.88 square inches; letting $\frac{1}{16}$ inch on the diagram represent 1 square inch of flange area, the distance $b\,r$ must be $37.88 \times \frac{1}{16} = 2.37$ inches, nearly.

Lay off to the same scale on the line $b\,r$ a distance $b\,n$, which represents 13.88 square inches, the net area of the two flange angles; also the distances $n\,o$, $o\,p$, $p\,q$, and $q\,r$, each representing 6 square inches, the net area of each of the flange plates. From the point r draw a horizontal line cutting the vertical line erected at a, thus locating the point b'. Divide the vertical line $a\,b'$ into the same number of equal parts as the line $a\,b$, thus locating the points c', d', e', f', etc.; and from these points draw the lines $c'\,r$, $d'\,r$, etc. Draw the curve $a\,s\,v\,r$ through the points where the vertical lines from c, d, e, etc. intersect the corresponding lines $c'\,r$, $d'\,r$, $e'\,r$, etc. Now from the points q, p, o, and n, draw horizontal lines as shown, cutting the curve in the points s, t, u, and v, and from each of these points of intersection, draw a perpendicular, extending it until it intersects the horizontal line next above. The rectangles $v'\,r\,q\,v$, $u'\,q\,p\,u$, $t'\,p\,o\,t$, etc. thus formed represent the flange plates and angles.

Now, in order to obtain the theoretical length of any flange plate, measure the length of the corresponding rectangle by the scale to which the half span was laid out on the line $a\,b$; this length multiplied by 2 gives the length of the plate in question. For example, if it is desired to obtain the length of the first, or top, plate, with the scale to which the half span was laid out, measure the length of the line $v'\,r$; as only one-half of the diagram is drawn, this gives one-half

of the length of the top, or first, plate, and by doubling this, the entire theoretical length of the plate in question is obtained. The length of the other plates may be determined in like manner. It is to be borne in mind that to the theoretical length given by the diagram it is necessary to add a length of about 1 foot at each end of the plate. From the diagram, the theoretical lengths of the flange plates of the girder shown in Fig. 12 are found to be 35 feet 2 inches, 50 feet 5 inches, 62 feet 2 inches, and 71 feet 11 inches, respectively. It will be found on checking these lengths by the formula of Art. 13 that they are approximately correct.

15. Application of the Graphic Method to Girders With Concentrated Loads.—The graphic method for determining the theoretical length of flange plates when the girder is loaded with concentrated loads, which is similar to that given for a uniformly loaded girder, will be illustrated by constructing a diagram for the lengths of the four flange plates required for a girder with a span of 80 feet and a depth of 6 feet, carrying a concentrated load of 185,000 pounds at 30 feet from one end.

The bending moment on the girder may be calculated by the method given in *Design of Beams* or by the formula

$$M = W \times \frac{a\,b}{L},$$

in which M = bending moment;
W = load on girder;
L = span;
a = distance that load is located from one abutment;
b = distance load is located from other abutment.

In Fig. 14, it is seen that the load is located at the point I 30 feet from R_1, and 50 feet from R_2. Substituting these values in the formula, just given, the bending moment is found to be

$$M = 185{,}000 \times \frac{30 \times 50}{80} = 3{,}468{,}750 \text{ foot-pounds}$$

From this bending moment, the required net flange area, assuming a safe unit stress of 15,000 pounds per square inch.

is found to be, approximately, 38 square inches, which is provided by the use of four $\frac{1}{2}'' \times 14''$ plates and two $6'' \times 6'' \times \frac{3}{4}''$ angles. The flange, therefore, has the section shown in Fig. 12.

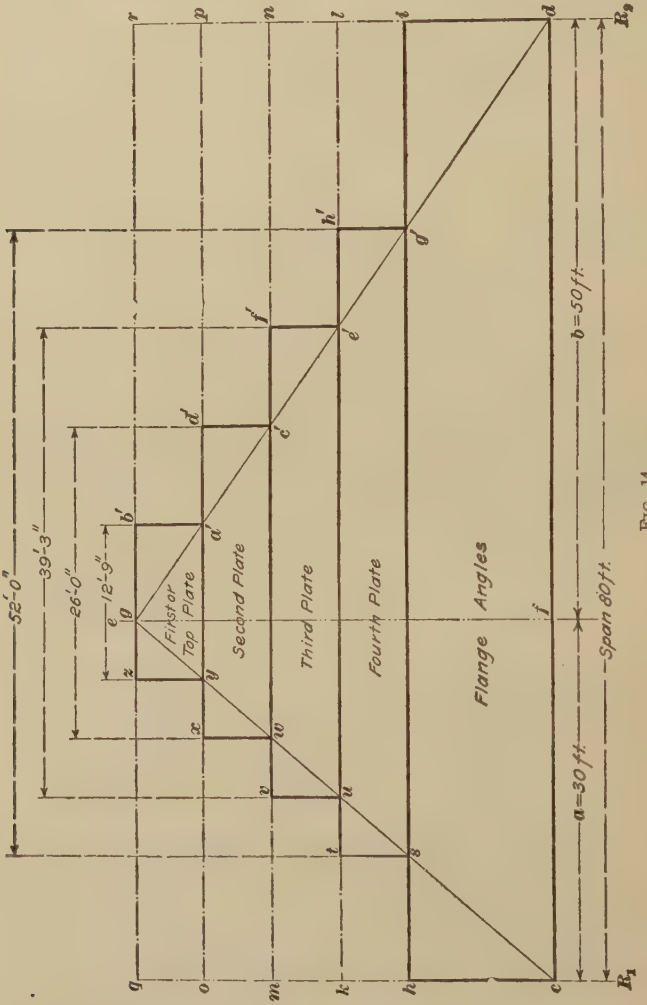


Fig. 14

To construct the diagram, which is shown in Fig. 14, draw the base line cd to any scale equal to the span of the girder in feet. (In this case, owing to the fact that the load

is not symmetrically placed, the center line of the girder will not divide the lengths of the plates in halves, and it will be necessary to draw the entire diagram.) Locate the point of application of the concentrated load at 30 feet from R_1, and draw the perpendicular line $e f$. On this line lay off, to any scale, a distance that represents the bending moment. For example, in this case, a scale has been used on which 50,000 foot-pounds is represented by $\frac{1}{32}$ inch; the distance to be laid off is, therefore,

$$\frac{346875 0}{50000} \times \frac{1}{32} = 2.168 \text{ inches}$$

This locates the point g, which is then connected by straight lines to the points c and d. Draw vertical lines from the points c and d, until they meet a horizontal line through the point g, at q and r.

In the flange section, the angles have a combined net area of 13.88 square inches, and each flange plate a net area of 6 square inches. The combined net sectional area of the flange is 37.88 square inches. Place the zero mark of any convenient scale, at the point f, and slant the scale until the marking on the scale that represents 37.88 square inches, the net area of the flange, falls on the horizontal line $q r$. Thus, in this case, a $\frac{1}{16}$-inch scale has been used, and the zero mark is placed at f, while the mark on the scale representing the division 37.88 is placed on the horizontal line $q r$. With the scale still in this position, begin at the zero mark and lay off a distance 13.88 to represent the net sectional area of the two angles; then lay off four distances, each equal to 6, to represent the net sectional area of each of the flange plates.

Through the points thus found, draw the horizontal lines $h i$, $k l$, $m n$, and $o p$. At the points s, u, w, y, etc. where these horizontal lines cut the oblique lines $g c$ and $g d$, draw the perpendicular lines $s t$, $u v$, $w x$, $y z$, $a' b'$, etc., extending each until it meets the next horizontal line above. Then the rectangles enclosed by the horizontal and vertical lines, shown heavy in the diagram, represent the cover plates and flange angles. By measuring the length of these rectangles with the scale to which the span was laid off on the line $c d$, the theoretical length of the plates may be determined.

In this case, the length of the angles is equal to the span, 80 feet, as marked on the diagram. The length of the first, or top, plate measures 12 feet 9 inches; of the second plate, 26 feet 0 inches; of the third plate, 39 feet 3 inches; and of the fourth, or last plate, 52 feet 0 inches. Slight errors may occur due to little inaccuracies in drawing. However, a variation of a fractional part of an inch in the length of a flange plate on a girder need not be considered.

16. Graphic Diagram for Several Concentrated Loads.—In order to illustrate this method still further, a more complicated problem, in which there are three concentrated loads, will now be presented. Assume that a girder having a span of 80 feet with a depth of 6 feet is loaded as shown in Fig. 15. What flange area is required, provided that a safe

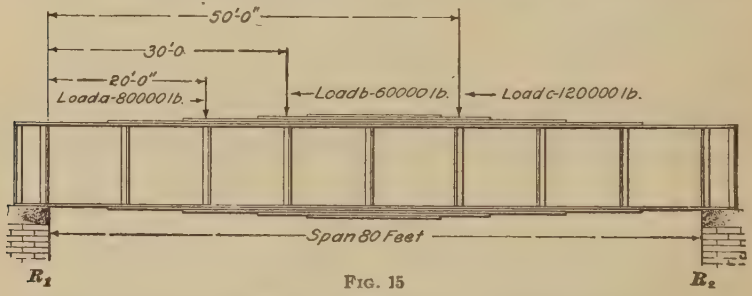


FIG. 15

unit fiber stress of 16,000 pounds per square inch is used, and of what sized plates and angles should the flange be constructed, also what will be the length of the several flange plates?

The reactions at R_1 and R_2 are found to be 142,500 and 117,500 pounds, respectively. It will first be necessary to calculate the bending moment, in foot-pounds, at the points a, b, and c, where the loads are concentrated.

The bending moment at a is

$$142{,}500 \times 20 = 2{,}850{,}000 \text{ foot-pounds;}$$

at b, the bending moment is

$$(142{,}500 \times 30) - (80{,}000 \times 10) = 3{,}475{,}000 \text{ foot-pounds;}$$

and the bending moment at c is

$$(142{,}500 \times 50) - [(80{,}000 \times 30) + (60{,}000 \times 20)]$$
$$= 3{,}525{,}000 \text{ foot-pounds}$$

Thus, it is seen that the greatest bending moment is under the load located at c, and its magnitude is 3,525,000 foot-pounds. From this, the flange area required may be

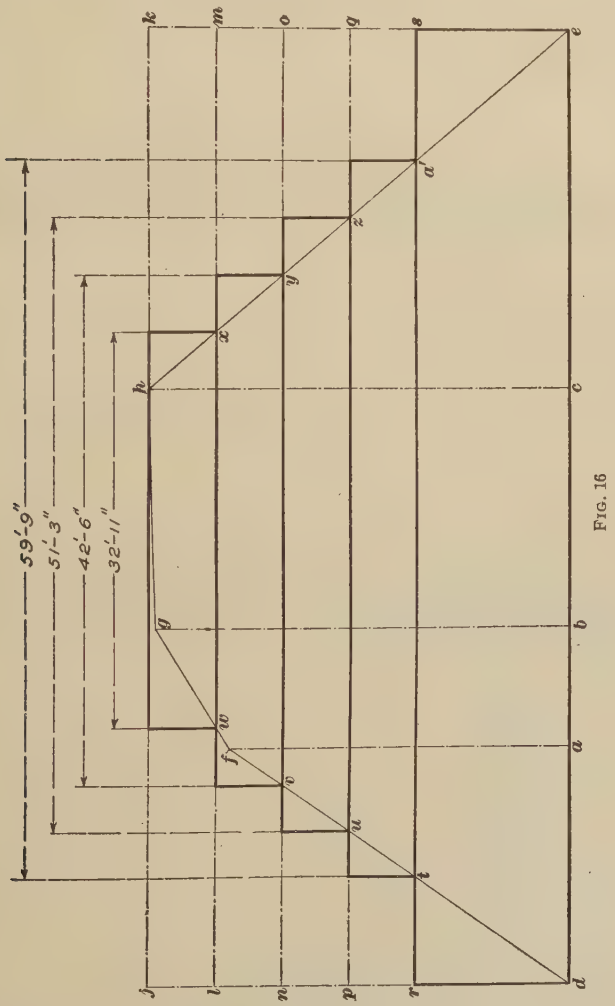


FIG. 16

calculated by applying the formula of Art. 12 as follows:

$$A = \frac{3{,}525{,}000}{6 \times 16{,}000} = 36.71 \text{ square inches}$$

Now, select a flange section that will have the required net sectional area. Referring to the section shown in Fig. 12, the area is found to be 37.88 square inches; this flange will therefore satisfy the requirements.

Having determined the bending moments and the greatest net flange area, begin the diagram shown in Fig. 16 by drawing to any scale the horizontal line de equal in length to the span of the girder; with the same scale locate the points of application a, b, and c of the concentrated loads. Upwards from the points d, a, b, c, and e, draw indefinite perpendicular lines, and on the perpendiculars from a, b, and c, lay off to some convenient scale distances af, bg, and ch, which represent the respective bending moments at these points.

For example, the bending moment at a is 2,850,000 foot-pounds; at b, it is 3,475,000 foot-pounds; and at c, it is 3,525,000 foot-pounds; therefore, assuming a scale on which each $\frac{1}{32}$ inch represents 50,000 foot-pounds of bending moment, the respective bending moments at the points a, b, and c are represented by lengths af of 57 thirty-seconds, bg of $69\frac{1}{2}$ thirty-seconds, and ch of $70\frac{1}{2}$ thirty-seconds.

Draw straight lines connecting the points d, f, g, h, and e. Through the highest point h, representing the greatest bending moment, draw the horizontal line jk.

The net area of the flange being 37.88 square inches, place the zero mark of any convenient scale on the line de, and slant the scale until the mark that represents 37.88 falls on the line jk.

Starting from the zero mark on the scale, lay off a distance that represents the net area of the two flange angles, in this case 13.88 square inches, then divide the remaining distance into equal parts, each of which represents 6 square inches, the net sectional area of the several flange plates. Through the points just found, draw the horizontal lines lm, no, pq, and rs. Where these horizontal lines intersect the oblique lines at the points t, u, v, w, x, etc., draw vertical lines until they intersect the next horizontal line above. Then draw in with heavy lines the rectangles representing the flange plates and flange angles. The theoretical length

of the flange plates may now be determined by measuring with the scale to which the span was laid off on the line *de*.

The length of the top, or first, plate in this case is found to be 32 feet 11 inches; of the second plate, 42 feet 6 inches; of the third plate, 51 feet 3 inches; and of the fourth, or last, plate, 59 feet 9 inches.

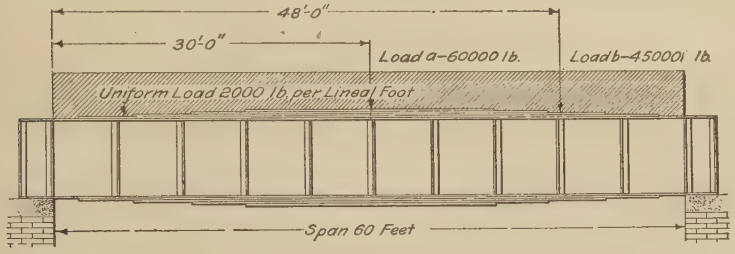


FIG. 17

17. Diagram for a Combination of Concentrated Loads With a Uniformly Distributed Load.—There is still another condition of girder loading that is frequently encountered in practical work, this condition is produced by a combination of a uniformly distributed load with several concentrated loads located at different points along the girder.

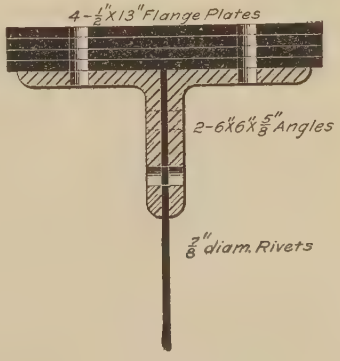


FIG. 18

In order to explain the method of obtaining the length of the flange plates in a girder loaded in this manner, the following problem will be assumed and the diagram will be constructed as was done in previous cases.

Assume the girder to be loaded, as shown in Fig. 17, with a uniformly distributed load and two concentrated loads. The flange section shown in Fig. 18 is sufficient to resist the bending moments due to these loads. It is required to determine, by the graphic method, the theoretical lengths of the several cover-plates making up the flange section.

Before starting to draw the diagram shown in Fig. 19, it is necessary to make the calculations for the following: The greatest bending moment; the maximum bending moment due to the uniformly distributed load; and the bending moment under each of the concentrated loads, neglecting the uniformly distributed load. These bending moments should

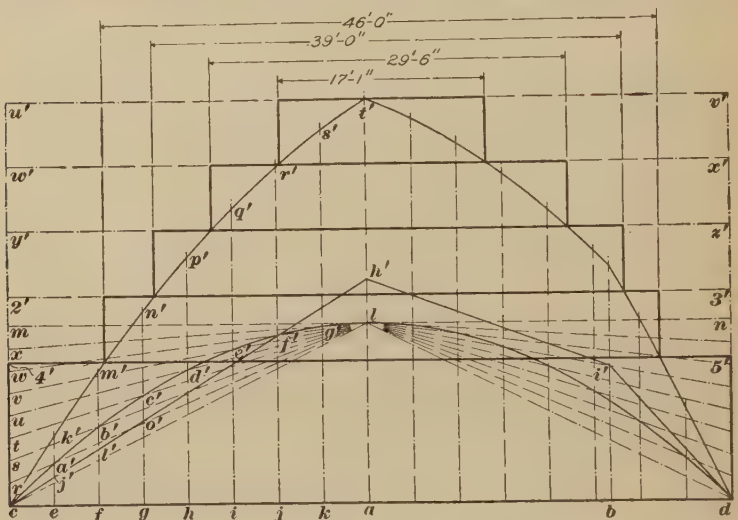


FIG. 19

be expressed in foot-pounds. The flange area required to successfully resist each of these bending moments should also be calculated.

The calculations in this case have been made in the usual manner, and the results are as follows:

Greatest bending moment . . . . . = 2,070,000 ft.-lb.
Bending moment due to a uniform load alone . . . . . = 900,000 ft.-lb.
Considering the concentrated load only, the bending moment under concentrated load *a* . . . . . = 1,170,000 ft.-lb.
Considering the concentrated load only, the bending moment under concentrated load *b* . . . . . = 792,000 ft.-lb.

Since the depth of the girder is 4 feet, if a unit fiber stress of 15,000 pounds is used, the flange area required to resist the greatest bending moment is

$$A = \frac{2{,}070{,}000}{4 \times 15{,}000} = 34.5 \text{ square inches}$$

The flange area required to resist the bending moment due to the uniform load is

$$A = \frac{900{,}000}{4 \times 15{,}000} = 15 \text{ square inches}$$

The flange area required to resist the bending moment at the point on the girder where the concentrated load a is situated, considering the concentrated loads only, is

$$A = \frac{1{,}170{,}000}{4 \times 15{,}000} = 19.5 \text{ square inches}$$

The flange area required to resist the bending moment at the point on the girder under the concentrated load b, considering as before only the concentrated loads, is

$$A = \frac{792{,}000}{4 \times 15{,}000} = 13.2 \text{ square inches}$$

As the loads on the girder are not symmetrically placed with regard to the center, it will be necessary to draw the complete diagram. Begin the diagram by drawing to any convenient scale the horizontal line $c\,d$, Fig. 19, equal in length to the span of the girder, and locate the points of application of the concentrated loads at a and b; upwards from the points c, a, b, and d, draw indefinite vertical lines.

Now, in accordance with the method explained in Art. **14**, make the construction to determine the curved line representing the bending moment due to the uniform load, as follows:

At the center of the girder draw a vertical line (in this case the center of the girder is found to be at the point where the load a is concentrated); divide one-half the span into any number of equal parts, as at e, f, g, h, i, etc., and from the points so obtained draw perpendiculars. Lay off on the vertical line passing through the center a distance $a\,l$, which may represent either the greatest bending moment at this point due to the uniform load, or the flange area required

to resist this bending moment, as they are proportional. In this case, the net area required in the flange will be used; hence, as the area of the flange required for the uniform load is 15 square inches, if $\frac{1}{16}$ inch is assumed to represent 1 square inch of flange area, the distance $a\,l$ will be $\frac{15}{16}$ inch.

Through the point l, draw the horizontal line $m\,n$. Divide the distance $m\,c$ into the number of equal parts that the half of the span was divided into, and from the points r, s, t, u, v, etc., thus obtained, draw converging lines to the point l; where these oblique lines intersect the vertical lines, mark the points a', b', c', d', etc., and through these points draw the curve $c\,l$. Draw the other half of the curve $l\,d$ in the same manner, thus completing the diagram for the uniform load.

Now draw the diagram for the concentrated loads. On the vertical line $a\,l$, extended, lay off the distance $a\,h'$ equal to the net flange area required to support the concentrated load a, and on the perpendicular line erected at b, lay off the distance $b\,i'$ equal to the flange area required at the point b to support the concentrated loads.

In drawing this diagram, it must be remembered that it is necessary to use the same scale as was used to lay off the flange area required for the uniform-load diagram; also, that if the vertical distances are laid off to represent the bending moment in the one case, the bending moment should be used in the other, while if the flange area required is used in the one case, it is self-evident that it should also be used in the other.

Having located the points h' and i', connect with straight lines the points c, h', i', and d, as was done in the previous similar diagrams of concentrated loads. This completes the diagram for the concentrated loads.

The next step in the process is to measure the distance $c\,j'$ with a pair of dividers, and from the point a' on the curve representing the uniform load, lay off on the vertical line the distance $a'\,k'$ equal to $e\,j'$, also lay off from the point b' the distance $b'\,m'$ equal to $f\,l'$, and from the point c' lay off on the vertical line the distance $c'\,n'$ equal to $g\,o'$; continue in this manner through the entire diagram. Having thus

determined the points k', m', n', p', etc. through the entire diagram, draw in the curve $c\,k'\,m'\,n'$, etc. The point t' is the highest point in the diagram, and its distance from the horizontal line $c\,d$ represents the entire flange area required in the girder to resist the greatest bending due to both the uniform and the concentrated loads.

Through the point t' draw the horizontal line $u'\,v'$, and lay off between the horizontal lines $u'\,v'$ and $c\,d$ the several distances representing the net area of the flange plates and flange angles. Through the points of these divisions, draw the horizontal lines w'–x', y'–z', $2'$–$3'$, and $4'$–$5'$. Where these horizontal lines intersect the curved line representing the net area required for the combined uniform and concentrated loads, draw short vertical lines to the next horizontal line above; draw with heavy lines the rectangles representing the flange plates and flange angles; scale the length of the flange plates with the scale to which the span $c\,d$ was laid off, and the theoretical length of the flange plates will be found.

In the girder under consideration, the theoretical length of the top, or first, plate is found to be 17 feet 1 inch; of the second plate, 29 feet 6 inches; of the third plate, 39 feet 0 inches; and of the last, or fourth, plate, 46 feet 0 inches.

RIVET SPACING

18. Rivets in the End Angles or Stiffeners Over the Abutment.—First, the allowable safe load on the rivet should be determined. Whether the double shear of the rivet or the bearing value of the plate around the rivet hole is greatest, should be found as explained in *Design of Roof Trusses*. The values for rivets in this Section will all be taken from Table III, *Roof-Truss Design*. Having obtained the safe allowable load for each rivet, a sufficient number should be placed in the end angles or stiffeners to take care of the entire shear at that point, which on a simple girder is equal to the end reaction at the abutment. For example, let the end reaction of a girder be 100,000 pounds; $\frac{7}{8}$-inch rivets are used and the thickness of the web is $\frac{3}{8}$ inch. With an allowable tensile stress of 12,000 pounds per square inch,

the web-bearing value of a $\frac{3}{8}$-inch plate is 6,825 pounds, which is the allowable load on the rivet. The number of rivets required in the two pair of end angles is, therefore, 100,000 ÷ 6,825 = 14.6, say 16, or 8 rivets in each pair.

19. Rivets in the Stiffeners Between the Abutments.—If possible, the rivets in the intermediate stiffeners are usually spaced the same as in the end stiffeners. It is hardly possible to state a rule that will be of practical value for spacing and finding the number of rivets required, and in fact no rule or formula is necessary; a practical rule is that the pitch of these rivets should never exceed 6 inches, nor should it exceed sixteen times the thickness of the leg of the angle.

20. Rivets Connecting Flange Angles With the Web.—It will readily be seen that when a plate girder is loaded, the tendency of the flanges and angles is to slide horizontally past the web; this tendency to slide induces a horizontal flange stress. The rivets connecting the angles to the web resist this tendency, and there must be a sufficient number of rivets to do it safely.

The stress that at any point is transmitted horizontally from the web to the flange is equal to the increment of the

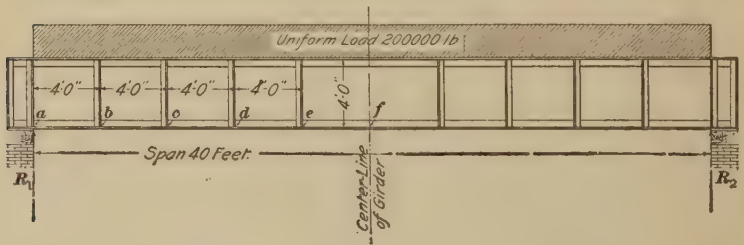


FIG. 20

flange stress at that point. This increment is found by dividing the maximum vertical shear at any point by the depth of the girder.

For example, assume a girder of 40 feet span, as shown in Fig. 20, with a depth of 4 feet, and a uniformly distributed load of 200,000 pounds. The shearing stress in the girder at the left reaction, or point a, is equal to R_1, in this case

100,000 pounds. At b, 4 feet from R_1, the vertical shear in the girder is

$$100{,}000 - (5{,}000 \times 4) = 80{,}000 \text{ pounds;}$$

at c, 8 feet from R_1, the vertical shear is

$$100{,}000 - (5{,}000 \times 8) = 60{,}000 \text{ pounds;}$$

at d it is

$$100{,}000 - (5{,}000 \times 12) = 40{,}000 \text{ pounds;}$$

at e the shear is 20,000 pounds; and at f the shear is zero.

From these results, the rate of increase, per inch of length, of the horizontal stress in the flange at the several points a, b, c, d, e, and f may be obtained by dividing the shear at those points by the depth of the girder, in inches. Thus, at the end a of the girder the increase in the horizontal flange stress is

$$\frac{100{,}000}{4 \times 12} = 2{,}083 \text{ pounds per inch of run}$$

at b, $\dfrac{80{,}000}{4 \times 12} = 1{,}667$ pounds per inch of run

at c, $\dfrac{60{,}000}{4 \times 12} = 1{,}250$ pounds per inch of run

at d, $\dfrac{40{,}000}{4 \times 12} = 833$ pounds per inch of run

and at e, $\dfrac{20{,}000}{4 \times 12} = 417$ pounds per inch of run

If $\frac{7}{8}$-inch rivets are used, the allowable safe load for each rivet, based on a fiber stress of 12,000 pounds per square inch, with web bearing in a $\frac{3}{8}$-inch plate, is 6,825 pounds. Then at the end, where the increase in stress is 2,083 pounds per inch of run, the rivets must not be spaced farther apart than $6{,}825 \div 2{,}083 = 3.27$ inches, from center to center. At b, the maximum allowable pitch of the rivets is $6{,}825 \div 1{,}667 = 4.09$ inches; at c, the pitch may be $6{,}825 \div 1{,}250 = 5.46$, or about $5\frac{1}{2}$, inches; and at d, $6{,}825 \div 833 = 8.19$ inches. Since, for practical reasons, the rivets in the vertical leg of the flange are spaced the same in both the upper and lower chords, and, since the greatest allowable pitch of rivets in a compression member is 6 inches, it is evident that it is needless to carry the calculation further.

In accordance with the foregoing calculation, the pitch of the rivets between a and b should be $3\frac{1}{4}$ inches; between b and c, the pitch should be about 4 inches; while between c and d, the theoretical pitch is $5\frac{1}{2}$ inches; since the theoretical pitch at d is more than 6 inches, which, for practical reasons, is not allowable, all the rivets between d and the center of the girder should be spaced 6 inches from center to center.

21. Rivets Spaced According to Stress Produced by Bending Moment.—The rivets that connect the flange angles with the web-plate may also be spaced according to the stresses produced on the flanges by the bending moment. The horizontal stress on the flanges diminishes either way from the point of greatest bending moment toward the end reactions, where it becomes zero, and for any point this stress may be calculated by the application of the principle of moments.

If the bending moment is obtained at any panel point and is divided by the depth of the girder, the stress on the flange at that point will be obtained; and, if this stress is divided by the allowable load on one rivet, the number of rivets required between that point and the end reaction will be obtained.

For example, in the girder used in the previous illustration (Fig. 20), the span being 40 feet and the load 200,000 pounds, the bending moment at the center is equal to

$$\frac{WL}{8} = \frac{200{,}000 \times 40}{8} = 1{,}000{,}000 \text{ foot-pounds}$$

then the depth of the girder being 4 feet, the flange stress at this point is $1{,}000{,}000 \div 4 = 250{,}000$ pounds. The allowable load on each rivet being 6,825 pounds, the number of rivets between the center and the end reaction is

$$250{,}000 \div 6{,}825 = 37 \text{ rivets, approximately}$$

Now, although the number of rivets between the end reaction and the center of the girder has been obtained, the pitch of these rivets is still unknown. The rate at which the horizontal stress in the flange increases varies, being greatest at the ends and least under the position of maximum bending moment; it therefore follows that the rivets should be spaced

nearer together at the ends, with an increase in the spacing toward the point of greatest bending moment.

In practical work, the rivet spacing is seldom varied in any one panel; if, however, the flange stress is obtained at each of the stiffeners *b*, *c*, *d*, *e*, and *f*, Fig. 20, the number of rivets required between each of these points and the end reaction may be ascertained; by finding the difference between these numbers for any two consecutive stiffeners, the number of rivets required in the panel between those stiffeners is arrived at. For example, the stresses on the flange at each of the stiffeners of the girder shown in Fig. 20 are as follows:

	Bending Moment, in Foot-Pounds		Depth of Girder, in Feet		Flange Stress, in Pounds
At *b*,	360,000	÷	4	=	90,000
At *c*,	640,000	÷	4	=	160,000
At *d*,	840,000	÷	4	=	210,000
At *e*,	960,000	÷	4	=	240,000
At *f*,	1,000,000	÷	4	=	250,000

The approximate number of rivets between each stiffener and the reaction R_1 is as follows:

Between *b* and R_1, 90,000 ÷ 6,825 = 13 rivets
Between *c* and R_1, 160,000 ÷ 6,825 = 23 rivets
Between *d* and R_1, 210,000 ÷ 6,825 = 31 rivets
Between *e* and R_1, 240,000 ÷ 6,825 = 35 rivets
Between *f* and R_1, 250,000 ÷ 6,825 = 36 rivets

Then the number of rivets that are required is:

Between *b* and *a*, 13 − 0 = 13 rivets
Between *c* and *b*, 23 − 13 = 10 rivets
Between *d* and *c*, 31 − 23 = 8 rivets
Between *e* and *d*, 35 − 31 = 4 rivets
Between *f* and *e*, 36 − 35 = 1 rivet

Consequently, the pitch between the stiffeners will be as follows:

Between *b* and *a*, 48 ÷ 13 = 3.69 inches
Between *c* and *b*, 48 ÷ 10 = 4.80 inches
Between *d* and *c*, 48 ÷ 7 = 6.85 inches

It is needless to continue the calculation further, because between d and c the theoretical pitch exceeds 6 inches, the limit allowable for a compression member.

By the first method, the pitch at each stiffener or panel point is determined; while by the second, the average pitch between two consecutive panel points is obtained; in order to compare the two methods, it will be necessary to reduce the results obtained by the first to the basis of the second. In the first method, the pitch at the several stiffeners or panel points was to be:

At a = 3.27 inches At c = 5.46 inches
At b = 4.09 inches At d = 8.19 inches

From these, the average pitch between the several points would be:

Between a and b, $(3.27 + 4.09) \div 2 = 3.68$ inches
Between b and c, $(4.09 + 5.46) \div 2 = 4.78$ inches
Between c and d, $(5.46 + 8.19) \div 2 = 6.83$ inches

These, on comparison, are found to be almost identical with the values 3.69, 4.80, and 6.85 inches obtained by the second method.

22. Rivet Spacing in Flange Plates.—In spacing the rivets that bind the several flange plates together, a sufficient number of rivets, spaced from $2\frac{3}{4}$ to 3 inches on centers, should be used at the ends of each plate to transmit the allowable stress in it to the members below. For the remainder of the plate, the rivets should have the greatest allowable pitch for a compression member; that is, sixteen times the thickness of the thinnest outside plate, provided such a distance does not exceed 6 inches. To illustrate:

An intermediate flange plate in a certain girder is $\frac{3}{8}$ in. $\times$ 12 in., the sectional area thus being $4\frac{1}{2}$ square inches. From this area is to be deducted the section cut out by two $\frac{7}{8}$-inch rivet holes, $(1 \times \frac{3}{8}) \times 2 = \frac{3}{4}$ square inch; then the net area of the cover-plate is $4\frac{1}{2} - \frac{3}{4} = 3\frac{3}{4}$ square inches. Assuming that a safe fiber stress of 15,000 pounds was used in calculating the strength of the girder, the safe strength of the cover-plate is $3\frac{3}{4} \times 15{,}000 = 56{,}250$ pounds. Now, the

safe load on a $\frac{7}{8}$-inch rivet depends, in this position, on the ordinary bearing value of a $\frac{3}{8}$-inch plate, which, calculated on the basis of a fiber stress of 12,000 pounds, is 5,119 pounds. Hence, the number of rivets required in the end of this cover-plate is 56,250 ÷ 5,119 = 10.9, say 12; this gives 6 on each side of the web, and they should be spaced about 3 inches from center to center. The remaining rivets in this plate may have the greatest allowable pitch until the next cover-plate is reached.

PRACTICAL PROBLEMS

23. In order to illustrate the application of the rules and formulas that have been stated, the following practical

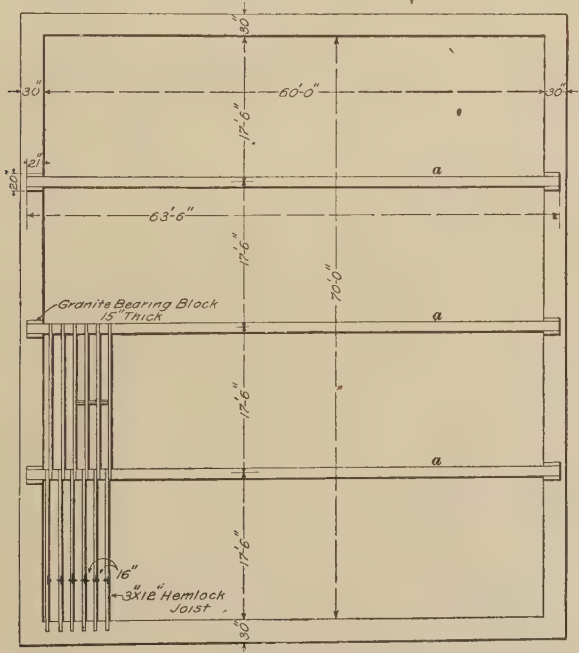


FIG. 21

problem will be assumed and worked out: The floor of a building used for light manufacturing purposes is to be supported by the three plate girders, as shown at *a*, *a*, *a*, Fig. 21.

The floor is composed of 1-inch yellow-pine flooring laid on $3'' \times 12''$ hemlock joists, spaced 16-inch centers; these joists are to carry a plastered ceiling on the under side. The live load on the floor will be 80 pounds per square foot. The

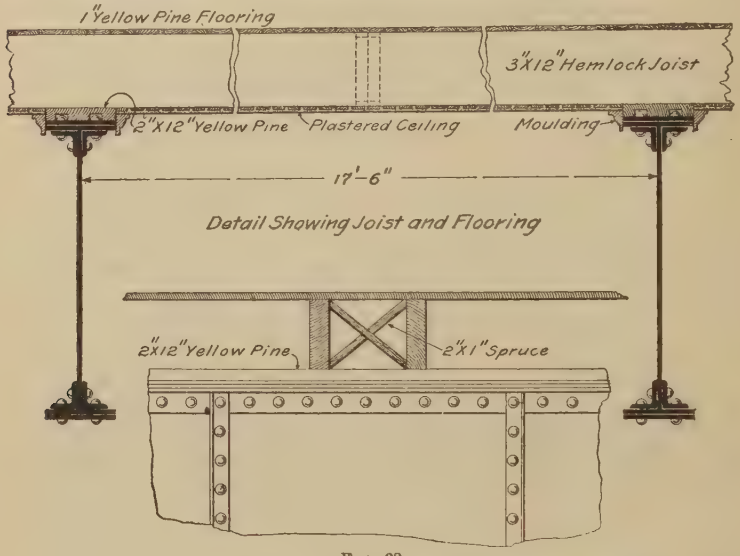


FIG. 22

girder itself is to extend below the surface of the ceiling and is to be painted, a detail of the construction being shown in Fig. 22.

The total load on each square foot of floor surface is as follows:

	POUNDS
Live load, per square foot of floor surface .	80
Lath and plaster, per square foot of floor surface	8
1-inch yellow-pine flooring, per square foot of floor surface	4
Hemlock joist flooring, per square foot of floor surface	6
Girder (assumed), per square foot of floor surface	8
Total	106

The floor area to be supported by one girder is $60 \times 17.5 = 1{,}050$ square feet; and the total uniformly distributed load on the girder is $1{,}050 \times 106 = 111{,}300$ pounds.

The greatest bending moment on the girder is

$$M = \frac{WL}{8} = \frac{111{,}300 \times 60}{8} = 834{,}750 \text{ foot-pounds}$$

The depth of the girder is 4 feet, and the allowable unit fiber stress to be used is 15,000 pounds; therefore, the required flange area may be determined by the formula of Art. **12.** Substituting the proper values in the formula, we have

$$A = \frac{834{,}750}{4 \times 15{,}000}$$

$$= 13.91 \text{ square inches}$$

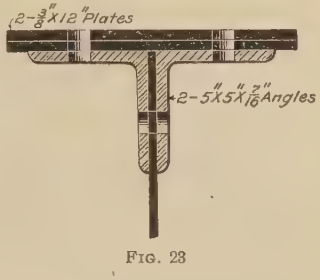


FIG. 23

Assume a flange composed of two $5'' \times 5'' \times \frac{7}{16}''$ special angles and two $\frac{3}{8}'' \times 12''$ flange plates; a sketch of the section with the location of the rivets is shown in Fig. 23. The entire area of the flange is

two $\frac{3}{8}'' \times 12''$ plates	= 9	sq. in.
two $5'' \times 5'' \times \frac{7}{16}''$ angles	= 8.36	sq. in.
Total,	17.36	sq. in.

The sectional areas cut out for rivet holes are:

four $\frac{7}{8}$-inch holes through $\frac{3}{8}$-inch plate	= 1.312	sq. in.
four $\frac{7}{8}$-inch holes through $\frac{7}{16}$-inch angles	= 1.531	sq. in.
Total,	2.843	sq. in.

The net area of the flange is, therefore, $17.36 - 2.84 = 14.52$ square inches, which, since the required area is 13.91 square inches, is ample, and this section will be adopted.

Now determine the thickness of the web-plate. The reaction at either end is equal to one-half of the load, or 55,650 pounds. Assuming that there are eleven $\frac{7}{8}$-inch holes cut in line through the web-plate, the net depth of the plate will be $48 - 11 \times .875 = 38.375$ inches. Using an allowable unit shearing stress of 11,000 pounds, the

theoretical thickness of the web-plate, from the formula of Art. **7**, is

$$T = \frac{55{,}650}{38.375 \times 11{,}000} = .131 \text{ inch}$$

It is not, however, practicable to use this thickness of metal for a web-plate, since it will not provide sufficient bearing value for the rivets. As it is never good practice to use a web-plate less than $\frac{5}{16}$ inch in thickness, this size will be adopted.

The lengths of the flange plates are now required; they may be determined either by the graphic method or by the formula of Art. **13**; using the latter method, the theoretical length of the outside plate is found to be

$$l = 60\sqrt{\frac{3.844}{14.52}} = 30.87 \text{ feet, or about 30 feet 10 inches,}$$

to which is to be added 1 foot at each end to allow for riveting. The total length of the plate is, therefore, 32 feet 10 inches, say 33 feet.

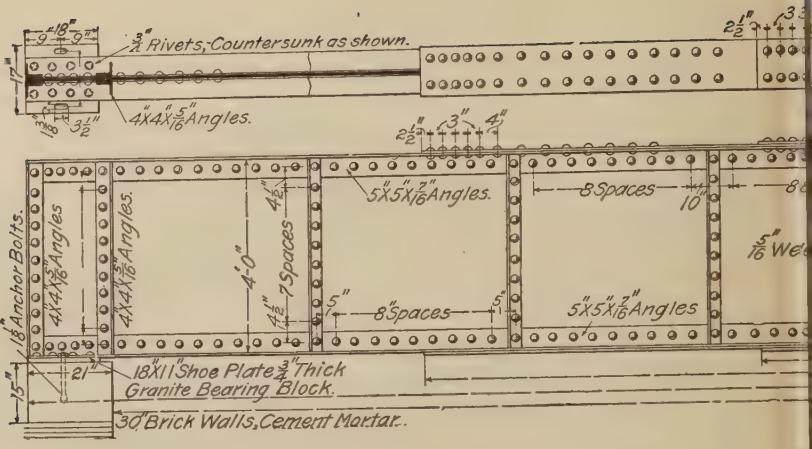
Applying the formula again, the length of the second flange plate is

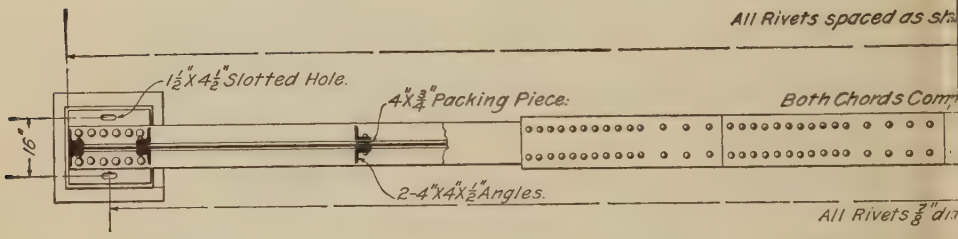
$$l = 60\sqrt{\frac{7.688}{14.52}} = 43.66 \text{ feet, or about 43 feet 8 inches;}$$

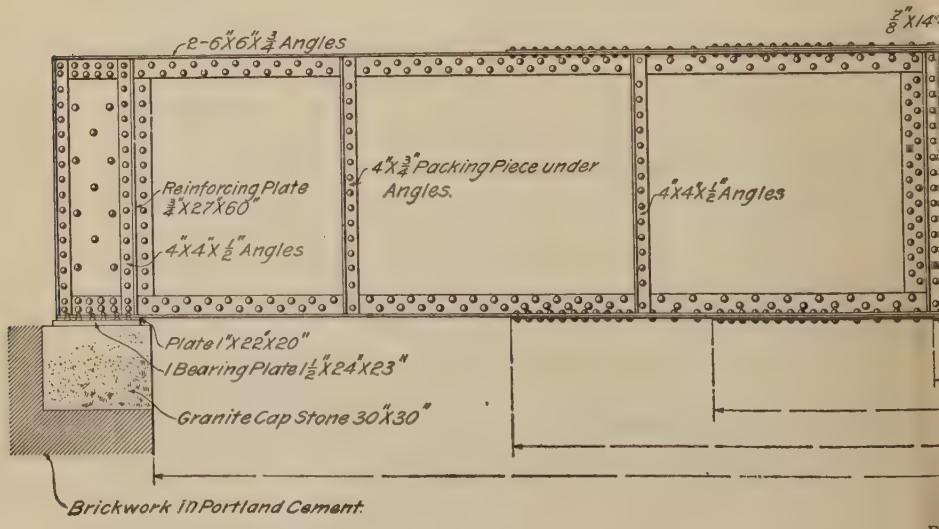
adding a foot at each end gives us 45 feet 8 inches.

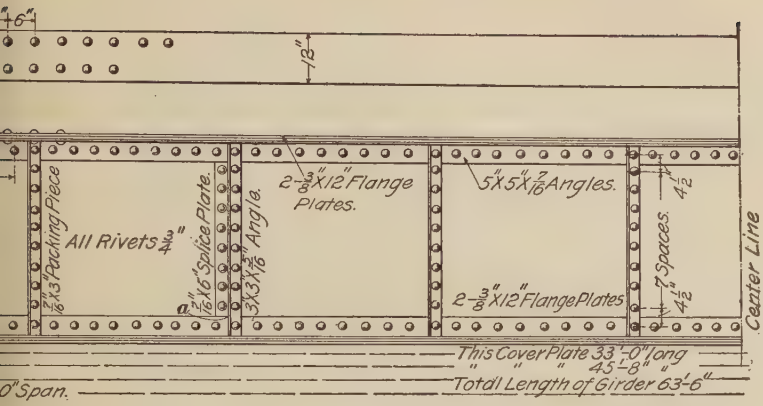
Consider now the size of the four stiffeners at the end of the girder. The reaction at the end of the girder is 55,650 pounds, and the allowable compressive strength of the material in the girder will be taken at 13,000 pounds. Then the sectional area required in the four angles composing the stiffeners on the plate girder over the abutments is $55{,}650 \div 13{,}000 = 4.28$ square inches. Since it would not be advisable to use smaller than a $4'' \times 4'' \times \frac{5}{16}''$ angle in this position, the sectional area of which is 2.4 square inches, it is evident that there will be ample strength in the four stiffeners. The other stiffeners may be made of $3'' \times 3'' \times \frac{5}{16}''$ angles, which is the smallest size that should be used for any girder requiring intermediate stiffeners.

The rivet spacing, etc. needs no explanation; it would be well, however, as a matter of practice, to calculate the

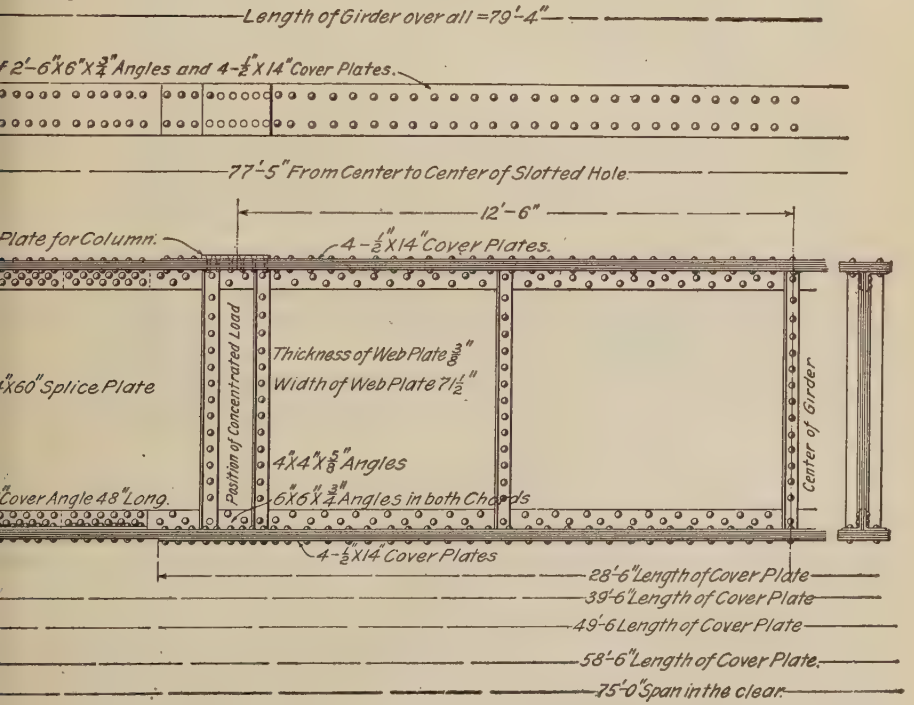

3/4" Rivets, Countersunk as shown.
4"X4"X5/16" Angles.
5"X5"X7/16" Angles.
8 Spaces
7 Spaces
8 Spaces
5"X5"X7/16" Angles
4"X4"X7/16" Angles
4"X4"X7/16" Angles
18" Anchor Bolts.
18"X11" Shoe Plate 3/4" Thick
Granite Bearing Block.
30" Brick Walls, Cement Mortar.


All Rivets spaced as sh
1 1/2"X4 1/2" Slotted Hole.
4"X3/4" Packing Piece.
Both Chords Comp
16"
2-4"X4"X1/2" Angles.
All Rivets 7/8" di


2-6"X6"X3/4" Angles
Reinforcing Plate
3/4"X27"X60"
4"X4"X1/2" Angles
4"X3/4" Packing Piece under
Angles.
4"X4"X1/2" Angles
Plate 1"X22"X20"
1 Bearing Plate 1 1/2"X24"X23"
Granite Cap Stone 30"X30"
Brickwork in Portland Cement.


6"
12"
All Rivets ¾"
⅜"x3" Packing Piece
⅜"x16" Splice Plate.
3"x3"x¼" Angle.
2-⅜"x12" Flange Plates.
5"x5"x⅞ Angles.
7 Spaces. 4½"
2-⅜"x12" Flange Plates
Center Line
This Cover Plate 33'-0" long
" " " 45'-8" "
Total Length of Girder 63'-6"
0" Span.

Drawing


Length of Girder over all = 79'-4"
f 2'-6"x6"x⅜" Angles and 4-½"x14" Cover Plates.
77'-5" From Center to Center of Slotted Hole.
12'-6"
Plate for Column.
4-½"x14" Cover Plates.
Thickness of Web Plate ⅜"
Width of Web Plate 7½"
"x60" Splice Plate
4"x4"x⅝" Angles
Cover Angle 48" Long.
Position of Concentrated Load
6"x6"x¾" Angles in both Chords
Center of Girder
4-½"x14" Cover Plates
28'-6" Length of Cover Plate
39'-6" Length of Cover Plate
49'-6 Length of Cover Plate
58'-6" Length of Cover Plate.
75'-0" Span in the clear.

number of rivets for the several parts and compare the results with the number actually used as shown by the detail drawing, Fig. 24. Undoubtedly it will be found that more rivets are used than are actually required, but it must be borne in mind that there are always practical considerations that influence more or less the design of structural work.

In Fig 24, it will be noticed that the web-plate is spliced at the point *a*. The shear at this point is equal to 55,650 pounds, the reaction at R_1, minus the load on the girder between R_1 and the point *a* under consideration, that is, 55,650 − 37,150 = 18,500 pounds. A sufficient number of rivets must be placed on the two sides of the joint to take care of this shear safely.

Fig. 25 shows the design of a heavily loaded girder with long span. This girder was designed to carry a uniformly distributed load of 2,400 pounds per lineal foot and two concentrated loads of 60,000 pounds, placed one on each side of the center of the girder and 12 feet 6 inches therefrom. The unit fiber stress allowable in calculating the flange section was taken at 15,000 pounds.

The splices on this girder should be particularly noted. In this case, it was necessary to splice the flange angles; when this is done, care must be taken to weaken the sectional area of the angle as little as possible by the punching of the rivet holes and a sufficient number of rivets must also be placed each side of the joint, so that the resistance of the rivet section may equal that of the net section of the flange angles. A careful study should be made of the various other details in this drawing, which represents excellent modern practice.

INDEX

NOTE.—In this Volume, each Section is complete in itself and has a number. This number is printed at the top of every page of the Section in the headline opposite the page number, and to distinguish the Section number from the page number, the Section number is preceded by a section mark (§). In order to find a reference, glance along the inside edges of the headlines until the desired Section number is found, then along the page numbers of that Section until the desired page is found. Thus, to find the reference "Accidental load, §9, p7," turn to the Section marked §9, then to page 7 of that Section.

A

Accidental load, §9, p7
Accidents caused by electric wiring, §6, p2
Alternating current, Calculations for, §6, p50
-current systems, §6, p29
Altitude of a parallelogram, §4, p35
of a pyramid, §4, p64
Ampere, Definition of, §5, p19
Angle, Acute, §4, p5
Inscribed, §4, p24
Obtuse, §4, p5
Right, §4, p3
Vertex of, §4, p2
Angles, Adjacent, §4, p3
and perpendiculars, §4, p2
Complementary, §4, p5
Supplementary, §4, p6
Vertical, §4, p7
Annunciator system, Return-call, §6, p81
Annunciators, §6, p79
Self-restoring, §6, p80
Apparatus, Arrangement of, §6, p89
Arc, Degree of, §4, p4
lamps, §5, p40
Minutes of, §4, p4
Seconds of, §4, p4
Arcs and angles, Measurement of, §4, p4
Area in circular mils, Calculation of, §6, p41
of any plane figure, §4, p51
of irregular figures, §4, p52
Arm, Lever, §7, p16
Armature, §5, p4
Artificial magnets, §5, p4
Automatic burner, §6, p88
cut-outs, §6, p16
Axis, Location of neutral, §8, p10
of symmetry, §4, p73

B

Balanced three-wire system, §6, p26
Batteries, Primary and secondary, §5, p43
Battery, Storage, §5, p42
Voltaic, §5, p11
Beam, Cantilever, §7, p42
construction, T-shaped, §7, p100
Continuous, §7, p42
Fixed, §7, p42
girders, §7, p74
Relation between span and depth of, §7, p74
Shear in a simple, §7, p48
Simple, §7, p42
Span of, §7, p42
with one strut, Stresses in a, §7, p88
Beams, §7, p42
Bending moments in simple, §7, p55
Calculating dimensions of steel, §7, p67
Deflection of, §7, p82
Design of, §7, p1
Design of trussed, §7, p91
Steel, §7, p66
Stiffness of, §7, p82
Stone, §7, p79
Strength of, §7, p61
Stresses in, §7, p47
Trussed, §7, p88
with two struts, Stresses in, §7, p90
Bearing value of rivets, §9, p48
values of rivets, Table of, §9, p52
Bell wiring, §6, p74
-wiring circuits, §6, p80
Bending moment, §7, p52
moments, General formulas for, §7, p60
moments in simple beams, §7, p55
stresses, §7, p52
Building materials, Strength of, §7, p35
materials, Weight of, §7, p25

Built-up section, Neutral axis of, §8, p12
Burglar alarms, §6, p85
Burners, Pendant, ratchet, and automatic, §6, p88
Buttons, Push, §6, p76
Buzzers, §6, p75

C

Camber rods, §7, p88
Cartridge-type fuse, §6, p17
Cast-iron columns, §8, p4
-iron columns, Design of, §8, p7
Center of gravity, §7, p24
of moments, §7, p16
Centers of distribution, §6, p21
Chemical action in a voltaic cell, §5, p10
Chord of circle, §4, p23
Circle, §4, pp4, 39
Chord of, §4, p23
Geometrical principles of, §4, p23
Circles, Concentric, §4, p26
Sectors and segments of, §4, pp23, 47
Circuit-breakers, §6, p19
External, §5, p15
Grounded, §5, p15
Internal, §5, p15
Short, §5, p15
Circuits, Bell-wiring, §6, p80
Circular-loom tube, §6, p60
mil, §6, p7
Circumference, §4, p4
Circumscribed polygons, §4, p23
Clocks, Electric, §6, p87
Watchman's, §6, p87
Column design, Examples of good, §8, p41
formulas for practical use, §8, p28
splices and connections, §8, p37
Columns and connections, Detailing of structural, §8, p35
Cast-iron, §8, p4
Design of cast-iron, §8, p7
Forms of steel, §8, p29
Formula for cast-iron, §8, p5
Formula for wooden, §8, p2
Formulas or steel, §8, p21
Long, §8, p2
Reinforced-concrete, §8, p46
Sections of, §8, p33
Securing ends of, §8, p4
Short, §8, p1
Steel, §8, p9
Types of, §8, p19
Wooden, §8, p1
Compass, Magnetic, §5, p4
Composition of forces, §7, p4
Compression members, §9, p85
Concealed knob-and-tube work, §6, p58
Concealed wiring, §6, p58
Concentric circles, §4, p26
Condensers, §5, p43
Conductance, §5, p14
and resistance, §5, p13
Conductor, Carrying capacity of, §6, p9
Resistance of, §6, p10
Conductors, §6, p7
and insulators, §5, p6
Conduit, Flexible metallic, §6, p69
Iron-armored, §6, p60
wiring, §6, p64
Conduits, Drawing wires into, §6, p69
Cone, Definition of, §4, p63
Frustum of a, §4, p65
Connection angles, §7, p79
Constant-current lamps, §5, p40
-current system, §6, p19
potential, §5, p39
-potential system, §6, p20
Conventional signs for rivets, §8, p45
signs for rolled shapes, §8, p45
Conversion tables, §4, p33
Cube, Definition of, §4, p59
Cubical contents, §4, p59
Current, Electric, §5, p18
for lamps, §6, p32
for motors, §6, p34
Sources of, §6, p77
Cut-out cabinets, §6, p23
-outs, Location of, §6, p19
-outs, Rating of, §6, p19
Cylinder, Definition of, §4, p59

D

Dead load, §7, p25
Deflection of beams, §7, p82
Design of a composite pin-connected roof truss, §9, p71
of a structural-steel roof truss, §9, p77
of beams, §7, p1
of cast-iron columns, §8, p7
of plate girders, Principles of, §10, p1
of trussed beams, §7, p91
Designing the members of a truss, §9, p65
Deterioration of materials, §7, p36
Diagonal of a quadrilateral, §4, p35
Diagram for a combination of concentrated loads with a uniformly distributed load, §10, p27
for a jib crane, §9, p18
for a small roof truss, §9, p17
for simple frames, §9, p12
for vertical loads, §9, p40
Diagrams, Lettering the, §9, p9
Diameter, §4, p4
Distance used in calculations, §6, p48

Distribution circuits, §6, p23
Door openers, §6, p85
Drawings, Reading, §4, p31
Dynamo, Description of a, §5, p12

E

Edison fuse plug, §6, p17
Efficiency of lamps, §5, p38
Elastic limit, §7, p34
Elasticity, Modulus of, §7, p34
Electric arc, §5, p40
bells, §6, p74
circuits and connections, §5, p15
clocks, §6, p87
current, §5, p18
gas lighting, §6, p88
horsepower, §5, p35
light and power wiring, §6, p1
motors, §5, p41
potential, §5, p7
power, §5, p32
resistance, §5, p13
time clocks, §6, p87
transformer, §5, p44
wiring, Accidents caused by, §6, p2
Electrical code, National, §6, p3
fires, Examples of, §6, p5
Electricity, §5, p1
and magnetism, Practical application of, §5, p36
Electrodynamics, §5, p6
Electrolyte, §5, p10
Electromagnets, §5, p6
Electromotive force, §5, p8
force, Methods of producing, §5, p9
Electrostatics, §5, p2
Elements of electricity and magnetism, §5, p1
Ellipse, Definition of, §4, p50
Elongation, Ultimate, §7, p35
Enclosed-arc lamps, §5, p41
Equiangular polygons, §4, p10
Equilateral polygons, §4, p10
Equilibrium, Meaning of, §7, p1
of moments, §7, p19
Experimental law for beams, §7, p54

F

Factor of safety, §7, p35
Factors of safety, §8, p26; §9, p84
Failure, Methods of, §9, p47
Feeder-and-main system, §6, p21
Field, Magnetic, §5, p5
Figure, Plane, §4, p9
Fink truss, Determination of stresses in the, §9, p39
Fire-alarm gongs, Wiring for, §6, p83
Fittings, National code standard, §6, p4
Flange plates, Graphic method of determining the length of, §10, p18
plates, Length of, §10, p15
stresses, §10, p11
Flanges of a riveted girder, §10, p11
Proportioning the, §10, p13
Flexible lamp cord, §6, p12
metallic conduit, §6, p69
Flitch-plate girders, §7, p85
Foot-pound, §7, p17
Force, Definition of, §7, p1
Effect of, on body, §7, p2
Representation of, §7, p2
Forces, Composition of, §7, p4
Moments of, §7, p15
Parallelogram of, §7, p4
Polygon of, §7, p8
Resolution of, §7, p11
Triangle of, §7, p6
Frame and stress diagrams, §9, p8
Frustum of a pyramid or cone, §4, p65
Fuse blocks, §6, p18
Cartridge-type, §6, p17
Ordinary link, §6, p16
plug, Edison, §6, p17
protection, §6, p51
Fuses, §6, p16

G

Geometry, §4, p2
Girder, Depth of, §10, p4
Girders with concentrated loads, Application of the graphic method to, §10, p21
Graphic diagram for several concentrated loads, §10, p24
statics, §9, p8
Gravity, Center of, §7, p24
Gusset plate, §9, p80
Gyration, Radius of, §7, p63

H

Hooke's law, §7, p34
Horsepower, Electric, §5, p35
Howe truss with an 80-foot span, §9, p28
Hypotenuse, Definition of, §4, p11

I

Incandescent lamps, §5, p36
Inscribed polygons, §4, p28
Inspection of castings, §8, p8
Installation of wires, §6, p53
Installing bell wires, §6, p84
Insulation for interior wiring, §6, p9
Slow-burning weather-proof, §6, p9
Insulators, §5, p7
Intensity of stress, §7, p33

Intersection, Point of, §4, p7
Irregular figures, Areas of, §4, p52

J

Jib crane, Diagram for a, §9, p18
Joints in wiring, §6, p11

K

Kilowatt, Abbreviation of, §5, p34
Definition of, §5, p34
-hour, §5, p34
Knife switches, §6, p12

L

Lamp bases, §5, p36
connections, §5, p39
cord, Flexible, §6, p12
Incandescent, §5, p36
ratings and efficiency, §5, p38
Lamps, Arc, §5, p40
Constant-current, §5, p40
Constant-potential enclosed-arc, §5, p41
Current for, §6, p32
Enclosed-arc, §5, p41
Open-arc, §5, p40
Lateral bracing, §9, p84
Lever arm, §7, p16
Definition of, §7, p21
Light and power wiring, Materials for, §6, p7
Line, Broken, §4, p1
Curved, §4, p1
Horizontal, §4, p7
Straight, §4, p1
Vertical, §4, p7
Lines, Parallel, §4, p7
Live and dead loads, §9, p1
load, §7, p29
loads for building construction, §7, p29
Loads carried by structures, §7, p25
Location of cut-outs, §6, p19
Long columns, §8, p2

M

Magnet, Definition of, §5, p3
Magnetic compass, §5, p4
field, §5, p5
Magnetism, §5, p3
Magnets, Artificial, §5, p4
Permanent, §5, p4
Material, Deterioration of, §7, p36
Materials for light and power wiring, §6, p7
Maximum shear, §7, p50
Mechanics, Elements of, §7, p1
Megohm, Definition of, §5, p15
Members in trusses subjected to transverse stresses, §9, p85
Members in trusses subjected to both transverse and direct stresses, §9, p86
Mensuration, §4, p2
of plane surfaces, §4, p34
of solids, §4, p59
Merriman's formula, §8, p24
Microhm, Definition of, §5, p15
Modulus of elasticity, §7, p34
Section, §7, p62
Molding work, §6, p71
Moment, Bending, §7, p52
of inertia, §7, pp61, 67; §8, p14
of resistance, §7, p54
Resisting, §7, p53
Moments, Center of, §7, p16
due to loads, Effect of, §7, p55
Equilibrium of, §7, p19
of forces, §7, p15
of inertia, Rules and formulas for, §8, p15
Positive and negative, §7, p17
Resultant, §7, p18
Motors, Current for, §6, p34
Electric, §5, p41
Multiple connections, §5, p16
-series connections, §5, p18

N

National electrical code, §6, p3
Neutral axis, §7, p54
axis, Locating, by means of principle of moments, §8, p11
axis, Location of, §8, p10
Non-conductors, §5, p7

O

Ohm, Meaning of, §5, p14
Ohm's law, §5, p20
law, Application of, §5, p23
Open-arc lamps, §5, p40
Outlet and junction boxes, §6, p64
Outlets, §6, p59
Arrangement of, §6, p67

P

Panel boards, §6, p23
Parallel connections, §5, p16
-series connections, §5, p18
system, §6, p88
Parallelogram, §4, p35
of forces, §7, p4
Parallelopiped, Definition of, §4, p59
Pendant burner, §6, p88
Perimeter of polygon, §4, p9
Permanent magnets, §5, p4
Pins and eyes, §9, p86
Resisting moments of, §9, p57
subjected to bending stresses, §9, p56

Plane figure, §4, pp2, 9
figures, Center of gravity of, §7, p24
surfaces and areas, §4, pp1, 47
surfaces, Mensuration of, §4, p34
Plate girders, Construction and design of, §10, p1
girders, Flitch-, §7, p85
Point of application, Changing the, §7, p3
of contact, §4, p26
of tangency, §4, p26
Poles of magnetic compass, §5, p4
Polygon, Definition of, §4, p9
of forces, §7, p8
Perimeter of, §4, p9
Regular, §4, p10
Polygons, §4, p38
Equiangular, §4, p10
Equilateral, §4, p10
Inscribed and circumscribed, §4, p28
Similar, §4, p19
Polyphase systems, §6, p30
Potential and electromotive force, §5, p7
Constant, §5, p39
Drop of, §5, p28
Electric, §5, p7
Power, Electric, §5, p32
Practical problems, §10, p37
Prism, Definition of, §4, p59
Prismoid, Definition of, §4, p69
Prismoidal formula, §4, pp68, 70
Proportioning the flanges, §10, p13
Pyramid, Definition of, §4, p63
Frustum of a, §4, p65

Q

Quadrilaterals, §4, p35

R

Radian, Meaning of, §4, p40
Radius, Definition of, §4, p4
of gyration, §7, p63; §8, p19
Rafter member, §9, p71
Ratchet burner, §6, p88
Rating of cut-outs, §6, p19
Reaction, §7, p42
Reading drawings, §4, p31
Rectangle, Definition of, §4, p35
Regular polygon, §4, p10
Reinforced concrete, §7, p95
concrete, Calculating the strength of, §7, p95
-concrete columns, §8, p46
Relation between span and depth of beam, §7, p74
Resistance and conductance, §5, p13
Electric, §5, p13
of a conductor, §6, p10
Resistance per 1,000 feet, Calculations based on, §6, p39
Resisting moment, §7, p53
Resolution of forces, §7, p11
Resultant moment of several stresses, §9, p60
moments, §7, p18
of several forces, §7, p6
Return-call annunciator system, §6, p81
Rhomboid, Definition of, §4, p35
Rhombus, Definition of, §4, p35
Right triangle, Principle of the, §4, p14
Rise, run, and pitch, §4, p13
Rivet signs, §8, p44
spacing, §10, p31
spacing in flange plates, §10, p36
Rivets and pins, Strength of, §9, p47
and rivet spacing, §8, p35
Bearing value of, §9, p48
connecting flange angles with the web, §10, p32
Conventional signs for, §8, p45
in the end angles or stiffeners over the abutment, §10, p31
in the stiffeners between the abutments, §10, p32
spaced according to stress produced by bending moment, §10, p34
Table of bearing values of, §9, p52
Roof truss, Design of, §9, p84
truss, Design of a composite pin-connected, §9, p71
truss, Design of a structural-steel, §9, p77
truss, Diagram for a small, §9, p17
truss with a 40-foot span, §9, p19
trusses, Loads on, §9, p1

S

Safety, Factor of, §7, p35
Section modulus, §7, p62
Sections of plate girders, Usual forms of, §10, p3
Sector of circle, §4, p23
Sectors and segments of circles, §4, p47
Segment of the circle, §4, p23
Segments of circles, Sectors and, §4, p27
Separators, §9, p83
for I beams, §7, p74
Series-connections, §5, p16
system, §6, p90
Shapes, Conventional signs for rolled, §8, p45
Shear, Meaning of, §7, p48
in a simple beam, §7, p48
Maximum, §7, p50
Positive and negative, §7, p50
Shearing stresses in web-plate, §10, p4

Short columns, §8, p1
Shunt, Meaning of, §5, p16
Similar figures, §4, p73
  polygons, §4, p19
  triangles, §4, p19
Single-phase system, §6, p29
Slant height of a pyramid, §4, p64
Snow and wind loads, §9, p3
Solid, Definition of, §4, p59
Solids, Mensuration of, §4, p59
Span of a beam, §7, p42
Speaking-tube system, Wiring for, §6, p82
Specifications for concealed electric-light wiring, §6, p61
Sphere, Definition of, §4, p71
Square, Definition of, §4, p35
Stamped-steel switch box, §6, p15
Standard symbols for wiring plans, §6, p92
Static charges, §5, p2
Statics, Meaning of, §7, p1
Steel beams, §7, p66
  columns, §8, p9
  columns, Forms of, §8, p29
  columns, Formulas for, §8, p21
  shapes, Tables of, §8, p49
  struts, §9, p74
Stiffeners, §10, p2
  Distribution of, §10, p7
  Practical rule for spacing, §10, p9
Stiffness of beams, §7, p82
Stone beams, §7, p79
Storage batteries, §5, p42
Strain, Unit, §7, p33
Strength of beams, §7, p61
  of building materials, §7, p35
Stress, Definition of, §7, p31
  Compressive, §7, p31
  Intensity of, §7, p32
  Shearing, §7, p31
  Tensile, §7, p31
  Transverse, §7, p32
  Unit, §7, p32
Stresses and strains, §7, p31
  Bending, §7, p52
  in a beam with one strut, §7, p88
  in beams, §7, p47
  in beams with two struts, §7, p90
Structural columns and connections, Detailing of, §8, p35
Structures, Loads carried by, §7, p25
Struts, Steel, §9, p74
Supporting and insulating fittings, §6, p53
Surface, Convex, §4, p59
  Entire, §4, p59
  Flat, §4, p1
  Lateral, §4, p59
  Plane, §4, p1
Surfaces and areas, Plane, §4, p1
Switch box, Stamped-steel, §6, p15
  boxes, §6, p15
  Electric, §6, p12
Switches and cut-outs, §6, p12
  Knife, §6, p12
  Location of, §6, p15
  Snap, §6, p14
Symmetrical and similar figures, §4, p73
Systems of distribution, §6, p19

T

Table, Analogies between the flow of water and electricity, §5, p27
  of polygons, §4, p55
  Properties and principal dimensions of standard T rails, §8, p61
  Properties of standard angles having equal legs, §8, p54
  Properties of standard angles having unequal legs, §8, p56
  Properties of standard channels, §8, p52
  Properties of standard I beams, §8, p50
  Properties of T bars, §8, p60
  Properties of Z bars, §8, p58
  Radii of gyration for two angles, having equal legs, placed back to back, §8, p62
  Radii of gyration for two angles, having unequal legs, placed back to back, §8, pp63, 65
  Square of the least radius of gyration, §8, p6
Tangency, Point of, §4, p26
Tangent to a circle, §4, p25
Tension members, §9, p85
  rods, §9, p73
Tests, Wiring, §6, p72
  Underwriters', §6, p73
Three-phase system, §6, p31
  -wire system, §6, p24
Transformers, Electric, §5, p44
Transverse stresses, Members in trusses subjected to, §9, p85
Trapezium, Definition of, §4, p35
Trapezoid, Definition of, §4, p35
Triangle, Altitude of, §4, p11
  Base of, §4, p11
  Definition of, §4, p37
  Equilateral, §4, p11
  Isosceles, §4, p11
  Oblique, §4, p11
  of forces, §7, p6
  Principle of the right, §4, p14
  Right, §4, p11
  Scalene, §4, p11
Triangles, Similar, §4, p19
Truss design, §9, p65

Truss, Designing the members of a, §9, p65
for a church roof, §9, p22
Trussed beams, §7, p88
Trusses with one end free, §9, p35
Two-phase system, §6, p30
-wire systems, §6, p21
Types of columns, §8, p19

U

Ultimate elongation, §7, p35
strength, §7, p35
Underwriters' tests, §6, p73
Unit strain, §7, p33
stress, §7, p32

V

Vector quantities, §7, p2
Volt, Meaning of, §5, p8
Voltaic battery, §5, p11
cell, §5, p9
cell, Chemical action in a, §5, p10
couple, §5, p10
elements, §5, p10
Volume, §4, p59

W

Watchman's clocks, §6, p87
Watt, Meaning of, §5, p33
Web-plate, Buckling of, §10, p7
-plate, Thickness of, §10, p5
-plates and stiffeners, §10, p4
Wedge, Definition of, §4, p67
Weight arm, §7, p21
of building materials, §7, pp25, 26
Wire for 1 volt drop, Size of, §6, p41
gauges, §6, p9
Rubber-covered, §6, p9
tables, §6, p8
Wires, Determining the size of, §6, p32
in parallel, Using several, §6, p50
Installation of, §6, p53
Installing bell, §6, p84
into conduits, Drawing, §6, p69
Size of, §6, p36
Wiring, Concealed, §6, p58
Conduit, §6, p64
for fire-alarm gongs, §6, p83
for flats, §6, p83
for minor electric appliances, §6, p74
for speaking-tube system, §6, p82
in damp places, §6, p56
in dry places, §6, p53
plans, Standard symbols for, §6, p92
Wooden columns, §8, p1
columns, Formula for, §8, p2